PARAMETRIC OPTIMIZATION OF REINFORCED CONCRETE BRIDGE DECK VIA LEONHARDT METHOD USING MATLAB

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PLAGIARISM

This work **PARAMETRIC OPTIMIZATION OF REINFORCED CONCRETE BRIDGE DECK VIA LEONHARDT METHOD USING MATLAB** by EGHOMWANRE, Emmanuel, Osahon, with Number ENG2006225, of the Department of Structural Engineering, Faculty of Engineering, University of Benin, Benin City, Edo State, Nigeria, has PASSED the PLAGIARISM TEST.

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DEDICATION

I dedicate this project work to God Almighty and my loving parents Mr. and Mrs. Eghomwanre.

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I am particularly grateful to my supervisor Engr. Musa Aaron Esekhagbona B.Eng (Hons) Benin, M.Eng (Benin) for his close and thorough supervision throughout the period of the project.

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ABSTRACT

This study aims to optimize reinforced concrete (RC) bridge decks using the Leonhardt and Makowski method. A MATLAB based GUI for live load distribution analysis was developed, allowing users to observe the impact of varying design parameters like span, slab thickness, and deck width. The analysis will follow BS5400 provisions, focusing on HA and HB loading combination.

The methodology involved creating a MATLAB program that integrates the Leonhardt method into an interactive GUI and MATLAB scripts to run batch inputs. This tool validates user inputs, apply load cases (UDL, KEL, HB vehicle loads), compute section properties, and generate both tabular and graphical outputs (e.g., bending moment diagrams). To verify accuracy, results from manual calculations for bridge of span 25m, deck width of 11m and slab thickness 230mm was compared.

The MATLAB tool is showed strong agreement with both manual calculations with 0.002% difference while the percentage difference compared to the STAAD.Pro analysis was 2.97% when computing the maximum longitudinal bending moments. The parametric study showed that the maximum moments appeared on the first support. The tool created will be able to provide engineers and students a flexible environment to explore design alternatives and understand how the inputs influence bridge behavior. Although the MATLAB GUI developed in this study performed excellently when compared to manual calculation, more comparative testing with other load distribution methods and with finite element based tool to further test the accuracy of the study.

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ACRONYMS

AASHTO	- American Association of State Highway and Transportation Officials
ACI	- American Concrete Institute
AISC	- American Institute of Steel Construction
ANOVA	- Analysis of Variance
BS	- British Standards
BM	- Bending Moment
FE	- Finite Element
FEM	- Finite Element Method
FDM	- Finite Difference Method
GUI	- Graphical User Interface
LRFD	- Load and Resistance Factor Design
MATLAB	- Matrix Laboratory
RC	- Reinforced Concrete
RSM	- Response Surface Method
SF	- Shear Force

CHAPTER ONE

INTRODUCTION

1.1 Background of the Study

Bridges are structures that spans over a physical obstacle and is designed to carry load from road traffic or other moving loads in order to allow passage through the obstacle or other structure. The first bridge, a gift to humanity and was likely a tree that fell across a small river and inspired early prehistoric builders to the possibility to overcome natural obstacles (Pipinato, 2022).

In this modern era, bridges are vital components of transportation infrastructure. They serve critical functions such as providing safe pedestrian crossings over busy highways as seen in the University of Benin pedestrian bridge and even improving traffic flow and connectivity between urban districts, as exemplified by the cable-stayed Lekki-Ikoyi Link Bridge in Lagos, Nigeria. This particular bridge was constructed to alleviate traffic congestion between the districts of Lekki and Ikoyi. The importance of bridges in facilitating transportation points to the need for efficient bridge design, which is essential for economic development, public safety, and reliable transportation networks.

Bridge structures are defined by measurable parameters, such as deck configuration and material properties, which determine their behavior and performance under various loading and environmental conditions (Lin and Yoda, 2017). Bridge structures are primarily composed of superstructure, bearing and substructure, Figure 1.1 and Figure 1.2 shows the various parts of a bridge at cross section and longitudinal section.

The deck is the part of the bridge superstructure that spans the bridge and directly receives the live or vehicular load (Naik and Gourav, 2023). The deck forms the surface over which traffic load travels with a primary function of distributing vehicular load to the girders, which are usually the primary load carrying members of the bridge superstructure. Bridge deck methods of load distribution are generally classified into classical and numerical approaches (Siekierski, 2019). Classical methods such as the Leonhardt and Makowski method, Guyon–Massonnet method, and Morrice and Little method are based on simplified assumptions and uses analytical formulas to provide insights into structural behavior of the decks. In contrast, numerical methods such as the finite element method and finite difference method rely on computational models to simulate more complex geometries and loading conditions. The Leonhardt and Makowski method offers a practical approach for evaluating the load distribution of the deck.

The design of reinforced concrete (RC) bridge decks requires careful consideration of several factors, including span length, deck thickness, reinforcement layout, and load distribution patterns. Due to the number of interdependent variables involved, many engineers find bridge deck design to be complex and time-consuming, especially when aiming for an optimal and efficient solution. In such optimization problems, where the optimal design varies by changing parameters, traditional non-parametric or deterministic methods become inefficient. These methods typically involve evaluating numerous combinations of individual parameters, resulting in high computational cost and considerable manual effort (Ravichandran, 2015). Parametric optimization has emerged as a powerful method to enhance the design process by allowing engineers to explore various design alternatives efficiently. In response to the increasing demand for structural efficiency, safety, and cost-effectiveness, parametric optimization has emerged as a valuable technique for systematically analyzing the impact of variable on structural performance.

In these present times with the availability of wide range of computer programs for structural analysis and a computer with analysis software on every desk allow bridge slab analysis to be performed with high accuracy and efficiency (Hołowaty, 2015). MATLAB is a high-level programming and numerical computing environment widely used for matrix operations, data visualization, algorithm development, and building user interfaces. MATLAB has become a widely used platform the development of custom engineering applications. In this study, MATLAB serves as the primary platform for implementing the Leonhardt and Makowsi method and automating parametric optimization processes in the design of reinforced concrete bridge decks.

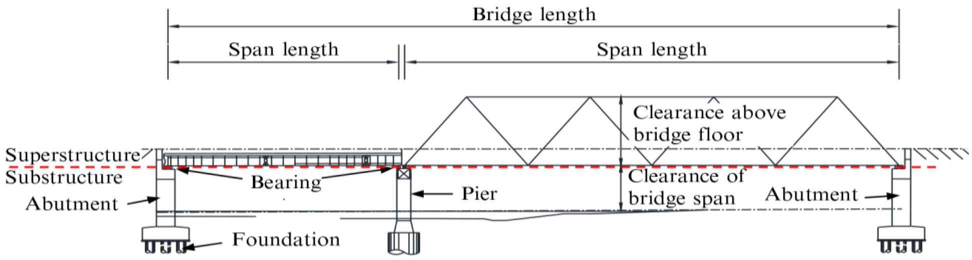


Figure 1.1: Longitudinal section of a bridge highlighting the major components of a bridge (Lin and Yoda, 2017)

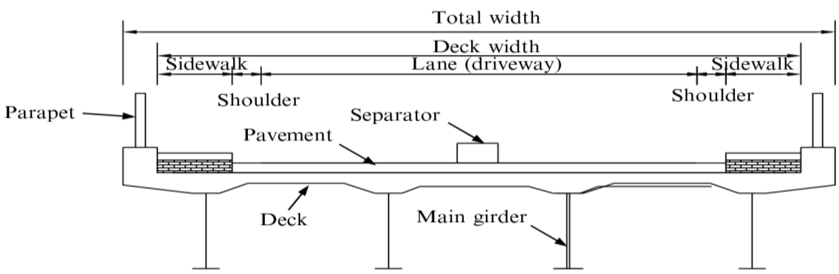


Figure 1.2: Cross sectional view of a bridge highlighting the major components of a bridge deck (Lin and Yoda, 2017).

1.2 Statement of the Problem

The demand for safer, more efficient, and economically sustainable bridge infrastructure has prompted a shift from classical design methods toward more advanced, computationally enhanced approaches. The Leonhardt and Makowski method has long been recognized as a reliable classical approach for analyzing bridge decks due to its precision. However, this method is rarely integrated into modern software tools. Most structural analysis and optimization software either overlook classical methods like Leonhardt or require advanced programming knowledge, making inaccessible to practicing engineers. Additionally, the current bridge design processes often involve repetitive, trial-and-error calculations to assess the effects of varying parameters such as slab thickness, span length, and reinforcement ratio. This makes the optimization process time consuming and inefficient, particularly when engineers need to evaluate multiple design alternatives manually.

Given these challenges there is a need for an accessible optimization framework that leverages the precision of the Leonhardt and Makowski method while automating the analysis process for practical engineering use. This project addresses that need by developing a MATLAB-based tool that enables parametric optimization of RC bridge decks. By automating load distribution calculations and allowing real time variation of key design parameters, the tool makes bridge design engineers to efficiently explore optimal design options.

1.3 Aim and Objectives

The main aim of this project is to develop a user-friendly MATLAB application that performs parametric optimization of reinforced concrete bridge decks using the Leonhardt and Makowski method.

The specified objectives are to:

1. To review the theoretical basis of the Leonhardt and Makowski Method for bridge deck load distribution.

2. To identify key design parameters for optimization.

3. To implement a MATLAB-based GUI for automated analysis that can be used for parametric input.

4. To validate the tool through comparative testing with manual design calculations and analysis from STAAD.Pro.

5. To study influence of the design parameters using the tool

1.4 Scope of the Study

This study focuses on the parametric optimization of solid slab RC bridge decks using the Leonhardt and Makowski Method as the primary analytical approach, implemented through a custom MATLAB-based graphical user interface (GUI). The project aims to integrate structural analysis theory with computational tools and design automation. The scope encompasses theoretical research, analytical modeling, software development, and validation of the proposed system.

The activities involved in the project include:

1. An extensive literature review of bridge deck systems and (traditional) design methods. Additionally, an exploration of parametric design principles and their application in structural optimization will be carried out.

2. Development of a computational model with MATLAB to implement the Leonhardt and Makowski method.

3. Development of MATLAB GUI will be developed to enhance user accessibility and interaction. Parametric optimization routine will be integrated into the system, allowing the tool to explore multiple design alternatives automatically.

4. To validate the model, selected results will be compared against manual design solutions and standard analysis software like STAAD Pro

5. Documenting the research process, tool development, methodology, and findings. The complete report will include chapters 4 and 5.

MATLAB is the primary environment for computation, modeling, and GUI development. MATLAB’s matrix capabilities make it ideal for structural analysis and iterative optimization. Microsoft Word and Microsoft Excel would be used for documentation, tabulation of results, and basic data presentation during testing phases. The British codes and is the relevant design code, used to provide standard loading requirement, promote uniformity and best practices.

1.5 Justification of the Study

The usefulness of this research lies in its development of a user-friendly, MATLAB based design tool that allows engineers to vary key design parameters such as slab thickness, deck width, and span length, while automatically analyzing and optimizing the structural performance of RC bridge decks. By automating this process, the tool not only saves time but also promotes better-informed decisions, especially during early-stage design. This project offers a practical benefit by empowering structural engineers with a flexible platform for exploring a broader range of design alternatives without compromising safety or compliance with codes. The system enhances design efficiency and supports cost-effective material use thereby making it highly applicable in developing regions where material economy and reliability are critical.

By automating the Leonhardt and Makowski method with MATLAB this project contributes to the body of knowledge by bridging the gap between structural theory and modern parametric modeling. It introduces an applied framework that integrates parametric design; optimization of structural elements, GUI based engineering tools and MATLAB in structural engineering. It also provides hands on learning tool for students and a practical design aid for engineers, contributing to safer and more efficient bridge design practices. In an era where computational tools are reshaping engineering workflows, this project aligns with the global shift toward digital transformation in engineering design.

CHAPTER TWO

LITERATURE REVIEW

2.1 Literature Review

This chapter reviews relevant to the parametric optimization of reinforced concrete (RC) bridge decks. It begins by providing a general overview of bridges, including their types and structural classifications, with a specific focus on bridge decks as a critical component. The chapter explores the different types of loading that act on bridges, as well as the various classical and numerical methods developed over time for bridge deck analysis.

Special attention is placed on the Leonhardt and Makowski method, which forms the foundation of this study. Furthermore, the chapter reviews the concept of parametric optimization in structural engineering and the use of computational tools, particularly MATLAB and its graphical user interface (GUI) capabilities, is examined in the context of automating structural analysis and optimization processes.

2.2 Structural Types of Bridges

Bridges are constructed to cross physical obstacles such as rivers, valleys, roads, or railways, ensuring a secure and uninterrupted passage. Bridges can be classified in several ways, including by structural form, construction material, span length, intended use, or geometric design. Among these classifications the classification based on structural form remains the most common. Bridges based on structural form include beam bridges, truss bridges, arch bridges, cable stayed bridges, and suspension bridges. This emphasis on structural form is due to the significant impact structural form has on the bridge's design process, construction, and long-term maintenance needs (Lin and

Yoda, 2017). For the purposes of this literature review, emphasis is placed on classification by structural form, as it has a direct impact on load distribution.

A Beam or Girder bridge is a structure that depends on girders to uphold the bridge deck and distribute loads to the supports. This type of bridge is the most frequently constructed and used globally. The widespread use of this structural form is largely due to its simplicity and suitability for short to medium span applications. At its core, the bridge design can be described as a basic beam across a gap, such as a river or stream. In modern bridge construction, steel plate girders and box girders are the most commonly employed types of girders. In bridge engineering, the term girder is frequently used interchangeably with beam (Mohan and Vijaykumar, 2016).

Arch bridges are designed in the form of an upward convex curved arch to support vertical loads. A basic arch bridge functions by distributing its weight and other loads partially into a horizontal thrust that is countered by robust abutments on either side. Typically, the arch must withstand compressive forces along its axis, as well as bending moments and shear forces. Arch bridges can be constructed as a single span with two abutments or as a series of continuous arches. Various types of arch structures exist, and they represent some of the oldest forms of modern bridges, continuing to be applied in numerous fields today (Lin and Yoda, 2017).

Truss bridge at its core utilizes a truss framework, where all structural members are interconnected through hinges. These members are connected in a way to create multiple triangles, which are recognized as the most stable form of two dimensional structures construction. The truss structure transmits the shear forces, stress, and bending moments, and converts these forces into simple axial tension and compression forces. Due to this unique design, truss bridges can accommodate greater load compared to other structures of the same length, thereby achieving enhanced strength. Furthermore, when comparing bridges of equivalent lengths, truss bridges tend to utilize fewer materials (Niu, 2022).

A cable-stayed bridge consists primarily of three components: the main beam, the tower, and the cables. In response to load, the main beam experiences bending, the tower is subjected to compression, and the cables are under tension. The cables serve as a crucial element of the cable-stayed bridge, as they are the primary load-bearing components. The quantity and configuration of the cables are vital for the overall stiffness and design of the structure. Due to their favorable mechanical properties and spanning capabilities, cable-stayed bridges are extensively employed in bridge construction projects around the world (Liu and Yi, 2018).

A suspension bridge consists of a continuous girder that is upheld by primary suspension cables and vertical suspenders. These cables span over the main towers and are firmly anchored at both ends of the bridge. In this structural form, the dominant forces are tensile forces acting on the suspension cables and vertical hangers, whereas the towers endure compressive forces. The bridge deck here is usually constructed as a truss or box girder and is linked to the suspension cables via hangers, which also function under tension. The loads from the deck are transfered through the hangers to the cables, subsequently to the towers, and finally to the ground. The overall design is efficient and strategically allocates loads to maintain structural stability (Aditya and Yadav, 2022).

2.3 Overview of the Reinforced Concrete Bridge Deck

The deck is the component of the bridge that spans the bridge and receives vehicular load directly (Naik & Gourav, 2023). The deck is the component of a bridge that forms the surface over which traffic load moves. The primary function of the deck is to distribute vehicular loads to the primary load carrying members on a bridge structure. In many cases, the deck and main girders act as a composite section to enhance the overall flexural strength and torsional stiffness of the bridge when properly reinforced (Keever and Fujimoto, 2000). The Common structural forms adopted in bridge deck design to achieve efficient distribution of bridge load includes beam decks, grid decks, beam and slab decks, slab decks and cellular decks (Hambly, 1991).

Beam decks functions as long and slender structural components where the cross section experiences bending and torsion with minimal deformation. This type of deck is commonly found in footbridges and long span bridges made from materials such as steel, reinforced concrete, or prestressed concrete. The primary load transfer occurs in the longitudinal direction, with minimal cross sectional deformation due to the concentric nature of the applied loads. Their structural behavior is relatively straightforward and is frequently modeled using fundamental beam theory, either as simply supported or continuous beam (Hambly, 1991).

Slab decks acts as flat plates that can distribute loads in various directions using both moment and torsional resistance. When loaded, slab decks bend in a dish-like shape, allowing nearby areas to help carry the load. Their stiffness can be the same in all directions (isotropic) or different along certain directions (orthotropic), depending on how they're built and reinforced. Figure 2.1 shows a typical cross-section of a slab deck bridge designed for short spans. This type of deck is typically used the construction of bridges with spans shorter than 15 meters, slab decks offers two dimensional load distribution and can be constructed in situ or combined with precast elements (Hambly, 1991).

Beam and slab decks combine longitudinal beams with a reinforced concrete slab, resulting in a lightweight system distinguished by high longitudinal stiffness. The load transfer mechanisms in these systems can be complex, with behavior ranging from orthotropic plate action to wave-like deflection, influenced by beam spacing and the torsional rigidity of the structure. This type of deck is appropriate for a various span lengths and accommodates both precast and prefabricated techniques (Hambly, 1991).

Cellular or box-girder decks are composed of enclosed thin slabs and webs that create cells, which offer significant torsional and transverse stiffness. These types of decks are particularly suitable for spans exceeding 30 m, especially in situations where temporary formwork is not feasible, and they are frequently constructed using cantilever or launching techniques. While load distribution is effective, it is not as uniform as in traditional slabs, since the top and bottom slabs can deflect independently (Hambly, 1991). Figure 2.2 presents a single cell box-girder deck, consisting of enclosed hollow cells formed by webs and slabs.

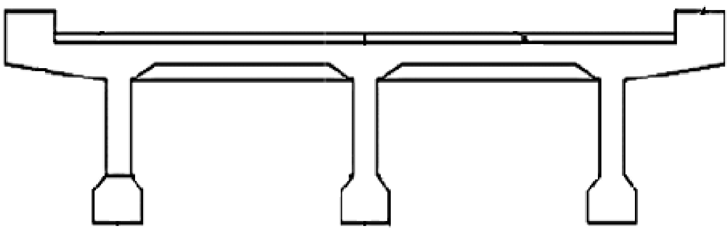


Figure 2.1: Cross-sectional view of a typical ordinary deck slab bridge (Upadhyaya and Sachin, 2016)

Figure 2.2: A single cell box-girder deck (Mone et al., 2022)

The most common materials used for bridge deck construction are concrete and steel. These materials can be combined to form a composite structure. Reinforced concrete decks are used for concrete bridges and as well as steel bridges. Reinforced concrete deck slabs are the most often constructed type of deck. Concrete decks are either constructed using cast-in-situ or precast methods and usually include mild steel as reinforcement in both longitudinal and transverse directions. (AISC, 2022)

2.5 Loading of Bridge Decks

The primary purpose of bridge load evaluation is to optimize the load carrying capacity of the bridge while ensuring of safety of people using the bridge (Alberta Transportation, 2016). There are numerous load types acting on bridges some include dead loads, live loads, construction loads, wind loads, friction forces and blast loading (AASHTO LRFD, 2014). However only dead loads and vehicular live load would be briefly mentioned here, other loads could be found in AASHTO and BS codes standards, specifically the AASHTO LFRD and BS5400 materials respectively.

The dead or permanent load represents the permanent forces acting on the structure and the bridge deck throughout its service life. These loads are divided into two categories: structural dead loads, which comprise the self-weight of significant structural components like the deck slab, girders, stiffeners, and floor beams; and superimposed dead loads, which include fixed additional elements such as earth fill, wearing surfaces, sidewalks, handrails, light poles, cables, and other embedded utilities (ACI, 2004).

The live loads or transient loads acting on the bridge decks are all other loads other than the permanent loads. They are the temporary forces that act on a bridge deck during its use. They include movable loads such as vehicles, pedestrians, bicycles, or construction equipment. One of the most prominent of live load is the vehicular load. The primary effects of vehicular loads are the vertical loads due to mass of traffic. The specification of bridge loading according to the BS codes is given in the BD37/01 and the BS5400 materials. The standard vehicular loads consists of the normal HA and abnormal HB loading. HA loading is a formula loading which is intended to represent normal actual vehicle loading. The HA loading consists of a uniformly distributed load plus a knife edge load the HA KEL load. HB loading is intended to represent an abnormally heavy vehicle. These load definitions form the basis for evaluating structural performance during the service life of the bridge (Clark, 1983). In this study, the effects of live loads as defined in BS 5400 will be embedded into the MATLAB computational model.

2.5 Methods of Bridge Deck Analysis

The aim of load distribution is to determine how traffic loads are shared among the main longitudinal components of the bridge, such as girders and beams, and how transverse elements like slabs or diaphragms contribute to this process (Ezi et al., 2018). Evaluating how loads spread across the width of the bridge helps in understanding force transfer to supporting structures. Accurate load distribution analysis is key to achieving both structural safety and design efficiency.

The methods for analyzing bridge deck load distribution can be broadly classified into Classical Methods and Numerical Methods (Siekierski, 2019; Mahadevan, 2005).

Classical Methods, often referred to as Simplified Methods, uses closed form equations and ideal assumptions to estimate load distribution. These methods are generally appropriate for regular bridge geometries and are extensively employed in preliminary design and manual calculations. Common examples include:

i. Guyon-Massonnet-Bares Orthotropic Plate Theory

ii. Leonhardt and Makowski's Method

iii. Morrice and Little's Orthotropic Plate Theory

iv. Courbon's Method

v. Equivalent Strip Method

vi. Westergard's Method

Numerical Methods, regarded as Rigorous Methods, employ computational techniques to simulate complex bridge behavior with enhanced accuracy. These methods are particularly beneficial for analyzing irregular geometries, non-uniform loading, and complex boundary conditions. Common examples include:

i. Finite Element Method (FEM)

ii. Finite Difference Method (FDM)

iii. Folded Plate Theory

iv. Grillage Analogy Method

In practice, the choice between classical and numerical methods depends on the design stage, available computational tools, required accuracy, and complexity of the structure. While classical methods provide a quick and intuitive understanding, numerical methods dominate detailed design and research applications due to their flexibility and precision

2.5.1 Guyon-Massonnet-Bares Orthotropic Plate Theory

Developed in the late 1940s, this theory represents the inaugural modern distribution coefficient method for assessing load distribution in bridge decks through the application of orthotropic plate theory (Holowaty, 2015). It conceptualizes a system of orthogonal beams as a continuous equivalent orthotropic plate, taking into account flexural and torsional rigidities in both directions as well as the width-to-span ratio of the bridge (Haddad & Edlebi, 2015). This method is most effective for systems comprising at least five beams (Jeziorski et al., 2024).

2.5.2 Morrice and Little's Orthotropic Plate Theory

The Morrice and Little method builds upon the work of Guyon and Massonnet, who introduced the concept of a distribution coefficient method for torsionless grillage and slab-type bridge decks. Guyon's initial method modeled the bridge deck as a quasi-orthotropic plate and used Lagrangian mechanics to develop equations for predicting its deflection behavior (Preera et al., 2015). The primary advantage of this method that a single set of distribution coefficients account for the two extreme cases of free torsion grillage and fully torsional slab allowing it to model the load distribution behavior of various bridge types effectively (Preera et al., 2015). A disadvantage of this method is it cannot be used for skew bridge (Mohan, et al., 2022).

2.5.3 The Leonhardt and Makowski Method

Deck load distribution studies by Leonhardt and Makowski have led to the following conclusions about grid deck bridges (Ezi, et al., 2018).

i. For most materials (excluding reinforced concrete), placing a cross-girder at mid-span is most effective.

ii. In concrete grillages, torsional stiffness reduces the influence of cross-girder placement, especially with multiple girders.

iii. More than five cross girders offer little added benefit; designers often use 1 or 3 for short spans, and a maximum of 5 for longer ones.

iv. Due to low torsional rigidity, steel cross girders have minimal impact on stress, and their joints act like hinges.

v. The efficiency of lateral load distribution depends on the number of longitudinal girders.

vi. Beyond 10 longitudinal girders, no further improvement in load distribution is observed.

vii. Ideal girder spacing matches the width of one traffic lane.

viii. Outer third girders carry more load than those in the middle third of the bridge deck.

2.5.4 Courbon's Method

Courbon's methods assume an entirely rigid cross beams, resulting in an even distribution of load when subjected to central loading. In cases of eccentric loads, the girders that are nearer to the load experience greater deflection. This technique is most effective for short-span, symmetrical bridges that possess rigid transverse components (Bani & Pardakhe, 2019). Courbon's method is simply applicable when the following conditions are satisfied:

a. The span-to-width ratio is between 2 and 4

b. At least five evenly spaced cross girders connect the longitudinal girders

c. Cross girders are at least 75% the depth of the longitudinal girders (Basilahamed, 2018).

2.5.5 Equivalent Strip Method

This method, outlined in LRFD specifications, models the bridge deck as a series of parallel continuous beams supported at regular intervals. The effective width for load analysis is based on LRFD tables. It applies to concrete decks that are at least 7 inches thick, have sufficient cover, and include four layers of reinforcement—longitudinal and transverse at both the top and bottom (AISC, 2022).

2.5.6 Westergard's Method

Westergaard modeled the pavement slab as a thin plate resting on a subgrade that responds elastically only in the vertical direction. The behavior depends on the relative stiffness between the concrete slab and the underlying soil (Al Ghafri & Javid, 2018).

2.5.7 Finite Element Method

Finite element method is a general approach to structural analysis. Its generality makes it the most suitable one for analysis of structures with arbitrary properties or geometries (Mohan, et al., 2022). Finite element modelling allows for a more rigorous analysis approach to be adopted that can often lead to significantly more accurate and economical results being obtained over some codified methods. By using finite element modelling structural members can be optimized and innovative cost-saving designs obtained (Margheriti and Icke, 2010). For particular applications, like bridge deck analysis The finite element method is therefore acceptable and clearly offers more attractions than the chart-based method of distribution coefficients presently in use in many design offices and the finite element method, being computer-based, is incomparably faster and less prone to errors (Onyia, 2008).

2.5.8 Finite Difference Method

The finite difference method developed by Nielson and later refined by Westergaards, this method is used for analyzing bridge decks with irregular shapes and complex boundaries. The finite difference method involves dividing the deck into mesh grid with deflection values at grid points treated as unknowns. The governing equation of the deck and boundary conditions are expressed of the unknowns and a system of equations is formed and solved to determine the deflections (Mahadevan, 2005).

2.5.9 Grillage Analogy Method

This method simplifies the bridge structure into an equivalent grid (grillage) system, analyzed using slope-deflection methods, compatibility conditions, and moment, shear, and reaction distributions. This method usually involves solving many simultaneous equations or performing extensive numerical calculations (Somani, 2021).

2.5.10 Folded Plate Theory

This method combines plane stress theory and plate bending theory to analyze stresses and bending in folded plate systems. This system involves connecting longitudinal plate elements along shared edges, with supports at each end and no intermediate membranes, allowing for a more refined analysis of the plane forces and bending moments (Al-Hadithy & Al-Masoudy, 2019).

2.6 Parametric Design and Optimization

Optimization refers to the search for the most favorable solution among all possible alternatives based on a defined optimality criterion (Kibkalo et al., 2016). In engineering, optimization is a vital tool for maximizing resource efficiency and minimizing environmental impact. Across various fields such as business, agriculture, and mechanical systems the goal is often to achieve the highest performance with the least input. For example, maximizing profit with minimal investment, obtaining higher crop yields with fewer fertilizers, or enhancing the strength and efficiency of machinery while reducing initial and operational costs (Kulkarni & Krishnasamy, 2017).

Parametric optimization involves determining the best values of certain decision variables (parameters) that either maximize or minimize a specific objective function (Gosavi, 2015). This process is achieved by systematically adjusting key parameters that influence the system's behavior, such as geometric dimensions or material properties (Kibkalo et al., 2016). Unlike conventional optimization which relies on fixed inputs, parametric optimization offers greater flexibility by exploring a range of design variables. This approach enables engineers to identify optimal solutions and also gain insights into how different parameters affect overall structural performance (Ravichandran, 2015).

2.6.1 Applications of Optimization in Structural Engineering

Traditionally, structural design and analysis process starts with an architect creating the drawings, which are subsequently passed to structural engineers for evaluation. If modifications are necessary, the process must revert to the architect before being reassessed by the engineer but this approach is time conusuming and inefficient. Parametric optimization enhances this workflow by enabling easy adjustments to key input parameters, which automatically refreshes the geometry and related structural characteristics, thus minimizing manual iterations and boosting overall efficiency (Weldegiorgis & Dhungana, 2020).

Formal optimization is already extensively utilized in structural design, particularly with the emergence of finite element methods and improvements in computing capabilities, leading to significant decreases in material consumption and costs (Birk, 2006). In comparison to traditional optimization, parametric methods have demonstrated greater efficiency in benchmark evaluations within structural engineering and provide an enhanced accuracy and lower computational demands (Ravichandran et al., 2019).

A real-world example of parametric optimization is illustrated in Birk's (2006) hydrodynamic design of offshore structures. By employing a Python-based parametric modeling framework, the research facilitated automated shape creation and formal optimization, simplifying the adaptation of designs across different scenarios. The findings underscored the advantages of merging parametric design with numerical analysis for improved design outcomes. In context of bridges, a study was performed a statistical parametric analysis using Response Surface Methodology (RSM) and Analysis of Variance (ANOVA), demonstrating how deck slab depth, bending moment, and reinforcement needs could be forecasted based on varying input parameters like span length and live load (Shashidhar and Ashwini, 2023).

2.7 MATLAB and GUI Applications in Structural Engineering

MATLAB is a powerful high-level programming environment widely used across educational institutions for solving mathematical and engineering-related problems (Navaee & Das, 2002). In structural engineering, MATLAB plays a crucial role in automating complex calculations, developing customized analysis tools, performing structural simulations, and creating user-friendly graphical interfaces.

As structural engineering evolves, the demand for accuracy, efficiency, and consistency in the design and analysis of structures especially bridges continues to grow. MATLAB supports this demand by enabling engineers to develop specialized programs customized to specific structural tasks. For instance, (Htwe and Khaing, 2014) employed MATLAB to analyze beam structures using the matrix stiffness method. The results closely matched those from SAP2000, with less than 1% difference in support reactions, validating MATLAB’s accuracy. Similarly, (Farhan and Dewangan, 2022) employed MATLAB based finite element analysis to a T-beam slab bridge deck. Their findings showed that MATLAB provided reliable dead load analysis and detailed insight into girder behavior, proving it effective for bridge deck modeling.

A key feature enhancing MATLAB’s practicality is its ability to support Graphical User Interfaces (GUIs). GUIs are visual platforms that allow users to interact with programs without writing code. GUI-based tools such as SAP2000, ETABS, and STAAD Pro are already standard in the industry, as they simplify complex calculations and visually present structural behavior, aiding in both learning and professional analysis.

Integrating MATLAB with GUI development brings additional benefits. As noted by (Marchand and Holland, 2003), GUIs increase productivity and accessibility by allowing users to interact with powerful analytical tools. In bridge deck optimization and related applications, GUIs reduce user error, improve design efficiency, and promote the adoption of both classical and parametric methods.

Several studies demonstrate the practical advantages of MATLAB-GUI integration. (Bhogade and Bolli, 2015) developed a MATLAB-based GUI for analyzing simply supported beams under various loading conditions. The tool computed support reactions, shears forces (SF), and bending moments (BM), and generated corresponding diagrams, streamlining structural analysis for both educational and real-world applications. Similarly, (Timmanagudar et al., 2018) created a GUI for the design and analysis of reinforced concrete (RC) simple span bridges, including solid slab and T-beam deck configurations. Their findings highlighted significant reductions in errors and computation time, making the tool particularly beneficial for students, instructors, and beginners in civil engineering.

2.8 Research gaps

The identified research gaps are;

i. There is limited integration of classical methods like Leonhardt and Makowski into user-friendly computational tools for structural engineers.

ii. Many existing optimization frameworks lack user-friendly interfaces, making them complex for everyday engineering applications.

iii. Most research tends to focus solely on manual calculation methods or high-end software solutions, leaving a gap for intermediate and interactive tools that bridge both worlds.

CHAPTER THREE

METHODOLOGY

3.1 Bridge geometry

The bridge to be analyzed has a deck width of 11 meters, designed to accommodate standard highway traffic. However, within the developed MATLAB based optimization tool, both the deck width, slab thickness and the bridge span are variable parameters. The bridge deck includes a 50 mm asphalt surfacing layer, which is accounted for in the dead load calculations. The primary load-carrying system consists of four standard precast I-girders, spaced equally across the deck width. These girders act compositely with a cast-in-situ grade C40/45 reinforced concrete slab, forming a T-beam superstructure. This composite action influences the effective breadth of the slab participating in flexural resistance. To accurately model this behavior, a modular ratio (n) is applied to account for the difference in elastic moduli between the precast concrete girders and the cast-in-place slab. As a result, the transformed effective breadth of the slab is expressed as:

$$b_{eff} = \frac{b_{eff}}{n} \tag{3.1}$$

Where

n is modular ratio

b_{eff} is effective breadth in mm

To provide sufficient overhang for parapets and guardrails, the deck features end cantilevers of 1.5 meters on each side. With four equally spaced girders supporting an 11-meter deck width (plus cantilevers), the spacing between adjacent girders is calculated as approximately 2.67 meters.

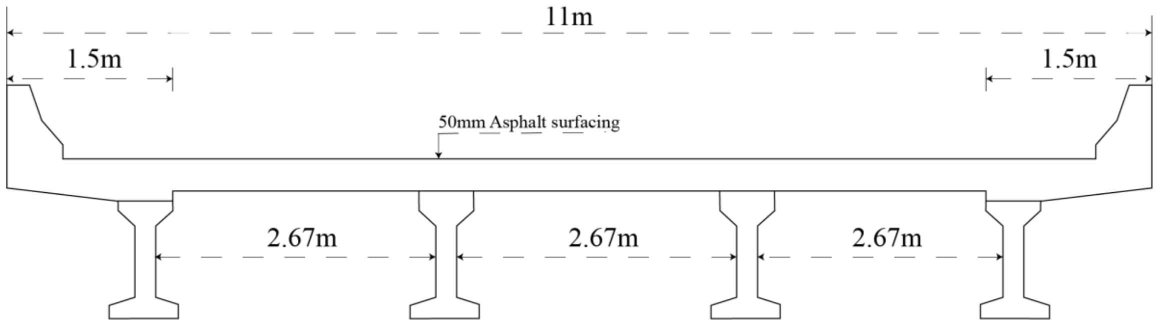


Figure 3.1: The cross-section of the composite bridge deck.

The section properties of the standard precast I-girders are provided in Table 3.1, and Figure 3.3 illustrates the cross-section of the composite T-beam system. Slab thickness is detonated as t(mm) and is a variable parameter.

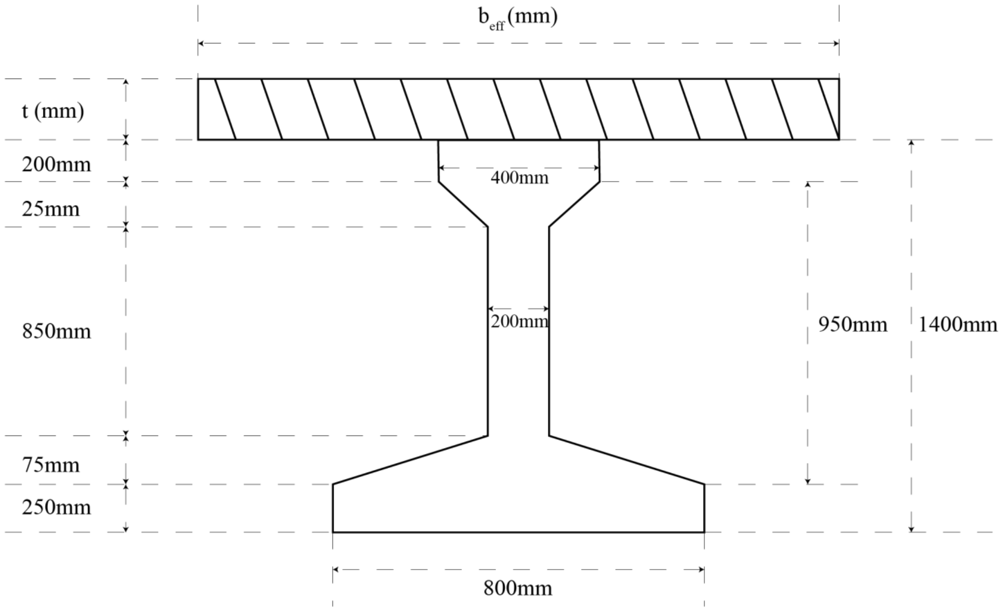


Figure 3.2: The cross-section highlighting the composite girder and slab structure

Table 3.1: Section properties of the girder

S/N	Shape	A(mm^2)	y(mm)	Ay(mm^3)	I_0(mm^4)	d(mm)	I(mm^4)
1	Top flange	8.0×10^4	100	8.0×10^6	2.67×10^8	742.42	4.4×10^{10}
2	Top triangular section	1.2×10^4	208	2.6×10^5	4.3×10^4	634.42	5.03×10^8
		1.2×10^4	208	2.6×10^5	4.3×10^4	634.42	5.03×10^8
4	Web	1.9×10^5	675	1.8×10^8	1.43×10^9	167.42	1.96×10^{10}
5	Bottom triangular section	1.1×10^4	1100	1.3×10^7	3.52×10^6	257.58	7.5×10^8
		1.1×10^4	1100	1.3×10^7	3.52×10^6	257.58	7.5×10^8
6	Bottom flange	2.0×10^5	1275	2.6×10^8	1.04×10^9	432.58	3.8×10^{10}
7	Total	4.95×10^5		4.2×10^8			1.04×10^{11}

$$\bar{y} = \frac{\sum Ay}{\sum A} \tag{3.2}$$

for the rectangular sections

$$I_0 = \frac{bh^3}{12} \tag{3.3}$$

for the triangular sections

$$I_0 = \frac{bh^3}{36} \tag{3.4}$$

$$I = I_0 + Ad^2 \tag{3.5}$$

Where

A is area in mm^2

Ay is moment of area in mm^3

y is y axis in mm

I_0 is moment of inertia in mm^4

d is neutral axis in mm

$\bar{y}$ is centroid axis in mm

The girder properties are given below:

I = 1.04×10^{11} mm^4

A = 4.95×10^5 mm^2

$\bar{y}$ = 842.42mm

3.2 Bridge loading

The structural analysis for this project considers the transient live loads acting on the bridge in accordance with BS 5400. These transient loads represent the effects of vehicular loads categorized under HA and HB loading classes and pedestrian traffic. According to code provisions, the bridge must be designed for the worst-case loading condition, either due to HA loading acting independently or a combination of HA and HB loadings. In this project, the governing load case is based on the combined HA and HB loading applied in accordance with BS 5400 guidelines. However, since the maximum bending effects typically occur when only one-third of the HA loading is applied in combination with the HB load, this scenario is selected for the critical load analysis. This conservative assumption is supported by code-based recommendations and ensures that the most severe response of the structure is captured. To aid understanding of how the loads are applied in the model, a diagram of the load positions including HA loads, Footpath loads and HB wheel loads is presented in Figure 3.3.

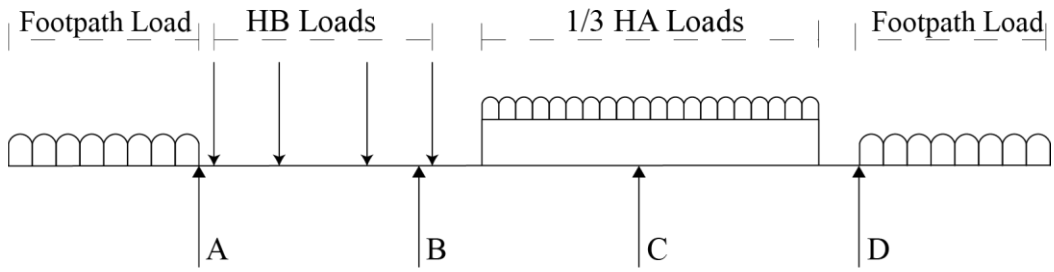


Figure 3.3: The positions of the various traffic loads to be considered

3.2.1 HA Loading

HA loading represents the effects of normal traffic on highways in Great Britain. For the application of HA loading, the bridge carriageway will be conceptually divided into notional lanes. These are theoretical divisions used solely for the purpose of applying specified live loads and do not necessarily correspond to physical lane markings on the road surface. Notional lanes are positioned at uniform spacing across the carriageway. HA loading consists of two primary components:

i. Uniformly Distributed Load (UDL): This load is applied over the loaded length of the bridge. Its intensity varies inversely with the loaded length. The loaded length or span is a parametric definition that would be varied throughout the project. And only short lengths > 50m would be considered

ii. Knife Edge Load (KEL): This is a concentrated load applied at a single point within the loaded length of each notional lane. The KEL is positioned to produce the most severe effect on the structural element under consideration.

Formulas for UDL and KEL:

i. Uniformly Distributed Load (UDL)

For loaded lengths (L) up to 50m:

$$W = 151(\frac{1}{L})^{0.475} \tag{3.6}$$

W is load (per notional lane)

L is loaded length

ii. Knife Edge Load (KEL):

KEL is 120kN (per notional lane).

For this study, only one-third of the full HA loading is applied in combination with the HB vehicle, as this configuration has been shown to produce the maximum bending moments in most bridge components. This aligns with BS 5400 provisions for conservative bridge design.

3.2.2 HB Loading

The HB loading is intended to represent abnormal or exceptionally heavy vehicular loads, such as industrial transporters carrying large machinery, transformers, or construction equipment. In BS 5400, this is modeled as an idealized 16 wheel vehicle, composed of four axles, each with four wheels equally spaced. The load intensity of the HB vehicle is defined in units, where 1 unit corresponds to 10 kN per axle (or 2.5 kN per wheel). For this project, an HB vehicle with 45 units is considered, resulting in a total axle load of 112.5kN per wheel. Figure 3.4 illustrates the standard spacing of the HB vehicle.

A critical parameter in HB loading is the distance between the two inner axles. BS 5400 provides standard spacing of 6, 11, 16, 21, and 26 meters. For this analysis, 6 meters spacing is adopted, as it typically produces the maximum bending moments and is therefore the most conservative for structural design.

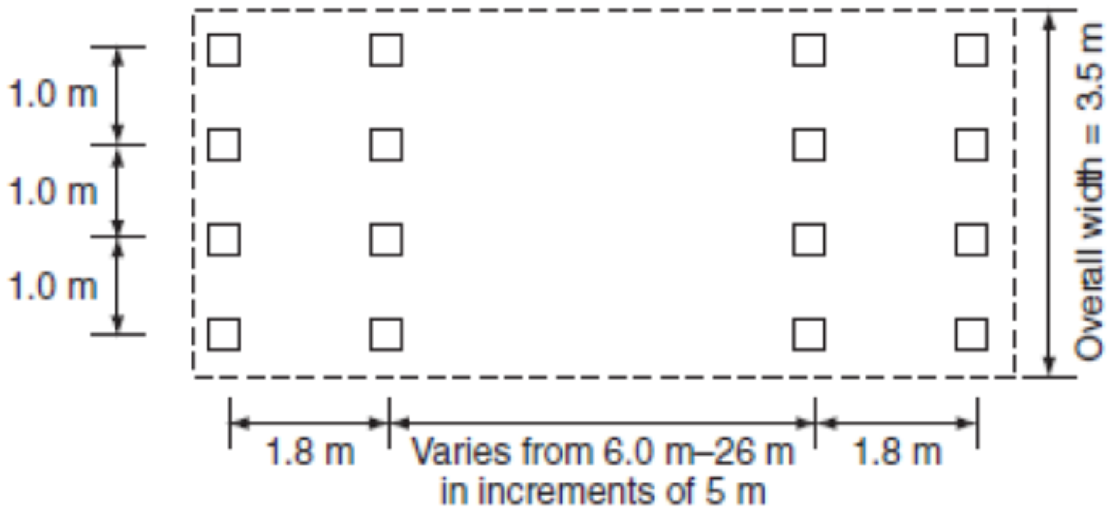


Figure 3.4: The standard spacing of the HB vehicle (Zakaria, et al., 2018)

3.2.3 Footpath live load

Beyond vehicular loading, the bridge deck design also considers pedestrian and cyclist loads on the footpaths and cycle tracks. In accordance with BS 5400, a uniformly distributed live load of 5KN/m² is applied over the full area of these peripheral zones. For the proposed bridge layout, the footpaths are located on the cantilevered sections extending from the main girders. The distributed load is applied during analysis to assess the additional bending moments and shear forces in both the cantilever and the composite deck. These areas are modeled as part of the overall bridge cross-section in the structural tool, ensuring the structural integrity of non-traffic zones is evaluated alongside the carriageway.

3.3 Leonhardt and Makowski Method Equations

The equations provided will be used for analysis of the bridge deck and will be implemented using MATLAB. The simplified grillage analysis approach will be discussed. In this approach, Leaonhardt and Makowski assumed that the deck lading can be expressed in the form of a harmonic series (see Figure 3.5, 3.6, 3.7 and 3.8)

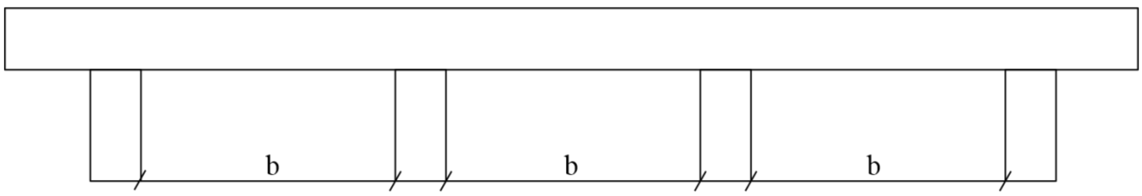


Figure 3.5: Bridge deck

Figure 3.6: Loading on deck

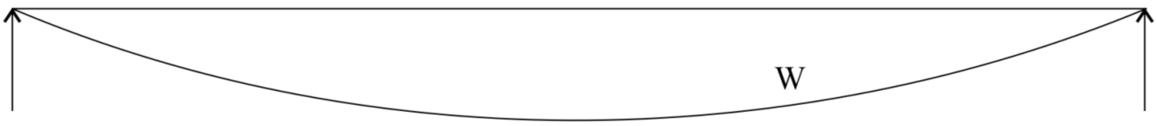


Figure 3.7: Moment on deck

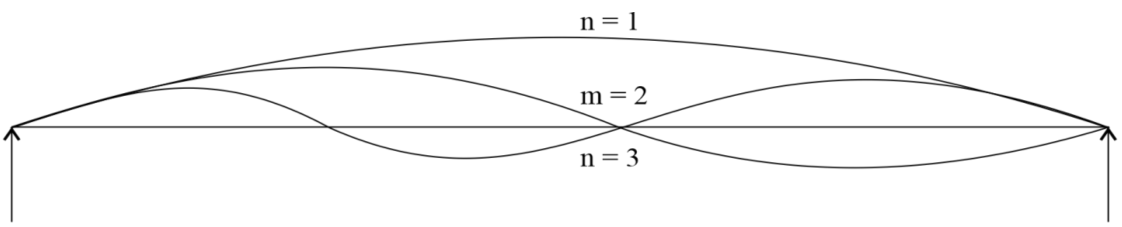


Figure 3.8: Response of deck

With

$$W = deflection \tag{3.7}$$

$$slope(\theta) = \frac{dW}{dX} \tag{3.8}$$

$$moment\ (M) = -\frac{Eld^2W}{dX^2} \tag{3.9}$$

$$shear\ (S) = -\frac{Eld^3W}{dX^3} \tag{3.10}$$

$$load\ (Q) = \frac{Eld^4W}{dX^4} \tag{3.11}$$

$$Q = \sum_{n-1}^{n\pm\infty} Q_n sin\frac{n\pi x}{L} = \frac{Eld^4W}{dX^4} \tag{3.12}$$

Where

Q is load

W is deflection

M is moment

S is shear

θ is slope

EI is flexural rigidity

Integrating the load function Eq. (3.12)

$$S = \frac{L}{n}\sum\frac{Q_n}{n}\cos\frac{\pi x}{L} \tag{3.13}$$

$$M = \frac{L^2}{\pi^2}\sum\frac{Q_n}{n^2}\sin\frac{n\pi x}{L} \tag{3.14}$$

$$\theta = \frac{L^3}{\pi^3}\sum\frac{Q_n}{n^3}\,sin\frac{n\pi x}{L} \tag{3.15}$$

Where

L is span of bridge

π is constant

and

$$W = \frac{L^4}{\pi^4}EI\sum_{n=1}^{n=\infty}\frac{Q_n}{n^4}\,sin\frac{n\pi x}{L} \tag{3.16}$$

giving

$$W = \frac{1}{K}\sum_{n=1}^{n=\infty}\frac{Q_n}{n^4}\,sin\frac{n\pi x}{L} \tag{3.17}$$

Where

k is stiffness of spring support with known load function

$$Q = Q_1 sin\frac{\pi x}{L} + Q_2 sin\frac{2\pi x}{L} + \ldots . Q_n sin\frac{n\pi x}{L} \tag{3.18}$$

multiplying both sides by $sin\frac{M\pi x}{L}$ and integrating both sides between 0 and L we have;

$$\int_0^L Q sin\frac{m\pi x}{L} = \int_0^L Q_1 sin\frac{\pi x}{L} sin\frac{m\pi x}{L} \ldots . \int_0^L Q_n sin\frac{n\pi x}{L} sin\frac{m\pi x}{L} \tag{3.19}$$

$$sin\frac{n\pi x}{L}sin\frac{m\pi x}{L} = 0 \quad (3.20)$$

It can be shown that

$$\int_0^L sin\frac{n\pi x}{L}sin\frac{m\pi x}{L} = \frac{L}{2}\ for\ M \neq n \quad (3.21)$$

applying Fourier series expansion

$$Q_n = \frac{2Q}{2}\int_0^L sin\frac{n\pi x}{L} \quad (3.22)$$

also applying a uniform distributed load as shown in Figure 3.9

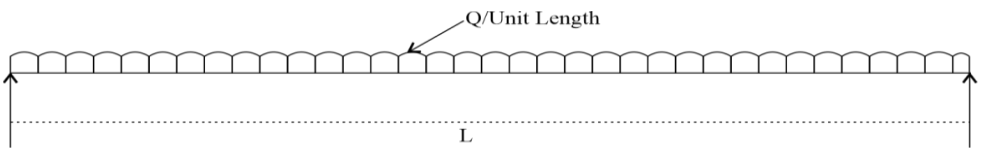


Figure 3.9: Uniformly distributed load on deck

Q_n is 0 when n is even.

$$Q_n = \frac{2}{L}\ \frac{QL}{n\pi}x2 = \ \frac{4Q}{n\pi}for\ n = 0\partial\partial \quad (3.23)$$

$$Q_n = \ \frac{4Q}{\pi}sin\frac{\pi x}{L} + \frac{4Q}{3\pi}sin\frac{3\pi x}{L} \quad (3.24)$$

Factorizing Eq. (3.18) we have that

$$Q_n = \frac{4Q}{\pi}\left[sin\frac{\pi x}{L} + \frac{1}{3}sin\frac{3\pi x}{L} \ldots.\right] \quad (3.25)$$

giving deflection function as

$$W = \frac{4QL^4}{\pi^5 EI}\left[sin\frac{\pi x}{L} + \frac{1}{3^5}sin\frac{3\pi x}{L} \ldots.\right] \quad (3.26)$$

For the case of a point load the relevant series was

$$Qx = \frac{2Q}{L}\left[sin\frac{\pi a}{L}sin\frac{\pi x}{L} + sin\frac{2\pi a}{L}sin\frac{2\pi x}{L}\dots.\right] \quad (3.27)$$

Using the load configuration as shown in Figure 3.10

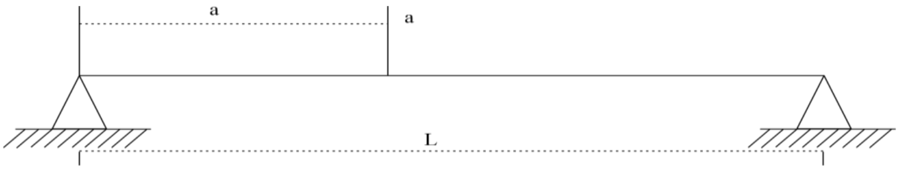


Figure 3.10: Point load on bridge deck

Integrating twice the above load function we have

$$M = \frac{2QL}{\pi^2}\left[sin\frac{\pi a}{L}sin\frac{\pi x}{L} + \frac{1}{2^2}sin\frac{2\pi a}{L}sin\frac{2\pi x}{L}\dots.\right] \quad (3.28)$$

Integrating equation Eq. (3.28) twice we have

$$W = \frac{2QL^4}{\pi^4 EI}\left[sin\frac{\pi a}{L}sin\frac{\pi x}{L} + \frac{1}{2^4}sin\frac{2\pi a}{L}sin\frac{2\pi x}{L}\dots.\right] \quad (3.29)$$

Where

I_x = 2nd moment of area of cross girder

I_{xf} = 2nd moment of area of fictitious cross girder

Then deflection constant caused by unit load on main girder;

$$\frac{1}{K} = \frac{L^4}{EI\pi^4} \quad (3.30)$$

Given that

$$\alpha = \frac{12L^4\left(\frac{EI}{unit\ width\ of\ concrete\ slab}\right)}{\pi^4 6^3\ (EI\ of\ longitudinal\ beam)} \quad (3.31)$$

Where

α = stiffness ratio

Then

$$K = \frac{12(EIx)}{\alpha b^3} \tag{3.32}$$

and

$$\frac{1}{K} = \frac{\alpha b^3}{12(EI_{xp})} \tag{3.33}$$

$$= \frac{b^3}{EI_{xf}}\left[\frac{L^4\ (EI\ per\ unit\ width\ of\ slab)}{\pi^4 6^3\ (EI\ per\ unit\ width\ of\ main\ girder)}\right] \tag{3.34}$$

If there is no cross girder

$$\frac{1}{K} = \frac{L^4}{\pi^4 EI(of\ one\ main\ girder)} \tag{3.35}$$

Or

$$K = \frac{\pi^4 EI}{L^4} \tag{3.36}$$

3.3.1 Illustration of load distribution

In his simplified grillage analysis approach, Leonhardt assumed that the load on any beam J can be represented by the following expression:

$$Q_j = Q_1 sin\frac{\pi x}{L} \tag{3.37}$$

after integrating Eq. (3.37) twice

$$M = \frac{Q_1 L^2}{2} sin\frac{\pi x}{L} \tag{3.38}$$

and replaces the girder by springs of equal stiffness where

$$K = \frac{12EIx}{\alpha b^3} \tag{3.39}$$

And

$$\alpha = \frac{12L^4}{\pi^4 6^3}\left[\frac{\frac{EI}{unit\ width\ of\ slab}}{\frac{EI}{unit\ width\ of\ main\ girder}}\right] \tag{3.40}$$

Where

L is span of bridge

B is Spacing of girders

I_f is Moment of Inertia of unit width of slab

I is Moment of Inertia of main girder

E is Young's Modulus of Elasticity

I_x is Moment of Inertia of cross girder

Considering a bridge deck with four (4) longitudinal girders as shown in Figure 3.11 and Figure 3.12

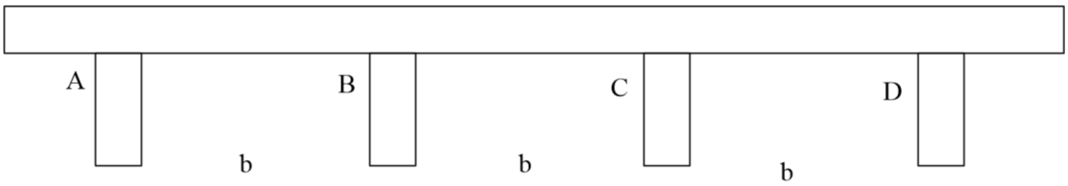


Figure 3.11: Actual bridge deck

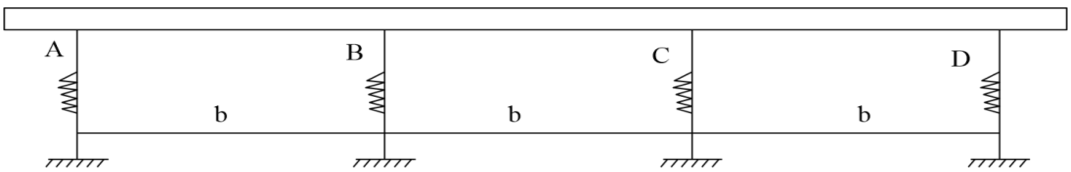


Figure 3.12: Idealized bridge deck

The lateral load distribution to the other girders, when the load is applied on one girder, can be determined by solving the system using the influence coefficient method, while accounting for axial forces. By introducing a release at the outer girder (as shown in Figures 3.11 and 3.12), the influence coefficient to be determined can be expressed in the following general form:

$$fij = \int \frac{M_i M_j}{EI} ds + \sum_{r=1}^{r} \frac{n_i^r n_j^r}{K_r} \tag{3.41}$$

$$\mu = \int \frac{M_i M_0}{EI} ds + \sum_{r=1}^{r} \frac{n_i^r n_0^r}{K_r} \tag{3.42}$$

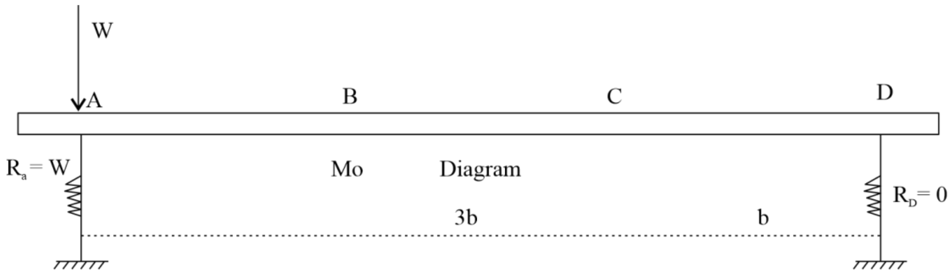


Figure 3.13: Moment and reaction due to applied load on released structure

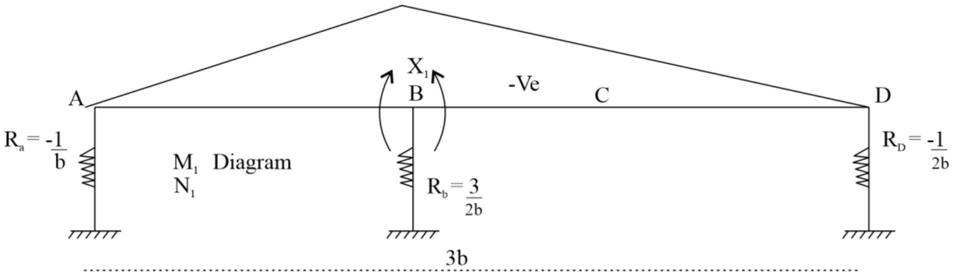


Figure 3.14: Influence coefficient diagram, moment & reaction due to unit load applied on release at B

And it can be shown that

$$R_A = Q - \frac{X_1}{b} - \frac{X_2}{2b} \tag{3.43}$$

$$R_B = \frac{3X_1}{2b} \tag{3.44}$$

$$R_c = \frac{3X_2}{2b} \tag{3.45}$$

$$R_D = \frac{-X_1}{2b} - \frac{X_2}{b} \tag{3.46}$$

3.3.2 DETERMINATION OF TRANSVERSE BENDING MOMENT

The same procedure is repeated for a load placed on girder B. The transverse bending moments in the bridge deck are then obtained by substituting the values of X_1 and X_2 into the appropriate figures and formulas. For bridge decks interconnected by cross girders, Leonhardt developed a table for determining the influence coefficients for lateral load distribution. The table is widely used and applies to grids with 3 to 10 main girders interconnected by a single cross girder at mid-span.

For decks with more than one cross girder, the same table can be used with a modification factor Z, where:

L = span of the bridge

A = spacing of main girders

I_i = second moment of area of the cross girder

I = second moment of area of the main girder

The factor γ is a numerical correction that accounts for the number of cross girders:

$\gamma = 1$ for 2 cross girders

$\gamma = 1.6$ for 3 or 4 cross girders

$\gamma = 2$ for 5 or more cross girders

This approach is based on replacing the actual cross girders with a single fictitious cross girder whose stiffness produces the same lateral load distribution as the real system. The second moment of area of the fictitious girder is directly proportional to the number of real cross girders it replaces.

3.4 Parametric optimization strategy

This project employs a parametric optimization approach to examine different design parameters and their effect on the structural performance of reinforced concrete (RC) bridge decks. The goal is to develop a flexible tool that enables engineers to efficiently assess multiple design alternatives and identify configurations that meet strength, serviceability, and economic criteria. The optimization process is driven by user-defined inputs range, which is processed through the MATLAB based tool analysis engine. These parameters include:

i. Span length

ii. Slab thickness

iii. Deck width

The ranges are defined in a MATLAB script, and loop through the values in the range then carries out analysis using the MATLAB tool analysis engine. The table below summarizes the key input parameters and their allowable ranges:

Table 3.2: Ranges of the parameters for optimization

S/N	Parameters	Ranges
1	Span (m)	25 - 30
2	Span thickness (mm)	200 - 300
3	Carriageway width (m)	11 - 15

3.5 MATLAB Implementation

The Leonhardt and Makowski method will be implemented using MATLAB, and an interactive and user-friendly Graphical User Interface (GUI). The MATLAB program automates the entire analysis process from accepting user inputs to performing structural computations and generating graphical and tabular outputs. This implementation would be used for parametric optimization of bridge deck using a MATLAB script while ensuring computational accuracy and compliance to the British Standards.

The tool is designed to calculate and display:

i. Section properties of the composite slab-girder system

ii. Bending moments resulting from applied live loads on the girders

iii. Transverse Bending moment diagrams for structural interpretation

iv. Display results of relevant computations in the process of the bridge analysis

The overall logic of the program follows a structured sequence as shown in Figure 3.15: Flowchart of the MATLAB program process, capturing each step from user input to analysis, output. The workflow comprises four main stages:

i. User Input and Validation

ii. Preprocessing and Structural Analysis

iii. Output Generation

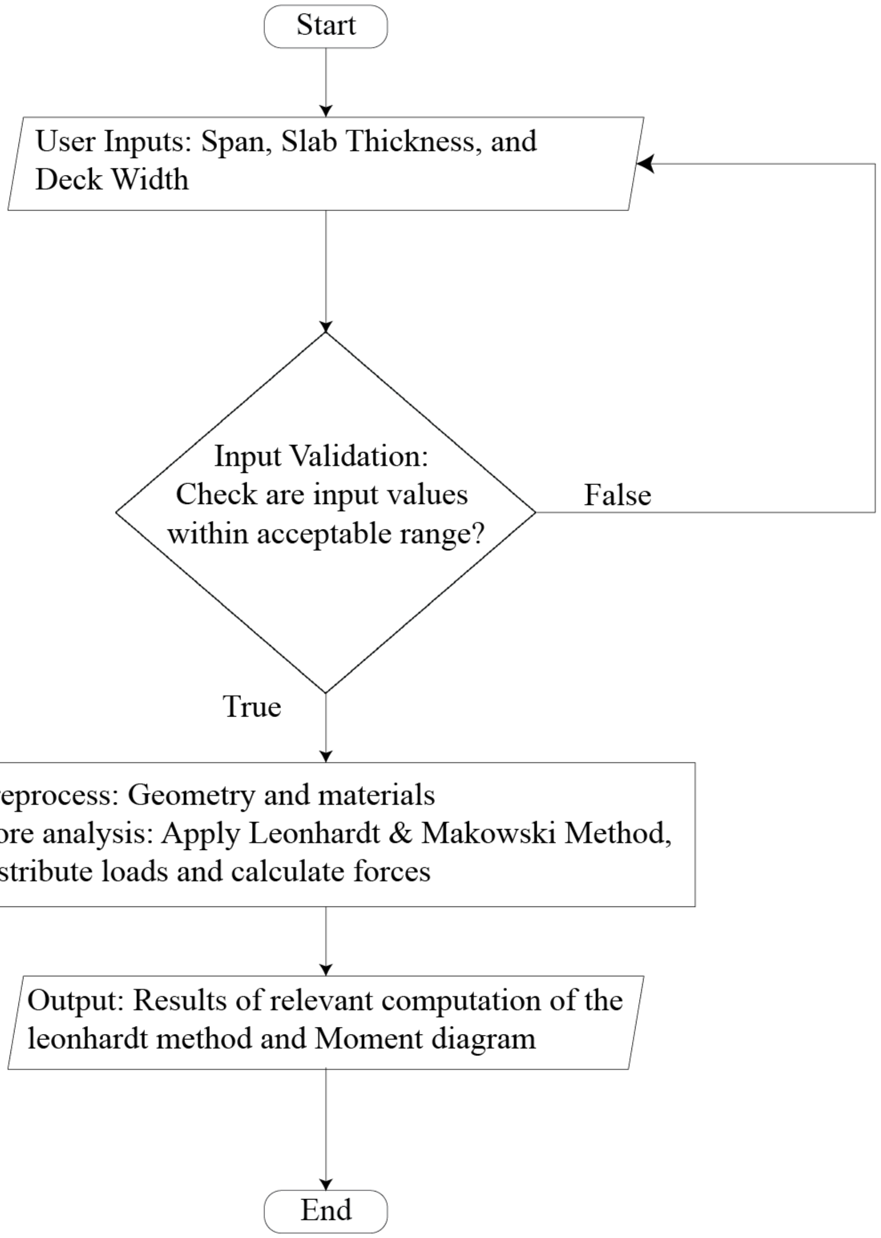


Figure 3.15: Flowchart of the MATLAB program process, capturing each step from user input to analysis, output.

3.6 Data analysis and verification method

A bridge with a deck width of 11m, span of 25m and slab thickness of 230mm would be analyzed manually using the standard loading configuration. The result from this analysis would be compared with the MATLAB tool and a STAAD.Pro analysis to validate accuracy. Then a MATLAB script would be developed to utilize the MATLAB tool to carry out parametric analysis. This parametric analysis studies the influence of the span, slab thickness and deck width on the maximum longitudinal bending moments, by keep two the parameters constant whiled the other is varied within the ranges defined in Table 3.2.

The MATLAB tool not only computes the loading, section properties and distribution of loads, but also presents results with well-labeled diagrams and structured tables. The MATLAB tool is designed to accommodate variations of the deck width, slab thickness, and span. This enables the user to examine the relationship of the parameter definitions and the structural configurations of the bridges. A MATLAB script is designed to capture this variation in real-time as part of the parametric optimization process.

To ensure the accuracy and engineering validity of the developed MATLAB based analysis tool the results from the program would be compared with both manual analysis. Key aspects of the MATLAB results to be verified include:

i. Comparison of results of manual calculations of moments for span of 25m, deck width of 11m and slab thickness of 230mm using the Leonhardt method and loading principles from BS 5400

ii. Comparison of results from the MATLAB tool and that of STAAD.Pro

CHAPTER FOUR

RESULTS AND DISCUSSION

4.1 Introduction

This chapter presents the results obtained from both the manual analysis and the MATLAB-based Graphical User Interface (GUI) developed using the Leonhardt and Makowski method. It discusses the bridge deck's structural response under combined HA and HB loadings, along with the verification of the developed tool against manual computation. The chapter also explores the effects of varying key parameters such as span length and slab thickness on the resulting maximum bending moments through a parametric study.

4.2 Transformed deck properties

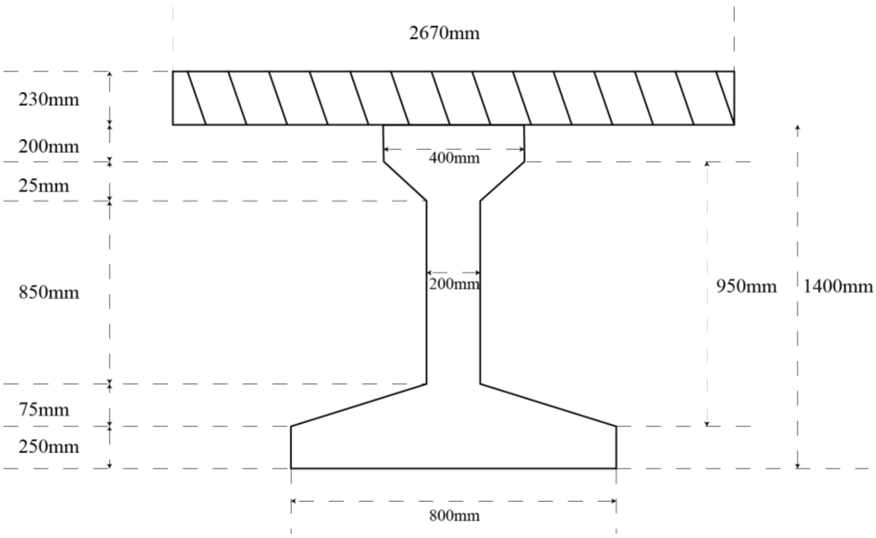


Figure 4.1: Composite deck and girder section

From 3.1 the results of the girder properties calculated are given below:

I = 4.58×10^{11} mm^4

A = 4.95×10^5 mm^2

$\bar{y}$ = 842.42mm

$\bar{y}_2$ = 1400 - 842.42 = 557.58mm

Section modulus of the girder

$$Z = \frac{I}{\bar{y}} \tag{4.1}$$

Where

Z is section modulus in mm^3

I is moment of inertia in mm^4

Y is in mm

$Z = \frac{1.04 \times 10^{11}}{557.58} = 1.88 \times 10^8$ mm^3

$Z = \frac{1.04 \times 10^{11}}{842.42} = 1.25 \times 10^8$ mm^3

The modular ratio of the transformed section = 0.87

Transformed slab width = $0.87 \times 2670 = 2322.9$ mm

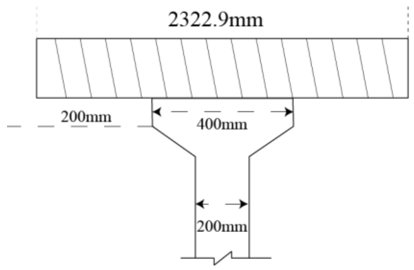


Figure 4.2: Transformed composite deck and girder section

Area of Slab (A_S)

230×2670 = 6.141×10^5 mm^2

Transformed area (A_T)

$0.87 \times 6.14 \times 10^5$ = 5.34×10^5 mm^2

Moment of Inertia of Slab (I_S)

$\frac{1}{12} \times 2670 \times 230^3$

= 2.71×10^9 mm^4

Transformed MOI of Slab (I_T)

$0.87 \times 2.71 \times 10^9$ = 2.36×10^9 mm^4

$$\bar{y}_3^T = \frac{\sum Ay}{\sum A} \tag{4.2}$$

$\bar{y}_3^T = \frac{(230\times2670\times0.87\times115)+(4.95\times10^5\times(842.42+230))}{4.95\times10^5+(230\times2670\times0.87)}$

= 575.45mm

$\bar{y}_1^T = (1400 + 230) - 575.45$ = 1054.55mm

$\bar{y}_2^T = 575.45 - 230$ = 345.45mm

$$I = I_0 + Ad^2 \tag{4.3}$$

= $1.04 \times 10^{11} + 4.95 \times 10^5(842.42 - 572.45)^2 + 2.36 \times 10^9 + 5.34 \times 10^5(575 - 115)^2$

= 2.55×10^{11} mm^4

$Z_1^T = \frac{2.55\times10^{11}}{1054.55}$ = 2.417×10^8mm^3

$Z_2^T = \frac{2.55\times10^{11}}{345.45} = 7.38 \times 10^8$mm^3

$Z_3^T = \frac{2.55\times10^{11}}{575.45} = 4.43 \times 10^8$mm^3

Summary

1. Section properties of girder

I = 4.58×10^{11} mm^4

A = 4.95×10^5 mm^2

$\bar{y}$ = 842.42mm

$\bar{y}_2$ = 557.58mm

Z $= 1.88 \times 10^8$mm^3

Z = 1.25×10^8 mm^3

2. Section properties of slab

A_S = 6.141×10^5 mm^2

A_T = 5.34×10^5 mm^2

I_S = 2.71×10^9 mm^4

I_T = 2.36×10^9 mm^4

3. Section properties of composite deck

A = 1.109×10^6 mm^2

I = 2.55×10^{11} mm^4

$\bar{y}_1^T$ =1054.55mm

$\bar{y}_2^T$ =345.45mm

$\bar{y}_3^T$ =575.45mm

$Z_1^T = 2.42 \times 10^8$mm^3

$Z_2^T = 7.38 \times 10^8$mm^3

$Z_3^T = 4.43 \times 10^8$mm^3

4.3 Bridge Loading

Carriageway width = 11m

Span = 25m

45 Units of HB loads

Only HB and $\frac{1}{3}$ HA loads would be considered for analysis as this is critical for this type of short span bridge

<u>HA Loads</u>

1. HA UDL

From BS5400 tables = 30.8 KN/m

= $\frac{1}{3}$ UDL = 10.27 KN/m

2. HA KEL = 120 KN

= $\frac{1}{3}$ KEL = 40 KN

HA load on deck

$= \frac{10.27}{2.2} = 4.67$ KN/m^2

$= \frac{40}{2.2} = 18.18$ KN/m^2

Total HA load = 18.18 + 4.67 = 22.85KN/m^2

<u>HB load</u>

45 units of HB load to be considered

Load per wheel = 2.5×45

= 112.5 KN

<u>Footpath load</u>

The load value considered for the footpath load is 5KN/m.

4.4 Leonhardt and Makowski Method

This method of analysis is carried out in two ways:

i. The Non-yielding support analysis

ii. The Yielding support analysis

4.4.1 Non-Yielding support analysis

The loading arrangements considering only live loads

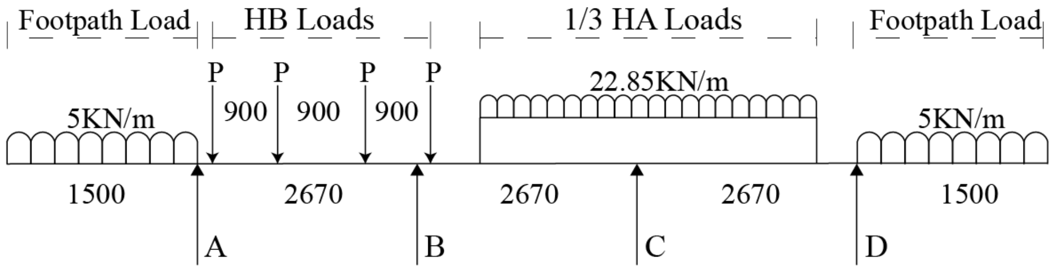
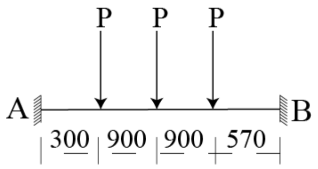


Figure 4.3: Loading arrangement of the live loads

4.4.2 Determination of Fixed End Moment

P = 112.5 KN

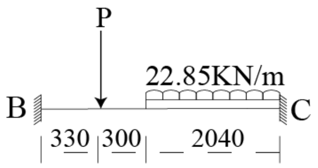


$M_{AB} = -\frac{112.5}{2.67^2}(0.3 \times 2.37^2) + (1.2 \times 1.47^2) + (2.1 \times 0.57^2)$

M_{AB} = -78.28 KNm

$M_{BA} = \frac{112.5}{2.67^2}(0.57 \times 2.1^2) + (1.47 \times 1.2^2) + (2.37 \times 0.3^2)$

M_{BA} = 76.44 KNm

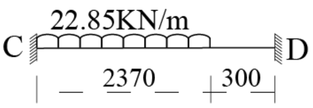


$M_{BC} = -(\frac{112.5 \times 0.33 \times 2.34^4}{2.67^2} + \frac{46.61}{12} \times 2.67 \times 0.764^2(1.7079)) - (28.52 + 10.23)$

M_{BC} = -38.75KNm

$M_{CB} = (112.5 \times 2.34 \times 0.33^2 + 46.61 \times 0.764(2.67 \times 2.2)$

M_{BC} = 21.45KNm

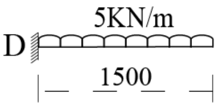


$M_{CD} = -\frac{22.85(2.37)^2}{12(2.67^2)}[6(2.67)^2 - 8(2.67 \times 2.370) + 3(2.37)^2]$

M_{CD} = -13.5 KNm

$M_{DC} = \frac{22.85(2.37)^3}{12(2.67^2)}[4(2.67) - 3(2.37)]$

M_{DC} = 12.7 KNm



FEM Cantilever = $\frac{5 \times (1.5)^2}{2}$ = 5.63 KNm

4.4.3 Moment distribution of Non-Yielding support

	A		**B**		**C**		**D**
0	1	0.5	0.5	0.5	0.5	1	0
5.63	-78.29	76.44	-38.75	21.45	-13.5	12.7	-5.63
	72.66	36.33			-3.64	-7.07	
5.63	-5.63	112.77	-38.75	21.45	-17.02	5.63	-5.63
		-37.01	-37.01	-18.51			
			3.52	7.04	7.04		
		-1.76	-1.76	-0.88			
			0.22	0.44	0.44		
		0.10	0.10				

5.63	-5.63	73.4	-73.9	9.54	-9.54	-5.63	5.63
5.63 KNm		73.9 KNm		9.64 KNm		5.63 KNm	

4.4.4 Reaction at support

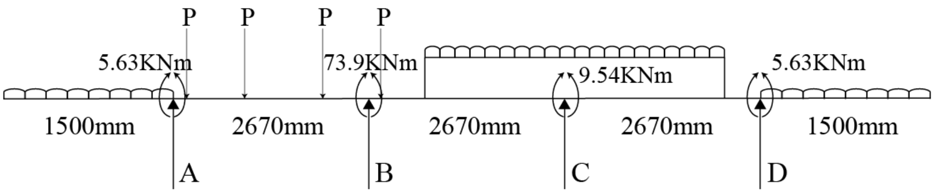


Figure 4.4: Hogging moments at supports

$R_A = \frac{112.5}{2.67}(0.57 + 1.47 + 2.37) + \frac{(5.63 - 73.9)}{2.67} + 7.5$

$R_A = 167.75$ KN

$R_B^1 = 7.5 + (3 \times 112.5) - 167.74$

$R_B^1 = 177.26$ KN

$R_B^{11} = \frac{112.5 \times 2.34}{2.67} + \frac{22.85 \times (2.04)^2}{2.67} + \frac{(73.9 - 9.54)}{2.67}$

$R_B^{11} = 140.50$ KN

$R_B = 177.26 + 140.50 = 317.78$ KN

$R_C^1 = \frac{112.5 \times 0.33}{2.67} + \frac{22.85 \times 2.04 \times 1.63}{2.67} - \frac{(73.9 + 9.54)}{2.67}$

$R_C^1 = 18.6$ KN

$R_C^{11} = 22.85 \times 2.37 \times \frac{1.185 + 0.3}{2.67} + \frac{(9.54 + 5.63)}{2.67}$

$R_C^{11} = 31.58$ KN

$R_C = 18.6 + 31.58 = 50.18$ KN

$R_D^1 = \frac{22.85\times(2.37)^2}{2.67} + \frac{(5.63-9.54)}{2.67}$

R_D^1 = 22.57 KN

R_D^{11} = 7.5 KN

R_D = 7.5 + 22.57 = 30.07 KN

4.4.5 Computation of Longitudinal Moments

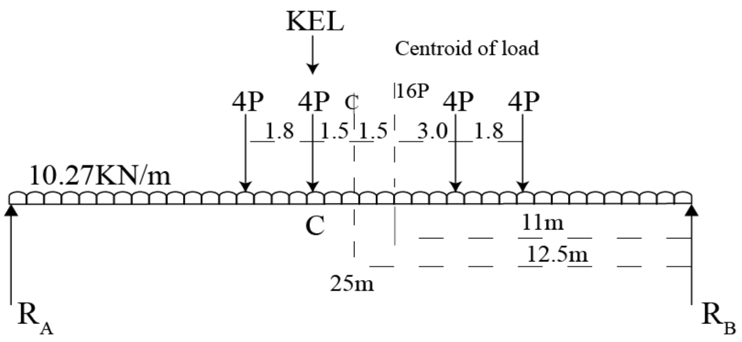


Figure 4.5: Longitudinal arrangement of loads

1. Moment due to HB load

$R_A = \frac{16P\times11^2}{25}$

The maximum bending moment will occur under the axle at C

$M_C = \frac{16P\times11^2}{25} - 4P\times1.8$

$= 77.44P - 7.2P$

$= 70.24P$ KNm

= 7902 KNm

2. Moment due to 1/3 HA load

M_C (KEL) = $\frac{40 \times 14 \times 11}{25}$

= 246.4 KNm

M_C (UDL) = $\frac{10.27 \times 25 \times 11}{2} - \frac{10.27(11)^2}{2}$

= 790.79 KNm

3. Foot path loading

M_C (UDL) = $2 \times (\frac{5 \times 25 \times 11}{2} - \frac{5 \times 1.5 \times 2 \times 11^2}{2})$

= 770 KNm

Total Deck B.M at C

M_C = 7902 + 246.4 + 790.79 + 770

= 9709.2 KNm

The above deck free moments is distributed to the girder in the proportion of the Non-yielding support transverse bending analysis to girder reaction giving

Table 4.1: Moment and reactions at support point

Girder	**A**	**B**	**C**	**D**	**Total**
Reaction KN	167.75	317.78	50.18	30.07	567.78
Moment KNm	2882	5462	833.2	531.6	9709.2

4.4.6 Determination of the influence coefficient

From Figure 3.12 and Figure 3.14 and when using equations 3.41 and 3.42 we have that

$F_{11} = \int_s \frac{M_1^2\, ds}{EI_f} + \frac{\sum n_1^2}{K}$

$= \frac{3b}{4EI_f} + \frac{1}{K}(\frac{1}{b^2} + \frac{9}{4b^2} + \frac{1}{4b^2})$

$F_{11} = \frac{3b}{4EI_f} + \frac{14}{4Kb^2}$

$F_{12} = \frac{3b}{4EI_f} + \frac{1}{Kb^2}$

$\mu_{01} = \frac{1}{k}(-\frac{W}{b} + 0)$

$= \frac{-W}{Kb}$

$\mu_{02} = \frac{-W}{2bk}$

Solving the equation

$X_1 F_{11} + X_2 F_{12} + \mu_{01} = 0$

$X_1 F_{21} + X_2 F_{21} + \mu_{02} = 0$

This process is repeated for load on position B. This gives

$F_{11} = \frac{3b}{4EI_f} + \frac{14}{4Kb^2} = F_{22}$

$F_{12} = \frac{3b}{4EI_f} + \frac{1}{Kb^2} = F_{21}$

$\mu_{01} = \frac{2bW}{4EI_f} \times \frac{2b}{3} + \frac{1}{E}(\frac{-2W}{3b} - \frac{W}{6b})$

$= -W(\frac{2b^2}{3EI_f} + \frac{5}{6bk})$

$\mu_{02} = \frac{-2bW}{3EI_f} \times \frac{1}{6} - \frac{2(b-2b)^2}{2\times 2b} + \frac{1}{K} \times \frac{2W}{3} \times \frac{1}{2b} \times \frac{W}{3} \times \frac{3}{1}$

$\mu_{02} = -W(\frac{3}{6EI_f} + \frac{2}{3bk})$

And the reaction can be determined from adding up the released structure giving

$R_A = \frac{2W}{3} - \frac{X_1}{b} - \frac{X_2}{2b}$

$R_B = \frac{3X_1}{2b}$

$R_C = \frac{3X_2}{2b}$

$R_D = \frac{W}{3} - \frac{X_1}{2b} - \frac{X_2}{b}$

4.4.7 Lateral distribution coefficient

$I_s = \frac{1}{12} \times (0.23)^3 = 1.0139 \times 10^{-3}$ m^4

b = 2.67 m

$I_b = 1.04 \times 10^{11}$ mm^4 = 1.04×10^{-1}m^4

$I_{st} = 0.87 \times 1.0139 \times 10^{-3} = 8.8209 \times 10^{-4}$ m^4

L = 25.0 m

For no cross Girder

$K = \frac{\pi^4 EI}{L^4}$

E = $4.5\sqrt{40} = 23.46$ KN/mm^4 = 28.46×10^6 MN/mm^4

K = $\frac{97.41 \times 28.46 \times 10^6 \times 1.04 \times 10^{-1}}{3.91 \times 10^5}$

K = 745.18 KN/m^2

$K = 2.6 \times 10^{-5} E$ KN/m^2

For Unit load on Girder A:

$F_{11} = F_{22} = \frac{3\times2.67}{4\times8.82\times10^{-4}\ E} + \frac{14}{(2.67)^2\ \times4\times2.6\times10^{-5}E}$

= 2106/E

$F_{12} = F_{21} = \frac{3b}{4EI_f} + \frac{1}{Kb^2}$

$= \frac{2.27\times10^3}{E} + \frac{1}{2.6\times10^{-5}(2.67)^2\ E}$

= 7638/E

$\mu_{01} = -\frac{1}{2.67\times2.6\times10^{-5}\times E}$

$= -1.43 \times 10^4$/E

$\mu_{02} = \frac{-1}{2bk} = -7.2 \times 10^3$/E

X_1 = 0.6416 KNm

X_2 = 0.108 KNm

$R_A = W - \frac{X_1}{b} - \frac{X_2}{2b}$

$= 1 - \frac{0.64}{2.67} + \frac{0.108}{2\times2.67} = 0.7496$ KN

$R_B = \frac{3X_1}{2b} = \frac{3\times0.64}{2\times2.67} = 0.3604$ KN

$R_C = \frac{3X_2}{2b} = \frac{3\times0.108}{2\times2.67} = 0.0604$ KN

$R_D = \frac{-X_1}{2b} - \frac{X_2}{b} = \frac{-0.64}{2\times2.67} + \frac{-0.108}{2.67} =$ - 0.1604 KN

$\sum(\mathrm{R_A} + \mathrm{R_B} + \mathrm{R_C} + \mathrm{R_D}) = 1$

<u>For Unit load on Girder B:</u>

$F_{11} = F_{22} = 7638/E$

$F_{12} = F_{21} = 2106/E$

$\mu_{01} = -(\frac{2b^2}{3EI_f} + \frac{5}{6bk}) = -(\frac{2\times2.67^2}{3\times8.821\times10^{-4}\times E} + \frac{5}{6\times2.67\times2.6\times10^{-5}E})$

$= -1.733 \times 10^4/\mathrm{E}$

$\mu_{02} = -(\frac{3}{6EI_f} + \frac{2}{3bk}) = -(\frac{3}{6\times8.821\times10^{-4}\times E} + \frac{2}{3\times2.67\times2.6\times10^{-5}E})$

$= -1.61 \times 10^4/\mathrm{E}$

$X_1 = 0.747$ KNm

$X_2 = 0.21$ KNm

$R_A = \frac{2W}{3} - \frac{X_1}{b} - \frac{X_2}{2b} = \frac{2}{3} - \frac{0.747}{2.67} - \frac{0.21}{2\times2.67} = 0.35$ KN

$R_B = \frac{3X_1}{2b} = \frac{3\times0.747}{2\times2.67} = 0.42$ KN

$R_C = \frac{3X_2}{2b} = \frac{3\times0.21}{2\times2.67} = 0.12$ KN

$R_D = \frac{W}{3} - \frac{X_1}{2b} - \frac{X_2}{b} = \frac{1}{3} - \frac{0.747}{2.67} - \frac{0.21}{2\times2.67} = 0.11$ KN

$\sum(\mathrm{R_A} + \mathrm{R_B} + \mathrm{R_C} + \mathrm{R_D}) = 1$

Using the Maxwell reciprocal theorem we have the following

Table 4.2: Influence coefficients

Load Location	A	B	C	D	Total
A	0.7496	0.3604	0.0604	-0.1604	1
B	0.35	0.42	0.12	0.11	1
C	0.11	0.12	0.42	0.35	1
D	-0.1604	0.0604	0.3604	0.7496	1

4.4.8 Girder moment distribution

Girder	Free B.M KNm	A	B	C	D
A	2882	2133	1037.5	172.9	421.12
B	5462	1911.7	2294	655.44	600.82
C	833.2	91.65	99.98	349.94	291.62
D	531.6	85.06	31.9	191.38	393.38
Total KNm	9709.2	4221.1	3463.4	1369.7	1746.9

4.4.9 Transverse Bending moment analysis

The transverse bending moment is calculated making use of the free bending moment on the individual girders and the distribution coefficient already obtained.

Considering the diagram shown below fig 4.3.2 the transverse bending moment caused by the relative deflection of girders when there is a free bending moment is given by

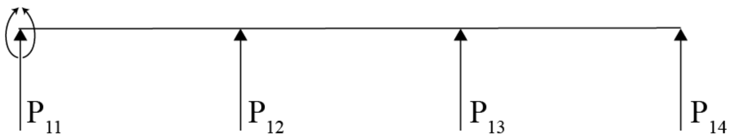


Figure 4.6: Moment due to relative deflections

With M_1 on girder 1 the transverse bending moment induced is given by

$M_{21} = M_{23} = \frac{\pi^2 b M_1}{L^2}(2P_{14} + P_{13})$

$M_{32} = M_{34} = \frac{\pi^2 b M_1}{L^2}(P_{14})$

When M_2 is on girder 2

$M_{21} = M_{23} = \frac{\pi^2}{L^2} b M_2 P_{21}$

$M_{32} = M_{34} = \frac{\pi^2}{L^2} b M_2 P_{24}$

When M_3 is on girder 3

$M_{32} = M_{34} = \frac{\pi^2}{L^2} b M_3 P_{34}$

$M_{21} = M_{23} = \frac{\pi^2}{L^2} b M_3 P_{31}$

When M_4 is on girder 4

$M_{21} = M_{23} = \frac{\pi^2}{L^2} b M_4 P_{41}$

$M_{32} = M_{34} = \frac{\pi^2}{L^2} b M_4 (2P_{41} + P_{42})$

$\therefore M_{21} = M_{23} = \frac{\pi^2 b}{L^2}[M_1(2P_{14} + P_{13}) + M_2(P_{21}) + M_3(P_{32}) + M_4(P_{41})]$

$M_{32} = M_{34} = \frac{\pi^2 b}{L^2}[M_1(2P_{14}) + M_2(P_{24}) + M_3(P_{34}) + M_4(2P_{41} + P_{42})]$

Free longitudinal B.M on Girder

Table 4.3: Free moment at supports

Girder	M_1	M_2	M_3	M_4
Free B.M (KNm)	4221.1	3463.4	1369.7	1746.9

Substituting values into the above formulae

we have that :

$M_{21} = \frac{\pi^2 b}{L^2}[-0.105M_1 + 0.2M_2 - 0.19M_3 - 0.047M_4]$

M_{21} = 49.59 KNm

$M_{32} = \frac{\pi^2 b}{L^2}[-0.047M_1 + 0.19M_2 + 0.2M_3 - 0.105M_4]$

M_{32} = 59.14 KNm

This moment is combined with the non-yielding transverse bending moment to obtain the actual transverse distribution of moments. This distribution of the Non-yielding support and the Yielding supports moment is shown in Figure 4.7 below.

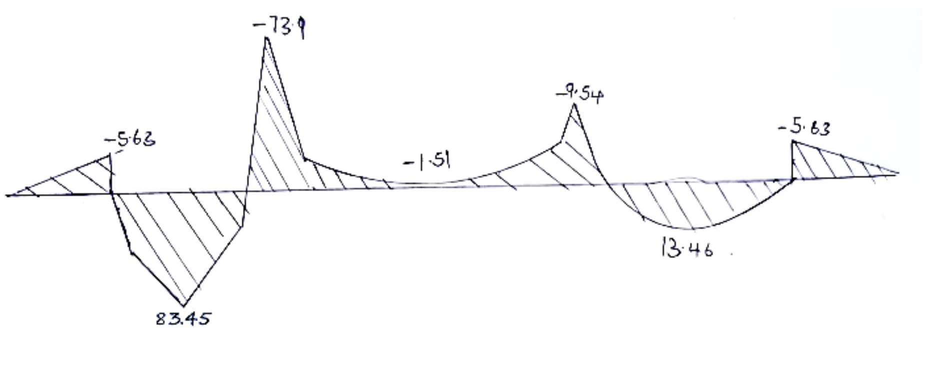


Figure 4.7: Non yielding support moment diagram

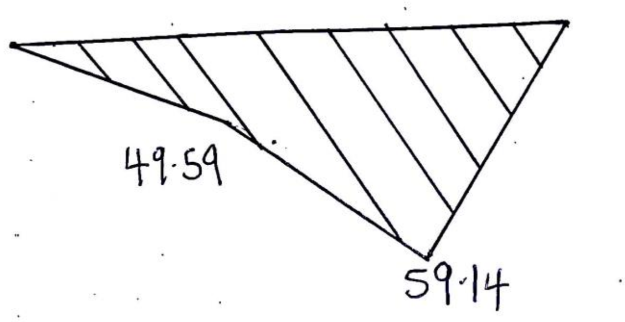


Figure 4.8: Yielding support moment diagram

4.5 Results from MATLAB GUI tool

The MATLAB GUI tool was executed using the same parameters applied in the manual computation (span = 25 m, deck width = 11 m, and slab thickness = 230 mm) to validate its reliability. The GUI automatically computed section properties, applied the HA + HB loading combination, and generated both tabular and graphical results such as support reactions, longitudinal bending moments, and influence coefficients. The various stages of the tool solution are shown below

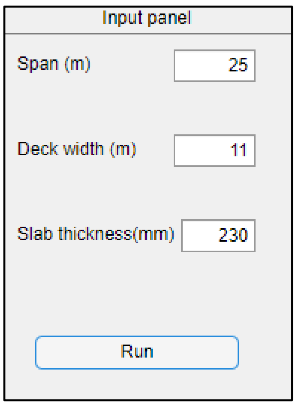


Figure 4.9: The input section showing the parameters being inputted

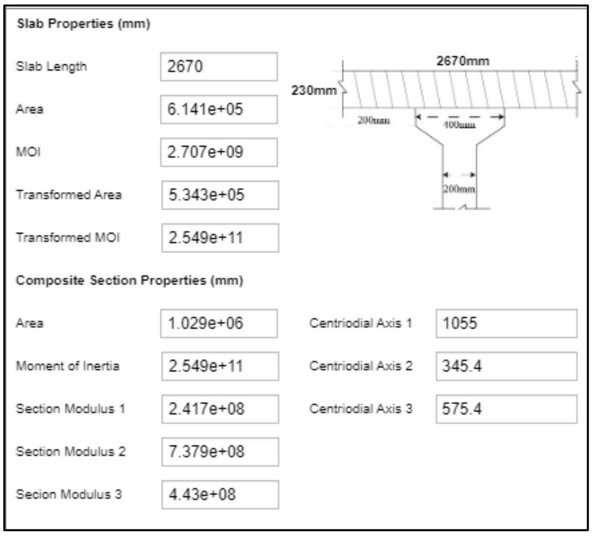


Figure 4.10: The output section showing the results from the slab properties and the composite section properties solutions.

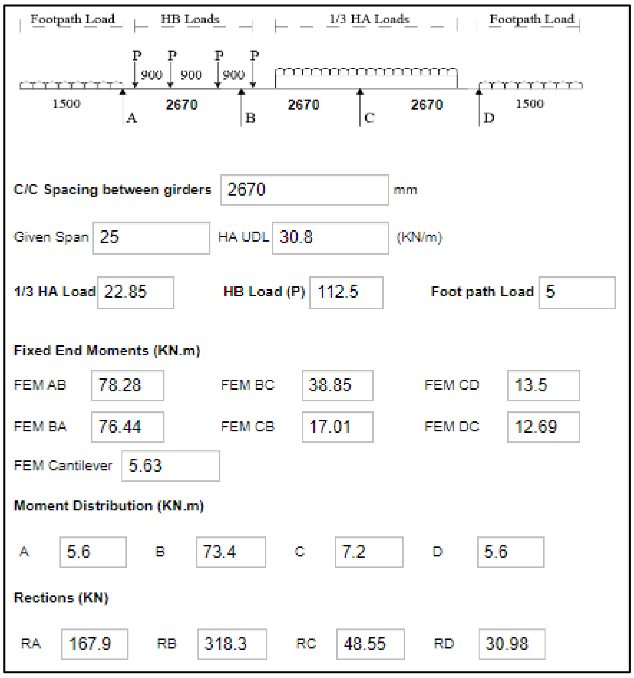


Figure 4.11: The output section showing the results from the loading, fixed end moments moment distribution and reactions

Longitudinal Moment (KN.m)

HA UDL	790.5
HA KEL	246.4
HB Loads	7902
Footpath Loads	770
Total Moments	9709

Girder Moments

Girder	A	B	C	D	Total
Reaction (KN)	167.9	318.3	48.55	30.98	565.8
Moment (KN.m)	2882	5462	833.2	531.6	9709

Figure 4.12: The output section showing the results from the longitudinal moments and girder moment distributed with the reaction.

Influence Coefficient

Load Location	A	B	C	D
A	0.74	0.36	0.06	-0.16
B	0.35	0.42	0.12	0.11
C	0.11	0.12	0.42	0.35
D	-0.16	0.06	0.36	0.74

Figure 4.13: The output section showing the results of the computed influence coefficient

Girder Distributed Moments (KN.m)

Girder	Free BM	A	B	C	D
A	2882	2132	1037	172.9	461.1
B	5462	1912	2294	655.5	600.9
C	833.2	91.65	99.98	349.9	291.6
D	531.6	85.06	31.9	191.4	393.4
Total	9709	4221	3464	1370	1747

Figure 4.14: The output section showing the results of the longitudinal distributed girder moments and the computed transverse moments.

Table 4.4: Free moment distributed to girders from the MATLAB tool

Girder	Free B.M KNm	A	B	C	D
A	2882	2132	1037	172.9	421.1
B	5462	1912	2294	655.5	600.9
C	833.2	91.65	99.98	349.9	291.6
D	531.6	85.06	31.9	191.4	393.4
Total KNm	9709	4221	3464	1370	1747

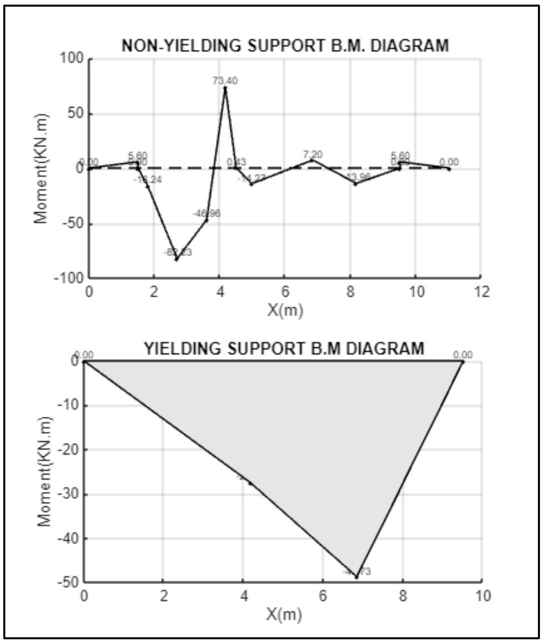


Figure 4.15: The output section showing the results of the plotted Non-yielding support and yielding support bending moment diagrams

4.6 Parametric Study

The developed MATLAB GUI enabled a development of a MATLAB script that allows for the systematic variation of the bridge parameters to examine their effect on the maximum bending moment after load distribution to each girder. Three principal parameters were studied: slab thickness, deck width and span length. Each parameter was adjusted within its defined range while keeping other parameters constant and the resulting maximum bending moments were recorded and plotted to visualize performance trends.

4.6.1 Influence of slab thickness

For this study, the span and deck width were kept constant at 25 m and 11 m respectively, while the slab thickness was varied from 200 mm to 300 mm.

Table 4.5: Results from Influence of slab thickness study

Thickness (mm)	A (KNm)	B (KNm)	C (KNm)	D (KNm)	Maximum B.M (KNm)
200	4112.5	3681.3	1078.3	1729.6	4112.5
210	4143.6	3622.9	1205	1761.4	4143.6
220	4174.7	3564.4	1331.8	1793.3	4174.7
230	4221	3463.5	1369.7	1747	4221
240	4221	3417.2	1416	1747	4221
250	4252.1	3350.4	1488.1	1778.8	4252.1
260	4252.1	3295.8	1479.8	1778.8	4252.1
270	4306.8	3249.5	1526.1	1787.1	4306.8
280	4283.2	3292	1614.8	1810.6	4283.2
290	4283.2	3237.4	1606.5	1810.6	4283.2
300	4337.9	3182.7	1598.2	1819	4337.9

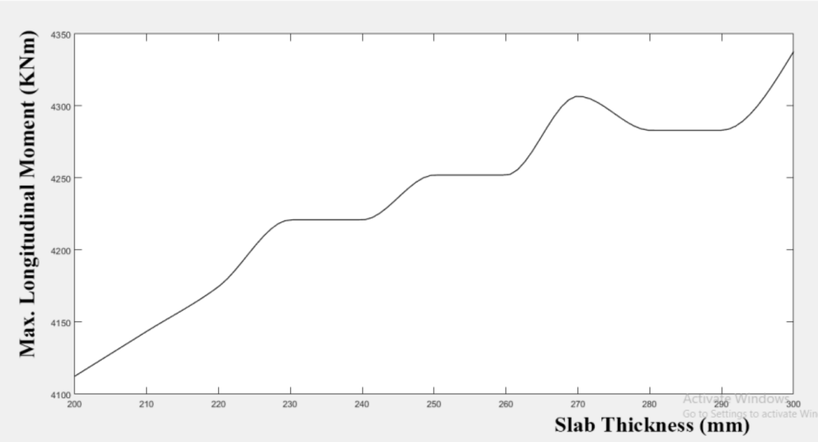


Figure 4.16: The graph of the slab thickness against the maximum longitudinal moments.

4.6.1 Influence of bridge span

In this study the slab thickness and deck width were maintained at 230 mm and 11 m, while the span length was varied from 22.5 m to 30 m.

Table 4.6: Results from Influence of span study

Span (m)	A (KNm)	B (KNm)	C (KNm)	D (KNm)	Maximum B.M (KNm)
22.5	3505.4	3137.9	919.1	1474.3	3505.4
23.3	3726	3257.8	1083.6	1583.9	3726
24.2	3975.8	3394.6	1268.3	1707.8	3975.8
25	4221	3463.5	1369.7	1747	4221
25.8	4423.6	3581.3	1484	1830.8	4423.6
26.7	4687.7	3693.6	1640.5	1961	4687.7
27.5	4895.1	3794.2	1703.5	2047.8	4895.1
28.3	5169.6	3900.5	1831.8	2145.2	5169.6
29.2	5379.9	4134.9	2028.3	2274.2	5379.9
30	5593.6	4227.8	2098	2364.5	5593.6

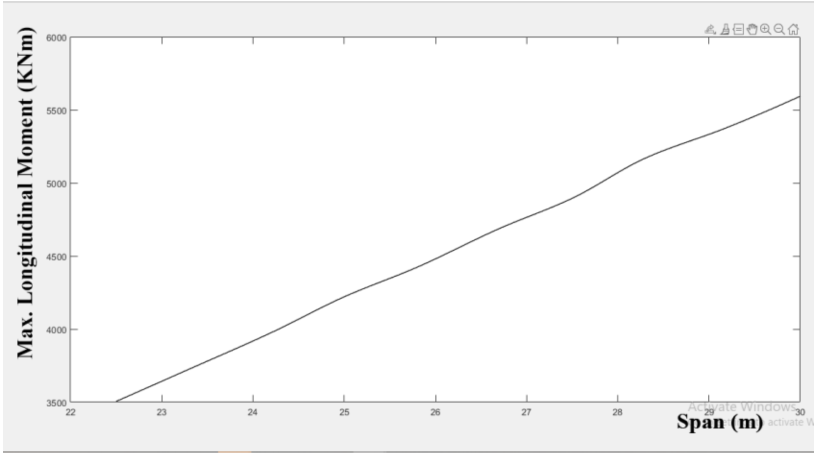


Figure 4.17: The graph of the span against the maximum longitudinal moments.

4.6.3 Influence of deck width

In this study the slab thickness and span were maintained at 230 mm and 25 m, while the deck width was varied from 11 m to 15 m.

Table 4.7: Results from influence of deck width study

Deck width (m)	A (KNm)	B (KNm)	B (KNm)	B (KNm)	Maximum B.M(KNm)
11	4221	3463.5	1369.7	1747	4221
11.4	4412.5	3279.6	1221.4	1856.1	4412.5
11.9	4642.6	3075.6	998.9	2022.7	4642.6
12.3	4756	2974.5	935.5	2064.6	4756
12.8	4884.9	2819.6	790.6	2190	4884.9
13.2	4988.4	2761.4	745.9	2227.8	4988.4
13.7	5092.8	2602.1	647.8	2265.8	5092.8
14.1	5148.7	2529.7	577.4	2326.1	5148.7
14.6	5237.2	2472.1	641.7	2315.1	5237.2
15	5287.2	2430.8	691.5	2371.5	5287.2

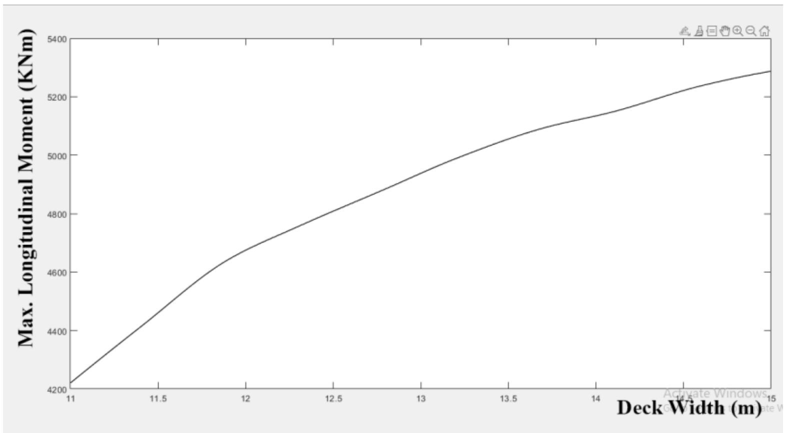


Figure 4.18: The graph of the deck width against the maximum longitudinal moments.

4.7 Verification of analysis using STAAD.Pro

This section presents the result of the maximum longitudinal bending moment generated from STAAD.Pro using the deck configuration where:

Span length = 25 m

Deck width = 11 m

Slab thickness = 230 mm

Girder spacing = 2.67 m

The bridge deck was modelled in STAAD.Pro using grillage analogy to replicate the conditions of the load distribution behaviour of the Leonhardt and Makowski method and to verify the validity of the result from the MATLAB tool and manual calculations.

The resulting analysis in STAAD.Pro indicates the maximum longitudinal moment as 4356.54 KN/m.

Table 4.9: Comparison of result table

Parameter	**MATLAB**	**Manual**	**STAAD.Pro**
Max. longitudinal moment (KNm)	4221	4221.1	4346.54

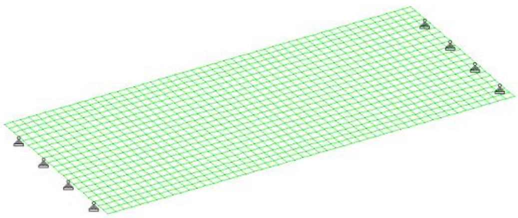


Figure 4.19: The bridge geometry in FE-model

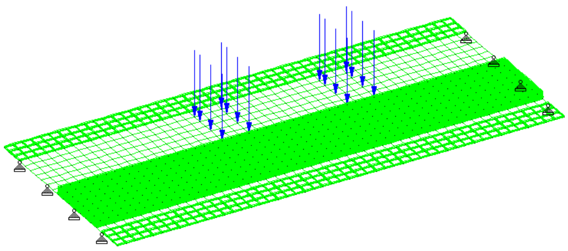


Figure 4.20: Bridge deck loading with the HB in blue, foot path and HA in green

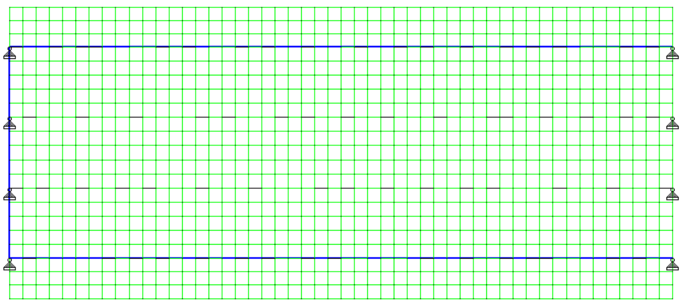


Figure 4.21: Bridge deck showing deck roadway

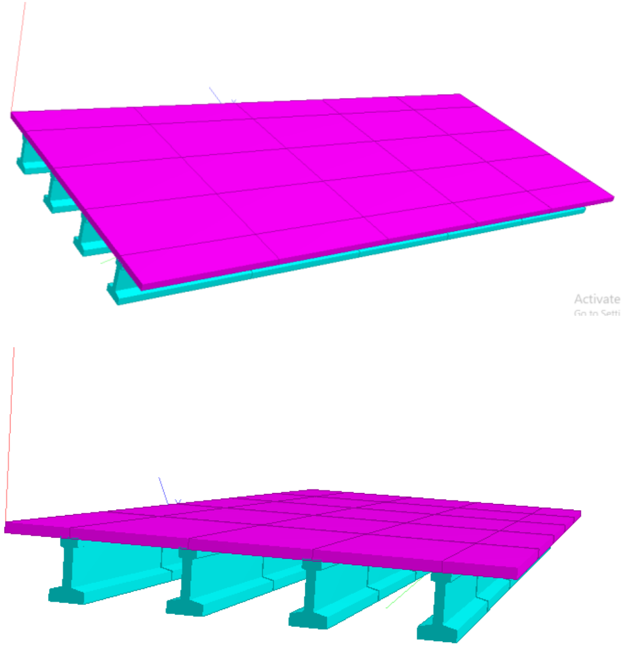


Figure 4.22: 3D Modelled bridge deck using STAAD.Pro

	Beam	L/C	Node	Fx kN	Fy kN	Fz kN	Mx kN-m	My kN-m	Mz kN-m
Max Fx	8	1 DEAD LOA	3	0.000	300.582	0.000	1.444	0.000	-11.087
Min Fx	8	1 DEAD LOA	3	**0.000**	300.582	0.000	1.444	0.000	-11.087
Max Fy	10	2 LIVE LOAD	5	0.000	**547.659**	0.000	14.876	0.000	-23.028
Min Fy	816	2 LIVE LOAD	718	0.000	**-574.581**	0.000	-17.902	0.000	-311.044
Max Fz	8	1 DEAD LOA	3	0.000	300.582	**0.000**	1.444	0.000	-11.087
Min Fz	8	1 DEAD LOA	3	0.000	300.582	**0.000**	1.444	0.000	-11.087
Max Mx	732	2 LIVE LOAD	676	0.000	354.562	0.000	**57.617**	0.000	-1777.906
Min Mx	804	2 LIVE LOAD	712	0.000	-394.712	0.000	**-66.184**	0.000	-1853.065
Max My	8	1 DEAD LOA	3	0.000	300.582	0.000	1.444	**0.000**	-11.087
Min My	8	1 DEAD LOA	3	0.000	300.582	0.000	1.444	**0.000**	-11.087
Max Mz	1115	2 LIVE LOAD	12	-0.000	-369.877	-0.000	-13.893	-0.000	**-8.607**
Min Mz	164	2 LIVE LOAD	186	-0.000	36.505	-0.000	14.675	-0.000	**-4356.543**

Figure 4.23: Summary of beam results from the bridge deck model in STAAD.Pro

CHAPTER FIVE

CONCLUSION AND RECOMMENDATIONS

5.1 Conclusion

This research examined the parametric optimization of reinforced concrete (RC) bridge decks using the Leonhardt and Makowski method implemented in a MATLAB-based Graphical User Interface (GUI). The tool was designed to automate manual computations, visualize structural behavior, and evaluate the effects of varying design parameters such as span length, deck width, and slab thickness under HA and HB loading conditions specified in BS 5400.

Manual analysis for a bridge deck of 25m span, 230mm deck slab thickness and 11m deck width was conducted and compared with the MATLAB GUI tool and a STAAD.Pro model of a bridge with same parameters to verify accuracy. The program achieved an excellent level of agreement with in terms of maximum longitudinal moment of 4221KN.m compared to 4221.1KN.m recording a 0.002% percentage difference. The results were then compared with the results of the STAAD.Pro model of 4346.54KN.m and the manual result of 4221.1KN.m recording a percentage difference of 2.97%. This confirmed that the mathematical formulations and structural assumptions of the Leonhardt and Makowski method can be correctly using MATLAB’s computational environment.

The parametric study revealed clear relationships between geometric parameters of slab thickness, span and deck width and maximum longitudinal moment of the bridge. It was observed that study showed that the maximum moment occurred at the support A for different parametric studies. Overall, the MATLAB GUI proved to be a reliable and user-friendly tool for both academic and professional use.

5.2 Recommendation

Although the MATLAB GUI developed in this study performed excellently when compared to manual calculation further improvements to this study include

1. Comparative testing with other load distribution methods and with finite element based tool

2. The tool can be expanded to accommodate variation of girder sizes and more parameters critical to the design of bridges

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APPENDIX

Workflow of the MATLAB tool

Figure B-1: Snapshots of the MATLAB GUI based tool work flow in order A – Input, B– Section properties output, C – Loads, Fixed end moments, moment distribution and reaction output, D – Longitudinal moments, E – Influence coefficient and distribution of girder moment, F – Distribution of girder moment and transverse moments and G – Plots of Non yielding support and yielding support moments