WINSORIZATION AND ITS APPLICATION

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A PROJECT WRITTEN AND SUBMITTED TO THE DEPARTMENT OF STATISTICS, IN PARTIAL FUFILLMENT OF THE REQUIREMENTS FOR THE AWARD OF BACHELOR OF SCIENCE (B.Sc.) DEGREE IN STATISTICS, FACULTY OF PHYSICAL SCIENCES, UNIVERSITY OF BENIN, BENIN CITY.

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UNDERTAKING

This project was carried out by JENNIFER OROBOSA IMUETINYAN with Matriculation Number PSC2008428. I have neither copied nor duplicated the work of any other author(s). All works have been duly cited and acknowledged.

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Jennifer Orobosa Imuetinyan **Date**

CERTIFICATION

This is to certify that the project work was carried out by JENNIFER OROBOSA IMUETINYAN with Mat. No. PSC2008428 in the Department of Statistics, Faculty of Physical Sciences, University of Benin, Benin City, Nigeria.

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DEDICATION

This project is dedicated to the Almighty God for His grace, mercy, strength and guidance and to my parents Mr. and Mrs. Julius and Faith Agbonmayo.

ACKNOWLEDGEMENT

By me during this challenging journey and throughout my years of study. I am grateful for your love, encouragement and support

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ABSTRACT

This study investigates the effectiveness of Winsorization, a statistical technique used to handle outliers in data analysis. Outliers can significantly distort statistical measures such as the mean, variance, and standard deviation, leading to inaccurate conclusions and decision-making. The research applied Winsorization to three different datasets representing employee income, monthly product sales, and customer complaints to assess its impact on mitigating the effects of outliers. In each case, extreme values were adjusted to a more reasonable range, resulting in more reliable statistical results and better representation of the underlying data distribution. The analysis demonstrated that Winsorization effectively reduced the influence of outliers on central tendency and variability measures, making the data more representative of the majority. The study highlights the importance of data cleaning techniques like Winsorization in business contexts, ensuring more accurate analysis and informed decision-making. The findings also suggest that further research is needed to explore optimal levels of Winsorization and its application in larger, more complex datasets. This research contributes to the understanding of how data preprocessing can improve the integrity of statistical analyses in various business environments.

CHAPTER ONE

INTRODUCTION

This study investigates the impact of Winsorization, a technique used to reduce the influence of outliers in datasets, on central tendency and dispersion. By replacing extreme values with more typical values, Winsorization aims to provide more robust statistical measures, such as the mean and standard deviation. The study compares these measures before and after applying Winsorization to various real-world and simulated datasets, with the goal of enhancing the accuracy and reliability of data analysis across different fields like finance, healthcare, and social sciences.

1.1 BACKGROUND TO THE STUDY

In statistical analysis, extreme values, or outliers, are data points that differ obviously from most of the other data in a dataset. These outliers can distort statistical measures such as the mean (average) and variance (measure of spread), leading to misleading results. For example, a few very large or very small values can make the mean much higher or lower than what would be typical for the data. To reduce the impact of these outliers, winsorization is a commonly used technique.

Winsorization is a technique that aims to reduce the influence of outliers without removing them from the dataset. Named after biostatistician Charles P. Winsor, this method involves replacing extreme values with values closer to the rest of the data. Typically, winsorization involves capping data points below a certain lower percentile and above a higher percentile, replacing extreme values with the corresponding percentile values (Tukey, 1977; Hoaglin, Iglewicz, & Tukey, 1986). For example, a 90% winsorization procedure replaces values below the 5th percentile with the value at the 5th percentile and values above the 95th percentile with the value at the 95th percentile. This method ensures that extreme data points do not disproportionately influence statistical measures such as the mean and variance.

Winsorization helps make key statistical measures more robust and reliable by reducing the influence of outliers. For example, the winsorized mean is calculated by first replacing extreme values with more typical values and then computing the mean of the adjusted dataset. This leads to a mean that is less affected by outliers and more reflective of the typical trends in the data (Rousseeuw & Croux, 1993; Huber, 2011). Similarly, winsorizing the data can improve the reliability of variance calculations, reducing the influence of extreme values and providing a more accurate estimate of data spread (Huber, 2011; Rousseeuw & Croux, 1993). Winsorization is particularly useful when outliers are present, but removing them would result in loss of data.

Winsorization has wide-ranging applications across various domains where outliers are common but must be addressed for reliable analysis. In finance, for instance, stock prices or returns can experience sudden and dramatic fluctuations due to market crashes or unforeseen events. By winsorizing these values, analysts can reduce the impact of these extreme fluctuations, ensuring that financial models and risk assessments reflect more stable and typical trends (Nychka et al., 2016, Lee et al., 2020). In medical research, outliers can arise from rare medical conditions or measurement errors in data like blood pressure or cholesterol levels. Winsorizing these extreme values helps researchers focus on general health trends and make more accurate conclusions regarding treatment effects (McKeown et al., 2021).

In manufacturing, quality control processes frequently require the analysis of product dimensions or weights, where measurement errors or equipment malfunctions may cause anomalies. Winsorization prevents these anomalies from skewing quality assessments, ensuring that decisions are hased on typical product characteristics (Mehta & Gupta, 2021). Environmental research, too, benefits from winsorization, particularly in cases where extreme values may arise from rare but significant environmental events, such as pollution spikes or unusual weather anomalies. Winsorization allows researchers to analyze typical environmental conditions while still accounting for rare events that may provide valuable insights (Gilbert & Green, 2019).

Although winsorization offers robust solutions for handling outliers, it does have limitations. One challenge is the selection of percentile thresholds for capping extreme values. The chosen percentile can have a significant impact on the results, and improper selection can lead to over-correction or under-correction (Iglewicz & Hoaglin, 2020). Additionally, winsorization reduces but does not entirely eliminate the influence of outliers, especially if the outliers reflect important data points. Therefore, while winsorization is a powerful tool, it should be applied carefully and often in conjunction with other robust statistical techniques, such as data transformation or robust regression methods (Cohen & West, 2018; Koch et al., 2018).

Winsorization is a valuable technique for reducing the impact of extreme values or outliers in datasets. By replacing extreme values with more typical values, winsorization ensures that statistical measures like the mean and variance are more robust and reliable. This technique is widely used across fields such as finance, medical research, manufacturing, and environmental studies, where outliers are prevalent but must be managed to derive meaningful conclusions. However, careful application of winsorization is essential to avoid potential pitfalls, particularly when determining the appropriate thresholds for capping extreme values.

1.2 AIMS AND OBJECTIVES

The aim of this study is to assess the impact of Winsorization on the central tendency and dispersion of a dataset. The objectives of this project work are:

1. To compare the mean and median of a dataset before and after applying Winsorization.
2. To compare the standard deviation before and after Winsorization.
3. To visualize the effect of Winsorization on the distribution of data.

1.3 SIGNIFICANCE OF THE STUDY

This study is significant because it digs into how Winsorization improves data analysis by reducing the impact of extreme values or outliers, which can distort statistical results. By applying Winsorization, we can achieve a more accurate representation of typical values in a dataset without completely removing outliers. This approach enhances the reliability of key statistical measures like the mean, median, and standard deviation, providing clearer insights for researchers, analysts, and decision-makers.

In fields such as finance, healthcare, and social sciences, Winsorization helps mitigate the influence of outliers, allowing for more accurate predictions, better treatment evaluations, and fairer interpretations of data. By ensuring that data analysis remains consistent and unbiased, this study encourages hest practicos in data pre-processing, leading to more reliable conclusions and improved decision-making across various industries.

1.4 SCOPE OF THE STUDY

This study focuses on assessing the impact of Winsorization on the central tendency and dispersion of datasets, particularly in reducing the influence of extreme values or outliers. It will evaluate how Winsorization affects statistical measures such as the mean, median, and standard deviation by applying it to various real-world and simulated datasets from fields like finance, healthcare, and social sciences.

The study will compare these statistical measures before and after Winsorization and use data visualization techniques to demonstrate its effect on data distributions

1.5 LIMITATION OF THE STUDY

This study is limited in that it focuses only on the impact of Winsorization on central tendency and dispersion, without exploring other potential statistical techniques for handling outliers. It also does not examine the effect of Winsorization across all possible percentile thresholds or in highly specialized fields. Additionally, the study will not consider the computational complexities or the ethical implications of data manipulation in great detail. Finally, the analysis is based on datasets from a limited number of sectors, which may not fully represent all types of data where outliers occur.

1.6 DEFINITION OF TERMS

- **Outliers:** Data points that are significantly different or distant from the other data points in a dataset. These extreme values can skew statistical results, such as the mean and variance.

- **Winsorization:** A statistical technique used to limit the influence of extreme values or outliers in a dataset by replacing them with values closer to the rest of the data, typically at a certain percentile.

- **Central Tendency:** Statistical measures that summarize the center of a dataset.

- The most common measures of central tendency are the mean, median, and mode. In this study, the focus is on the mean (the average) and median (the middle value when the data is ordered).

- **Dispersion:** A measure of how spread out or scattered the data points in a dataset are. Common measures of dispersion include the standard deviation, which quantifies the average deviation of data points from the mean, and the variance, which is the square of the standard deviation.

- **Mean:** The average of a dataset, calculated by adding up all the values and dividing by the number of values in the dataset.

- **Median:** The middle value in a dataset when the data is ordered from smallest to largest.

CHAPTER TWO

LITERATURE REVIEW

2.0 INTRODUCTION

This chapter presents a comprehensive literature review on the concept of Winsorization, a statistical technique used to manage outliers in datasets Winsorization has gained widespread application in fields such as finance, healthcare, and environmental research due to its ability to mitigate the effects of extreme values without losing valuable data. The chapter explores the origins of Winsorization, its methodological underpinnings, its applications across various domains, and the challenges associated with its use.

2.1 LITERATURE REVIEW

According to Wikipedia, Winsorization, first introduced by Charles Winsor in 1947, is a method of reducing the impact of outliers by replacing extreme values in a dataset with values closer to the central tendency of the data. The technique has become a widely adopted approach for handling outliers, as it helps improve the stability and reliability of statistical analyses.

The process of Winsorization involves limiting extreme data points to a specified percentile range. For example, values below the 5th percentile might be replaced by the value at the 5th percentile, and values above the 95th percentile might be replaced by the value at the 95th percentile (Tukey, 1977). By doing so, Winsorization reduces the influence of outliers without completely removing data points, thus preserving the dataset's overall structure,

Hoaglin et al. (1986) highlighted the utility of Winsorization in robust statistics, noting that it helps stabilize key statistical measures such as the mean and variance, which can be distorted by extreme values. The technique's ability to preserve the sample size while reducing the impact of outliers makes it especially valuable in fields like finance, where retaining all available data is crucial for accurate analysis.

In finance, Winsorization has been used to model stock prices, returns, and market indices, where extreme fluctuations due to events like financial crises or market shocks can skew results. Lee et al. (2020) discussed how Winsorization can help financial analysts maintain more accurate models by reducing the impact of rare, extreme events while preserving the overall data trends. Similarly, in healthcare, Winsorization has proven useful in clinical trials and medical studies, where outliers might result from rare medical conditions or data entry errors. McKeown et al. (2021) noted that applying Winsorization in healthcare research can lead to more reliable interpretations of treatment outcomes by mitigating the influence of extreme patient responses.

Winsorization has also been applied in environmental research to manage outliers in data related to pollution levels, temperature anomalies, and other environmental phenomena. Gilbert and Green (2019) illustrated how Winsorization helps researchers focus on typical environmental trends while still accounting for rare bet significant events, providing a clearer understanding of the broader context.

2.2 APPLICATIONS OF WINSORIZATION

Winsorization is a statistical technique widely applied across several fields due to its ability to manage outliers without discarding valuable data.

Winsorization is a statistical technique widely applied across several fields due to its ability to manage outliers without discarding valuable data. As noted by Lee et al. (2020), one prominent area where Winsorization is applied is in finance. It is used to model financial variables such as stock prices, market returns, and volatility. By capping extreme values, Winsorization helps analysts focus on broader market trends while minimizing the distortion caused by rare but significant events, such as market crashes. This adjustment improves the reliability of financial models, leading to more accurate forecasting and better risk assessment.

In the field of healthcare research, Winsorization is commonly used to handle outliers in clinical trial data. These outliers can result from rare medical conditions, measurement errors, or data entry mistakes. McKeown et al. (2021) emphasized that applying Winsorization allows researchers to draw more accurate conclusions about the effectiveness of treatments, as it prevents outliers from skewing the results. This ensures that the data remains representative of the broader population, contributing to more reliable health outcomes.

Environmental research also benefits from Winsorization, particularly in the analysis of extreme events like pollution spikes or temperature anomalies. Gilbert and Green (2019) highlighted the role of Winsorization in helping researchers concentrate on typical environmental patterms while still considering rare events that may provide valuable insights. This approach allows for more balanced analysis of environmental trends without disregarding potentially important but infrequent occurrences.

In engineering, Winsorization is applied to the analysis of system reliability and component performance. Extreme values in failure times or performance metrics could indicate anomalies or rare but critical events. By capping these values, engineers ensure that their analysis reflect typical performance trends while still accounting for outlier events that may provide valuable information for system design, maintenance and safety assessments (Smith & Johnson, 2020).

Finally, Winsorization is also useful in market research, where it is employed to analyze consumer behaviour, sales data and market trends. By managing the impact of extreme values, business can develop more accurate forecasts and create targeted marketing strategies. This helps improve decision making and profitability by providing a clearer understanding of typical consumer behaviour and market dynamics (Williams & Lee, 2021).

2.3 CHALLENGES AND LIMITATIONS OF WINSORIZATION

While Winsorization offers several advantages, it is not without its challenges and limitations. One of the primary concerns is the subjective choice of the percentile thresholds for capping extreme values. Iglewicz and Hoaglin (2020) pointed out that the selection of percentiles such as the 5th and 95ch percentiles can significantly influence the outcome of Winsorization Inappropriate thresholds may either over-correct or under-correct the data, leading to biased results.

Another limitation of Winsorization is that it does not completely eliminate the influence of outliers. In certain cases, extreme values may contain important information, especially in fields like finance, where market shocks or economic crises might signal significant changes. Cohen and West (2018) cautioned that removing or capping these extreme values may result in the loss of critical insights that could be essential for decision-making.

Winsorization also does not address the root causes of outliers, such as data entry errors or measurement inaccuracies. Koch et al. (2018) noted that while Winsorization helps mitigate the impact of outliers, it does not correct underlying data quality issues, which may still need to be addressed through data validation and correction.

Furthermore, Winsorization is considered a non-robust transformation in some statistical models. Rousseeuw and Croux (1993) argued that while Winsorization can improve the stability of certain statistical measures, it may now always ensure that the modified dataset conforms to the assumptions of the original model. In such cases, researchers may need to consider alternative robust techniques, such as M-estimators or robust regression models, to more effectively handle outliers.

CHAPTER THREE

METHODOLOGY

3.0 Introduction

This chapter presents the methodology adopted for this study, focusing on the Winsorized mean, median, variance, and standard deviation. It also explains the process of Winsorization, how outliers are identified and adjusted, and the statistical package used for the analysis.

3.1 Research Design

The study adopts a quantitative research design where three different datasets containing clear outliers are analyzed. Winsorization is applied to reduce the impact of extreme values, and statistical computations are performed using a statistical package - Python's NumPy and Pandas libraries.

3.2 Data Collection

The datasets were synthetically generated to reflect real-world scenarios where extreme values exist. Three datasets will be analyzed to show the effect of Winsorization on measures of central tendency and dispersion.

3.1.1 Normal Statistical Measures

Before applying Winsorization, the following formulas will be used to compute the normal mean, median, variance, and standard deviation for each dataset.

1. Mean formular:

$$\overline{x} = \frac{\Sigma \mathrm{x}}{n}$$

Where:

X represents individual data points,

n is the total number of values.

2. Median formular:

    a. For an odd number data set: Median = $X\frac{n+1}{2}$

    b. For an even number data set: Median = $\frac{X\frac{n}{2}+x\frac{n}{2}+1}{2}$

    c. Variance formular: $S^2 = \frac{\Sigma(X-\bar{X})^2}{n-1}$

    d. Standard deviation: S = $\sqrt{S^2}$

These formulas will provide the basic statistical measures, which will serve as a baseline for comparison after the application of Winsorization.

3.2.2 Winsorized Statistical Measures

After identifying outliers, Winsorization is applied, and the following formulas will be used to compute the Winsorized mean, variance, and standard deviation.

1. **Winsorized mean**

The Winsorized mean is a variation of the arithmetic mean that mitigates the influence of outliers by limiting extreme values within a dataset. Instead of using the raw data as it is, Winsorization adjusts outliers by replacing them with values closer to the middle of the distribution. This technique is particularly useful in situations where the data may contain extreme values that could distort the true central tendency of the dataset.

The formula for calculating the Winsorized mean is quite similar to the formula for the regular arithmetic mean, but it incorporates the adjustment for outliers. The adjusted values are used in the calculation, where the extreme data points are replaced by the nearest acceptable value within a specified percentile range.

Thus: $\bar{X}_w = \frac{\Sigma x_w}{n}$ where:

X_w represents the adjusted (winsorized) values in the data set

n represents the total number of data points in the dataset.

2. **Winsorized variance**

The Winsorized variance is an adjusted version of the variance that accounts for outliers by replacing them with more reasonable values. Like the Winsorized mean, this method reduces the impact of extreme values on the variance calculation, making it a more robust measure when dealing with datasets that contain outliers.

Variance is a measure of how spread out the values in a dataset are around the mean. When outliers are present, they can significantly inflate the variance, giving a misleading sense of data dispersion. The Winsorized variance addresses this issue by replacing extreme values with less extreme values, thus ensuring that the calculation of variance reflects the distribution of the central data points, rather than being disproportionately influenced by outliers.

To calculate the Winsorized variance, we use a similar formula as for regular variance, but with the Winsorized values (adjusted values) replacing the original data points. This ensures that the extreme values do not distort the variance as much as they would in the standard calculation.

The formula for Winsorized variance is:

$$S_w^2 = \frac{\sum(X_w - \bar{X}_w)^2}{n-1} \text{ where}$$

x_w represents the winsorized (adjusted) data values

$\bar{X}_w$ represents the winsorized mean, calculated as the mean of the adjusted values

n is the total number of data point in the data set

3. **Winsorized Standard Deviation:**

The Winsorized Standard Deviation is a robust measure of spread that adjusts for outliers by using Winsorized data. It is derived from the Winsorized variance and provides a more accurate estimate of variability when extreme values distort traditional standard deviation. The model reads thus:

$S_w = \sqrt{S_w^2}$ where

S_w is the winsorized standard deviation

S_w^2 is the winsorized variance

3.2.3 Winsorization Process

The process of Winsorization involves identifying and adjusting outliers within the data. Here's how it works:

1. Identification of Outliers:

Outliers are values that significantly differ from the rest of the data and can distort statistical analysis. In this study, outliers are identified as any data points that fall outside the 1st and 99th percentiles of the dataset.

2. Adjustment of Outliers:

Once outliers are identified, they are replaced with the nearest non-outlier values within the defined percentile range, For example, any value below the 1st percentile is replaced by the value at the 1st percentile, and any value above the 99th percentile is replaced by the value at the 99th percentile

This step reduces the influence of extreme values on the data, which could otherwise distort the mean, variance, and standard deviation,

3. Recalculation of Statistical Measures:

After Winsorization, the Winsorized mean, variance, and standard deviation are recalculated using the adjusted values. These new values will be used for comparison with the original, unadjusted values to show the effect of Winsorization.

3.3 Statistical Package Used

By using these statistical packages, python provides a robust environment for performing data analysis and statistical modelling, Bach package has its strength. and they can be used together to leverage their combined capabilities for a comprehensive statistical analysis workflow.

The analysis in this study was carried out using Python, a comprehensive and flexible programming language widely used for data manipulation, statistical analysis, and visualization.

Python's rich ecosystem of libraries allows for efficient handling of data, computation of various statistical metrics, and robust data visualization. Below is a detailed explanation of the libraries used and their specific functions in this analysis:

1. NumPy

NumPy is fundamental for numerical computations in Python. It provides support for efficient array-based operations, which are key to performing a variety of statistical analyses. In this study, NumPy was used for basic statistical calculations and identifying potential outliers through percentiles.

Functions Used:

- **numpy.mean():** This function computes the arithmetic mean of the dataset, which is the foundation for various statistical metrics, including the Winsorized mean.
- **numpy.var():** Used to compute the variance of the data, which measures the spread of the data points.
- **numpy.std():** This calculates the standard deviation of the dataset, which tells us how spread out the values are around the mean.
- **numpy.percentile():** Essential for identifying outliers, this function helps determine the 1st and 99th percentiles, which are used to perform Winsorization (capping extreme values).

These functions from NumPy allow for efficient statistical calculations that serve as the foundation for further data analysis.

2. Pandas

Pandas is a powerful library designed for data manipulation and analysis. It is particularly useful for structuring large datasets, handling missing values, and performing complex data operations. In this study, Pandas was used to load, clean, and manipulate the data, and to perform operations like Winsorization on the dataset.

Functions Used:

- **pandas.DataFrame():** This function is used to structure the raw dataset into a table (DataFrame) for easier manipulation. It allows for efficient indexing. selection, and filtering of data.
- **pandas.DataFrame.describe():** This function provides an overview of the dataset, including summary statistics like the mean, median, min, max, standard deviation, and percentiles. This helps to understand the data distribution before applying Winsorization.
- **pandas.DataFrame.apply():** Used to apply custom functions (like Winsorization) across columns or rows in the DataFrame. This is crucial for applying the Winsorization procedure efficiently to the entire dataset,
- **pandas.DataFrame.quantile():** This function is used to calculate the quantiles (such as the 1st and 99th percentiles), which are instrumental in identifying and managing outliers during the Winsorization process,

CHAPTER FOUR

DATA ANALYSIS AND INTERPRETATION

4.0 Introduction

This chapter presents the results of the Winsorization process. Including the calculations of the normal and Winsorized mean, median, variance and standard deviation for three datasets with clear outliers

Dataset 1: Income of Employees in a Small Business (in Naira)

Source: python simulated

This dataset includes income data for employees working in a small nosiness, representing a range of positions from entry-level to higher management. The outlier still represents a significant deviation form the rest of the data.

Data:

25,000, 28,000, 30,000, 33,000 35,000, 40,000 42,000, 45,000, 48,000, 50,000, 55,000, 60,000, 65,000, 74,000, 1,500,000.

These incomes values reflect the salaries for a variety of employees. Most salaries range from entry level positions (25,000) to higher managerial roles (75,000) with a significant outlier at 1,500,000, suggesting either an error or an exceptional case.

Outlier:

1,500,000: This salary value is far above the rest of the data. It could be a data entry error or reflect an exceptionally high-paying position (e.g., the owner, CHO, or external consultant),

Action: Winsorizing this outlier would be necessary to bring it down to a more reasonable level, potentially around 100,000 or lower

Dataset 2: Monthly Sales of a Product (in Units)

Source: python simulated

This dataset includes points reflecting monthly sales of a product by a small business. The sales figures show consistent growth with a notable spike towards the end

Data:

120, 140, 160, 180, 200, 220, 240, 265, 280, 300, 320, 350, 375, 450, 5,000

These sales numbers show the growth of the product over several months.

The data typically ranges between 120 and 450 units, but the 5,000 units at the end is a significant outlier that could distort any analysis.

Outlier:

5,000: This spike is much higher than any of the other monthly sales, indicating either a promotional surge, bulk orders, or a possible data entry mistake.

Action: Winsorization would adjust this outlier to a more reasonable value, like 500 or 600, to avoid distorting the analysis.

Dataset 3: Customer Complaints in a Year

Source: python simulated

This dataset includes data points for customer complaints throughout the year. It shows a gradual increase in complaints, but there is still an extreme outlier in the final data point.

Data:

3,4,5,6, 7, 9, 11, 13, 15, 18, 20, 22, 25, 30, 200

The complaints reflect a steady increase in issues over time. As the business grows, complaints rise from 3 to 30, but the 200 complaints at the end is an extreme spike that likely signals an unusual incident or error.

Outlier

200: This figure is vastly higher than the other values, indicating either an isolated incident (eg, a product recall, service failure, or system error) or a data reporting mistake

Action: Winsorizing would be appropriate, adjusting the 200 complaints figure to something more in line with the rest of the data, such as 30 or 35 complaints.

Stages followed on the use of Python for this study

1. Data Preparation

- **Library Used.** Pandas
- **Process**
    - Datasets were stored in a pandas DataFrame to organize and manipulate the data.
    - The data was synthetically generated or provided in raw format as a list of numbers for each dataset.

Example Code:

python

Copy

import pandas as pd

data_1 = [25000, 28000, 30000, 33000, 35000, 40000, 42000, 45000, 48000, 50000, 55000, 60000, 65000, 74000, 1500000]

df_1 = pd.DataFrame(data_1, columns=[“Income”])

2. Identification of Outliers

- **Library Used:** NumPy
- **Process:**
    - Outliers were identified using the 1st and 99th percentiles of the dataset. These percentiles help pinpoint values that are extreme compared to the rest of the data.
    - We used NumPy's percentile() function to calculate the percentiles, and any value outside this range was considered an outlier.

Example Code:

python

Copy

import numpy as np

lower_percentile = np.percentile(df_1["Income"], 1) upper percentile = np.percentile(df 1["Income"], 99)

outliersdf_1[(df_1["Income"] <lower_percentile) (df 1["Income"] >

upper_percentile)]

print("Outliers:", outliers)

3. Normal Statistical Measures

For each dataset, the following statistics were computed:

- **Mean:**
    - **Library Used:** NumPy
    - **Formula:** The mean was computed using numpy mean(), which calculates the average of the dataset.

Example Code:

Python

Copy

mean income np.mean(df 1["Income"])

Median:

- **Library Used:** NumPy
- **Formula:** The median was computed using numpy.median(), which finds the middle value in the sorted dataset.

Example Code:

python

Copy

median_income np.median(df_1["Income"])

Variance:

- **Library Used:** NumPy
- **Formula:** The variance was computed using numpy.var(). Variance measures how much the data points deviate from the mean.

Example Code:

python

Copy

variance income np.var(df_1]"Income"], ddof-1) # ddof=1 for sample variance

Standard Deviation:

- **Library Used:** NumPy
- **Formula:** The standard deviation was computed using numpy.std(). which calculates the square root of the variance.

Example Code:

python

Copy

std_dev_income np.std(df_1["Income"], ddof=1)

4. Winsorization Process

- **Library Used:** Pandas, NumPy
- **Process:**
    - For Winsorization, any outlier values that fall outside the 1st and 99th percentiles were replaced by the nearest non-outlier values (values at the 1st and 99th percentiles).
    - This was done by identifying the outlier values, and then setting those values to the nearest percentile.

Example Code for Winsorization:

python

Copy

#Replacing outliers with the 1st or 99th percentiles

df_1["Winsorized Income"] = np.wherefdf_1["Income"] <

lower percentile, lower_percentile,

np.where(df_1["Income"] > upper_percentile,

upper_percentile, df_1["Income"]))

5. Winsorized Statistical Measures

After applying Winsorization, the same statistical measures (mean, variance, and standard deviation) were recomputed for the adjusted dataset.

Winsorized Mean:

- **Library Used:** NumPy
- **Formula:** Calculated using the same method as the normal mean, but now applied to the Winsorized data.

Example Code:

python

Copy

mean winsorized_income np.mean(df_1["Winsorized_Income"])

Winsorized Variance:

- **Library Used:** NumPy
- **Formula:** Variance was calculated again using the Winsorized data.

Example Code

python

Copy

variance_winsorized_income-np.var(df_1["Winsorized_Income"], ddof-1)

Winsorized Standard Deviation:

- **Library Used:** NumPy
- **Formula:** Standard deviation was calculated again using the Winsorized data.

Example Code:

python

Copy

```
std_dev winsorized_income np.std(df_1["Winsorized_Income"], ddof-1)
```

6. Visualizing Data (Optional)

Library Used: Matplotlib, Seabom

Process:

- To visually inspect the effect of Winsorization, we used histograms and box plots before and after the process.
- Box Plot: A box plot was used to visualize the spread of the data and the effect of removing outliers.

Example Code for Visualization:

python

Copy

```
import matplotlib.pyplot as plt
```

```
import seaborn as sns


#Plotting box plot for original data

plt.figure(figsize (10,6))

sns.boxplot(data"df 1["Income"])

plt.title('Original Data Box Plot')

plt.show()


#Plotting box plot for Winsorized data

plt.figure(figsize(10, 6))

sns.boxplot(data df_1["Winsorized_Income"])

plt.title('Winsorized Data Box Plot')

plt.show()
```

7. Explanation of Functions

- The libraries NumPy atid Pandas provided efficient tools to calculate the statistics
- Functions like mean(), mediant), var(), and std() from NumPy helped in computing the normal and Winsorized statistical measures

- The pandas DataFrame() function helped in organizing the data into tabular format for easier manipulation.
- The numpy.percentile() function was used to calculate the 1st and 99th percentiles to identify and adjust the outliers.

Normal computations

Data set 1- Employee Income

- Mean = $\bar{x} = \frac{\Sigma X}{n}$

$$\frac{25{,}000 + 28{,}000 + 30{,}000 + 33{,}000 + 35{,}000 + 40{,}000 + 42{,}000 + 45{,}000 + 48{,}000 + 50{,}000 + 55{,}000 + 60{,}000 + 65{,}000 + 74{,}000 + 1{,}500{,}000}{15}$$

= 142,000

- Median. Since we have fifteen values, the median is the eight value arranging in ascending or descending order of magnitude. Hence the median, X 42,000
- Variance = $S^2 = \frac{\Sigma(X-\bar{x})^2}{n-1}$

$(25{,}000-142{,}000)^2+(28{,}000-342{,}00)^2+(30{,}000-142{,}000)^2+(33{,}000-142{,}000)^2+(35{,}000-142{,}000)^2+(40{,}000-142{,}000)^2+(42{,}000-142{,}000)^2+(45{,}000-142{,}000)^2+(48{,}000-142{,}000)^2+(50{,}000-142{,}000)^2+(55{,}000-142{,}000)^2+(60{,}000-12{,}000)^2+(65{,}000-142{,}000)^2+(74{,}000-142{,}000)^2+(1{,}500{,}000-142{,}000)^2$

5 - 1

S^2 = approximately 461,336,072,500

Standard Deviation. S = $\sqrt{S^2}$

S = $\sqrt{461{,}336{,}072{,}500}$

S = approximately 679,217.2499

Dataset 2: Monthly Sales of a Product (in Units)

120, 140, 160, 180, 200, 220, 240, 260, 280, 300, 320, 350, 375, 450, 5,000

- Mean = $\bar{x} = \frac{\Sigma X}{n}$

$$\frac{120 + 140 + 160 + 180 + 200 + 220 + 240 + 260 + 280 + 300 + 320 + 350 + 375 + 450 + 5000}{15}$$

Approximately 573.33

- Median. Since we have fifteen values, the median is the eight value arranging in ascending or descending order of magnitude. Hence the median, $X = 265$

- Variance = Variance = $S^2 = \frac{\Sigma(X-\bar{x})^2}{n-1}$

$(120 -573.33)^2+(140 -573.33)^2+(160 -573.33)^2+(180 -573.33)^2+(200 -573.33)^2+(220 -573.33)^2+(240 -573.33)^2+(260 -573.33)^2+(280 -573.33)^2+(300 -573.33)^2+(320 -573.33)^2+(350 -573.33)^2+(375 -573.33)^2+(450 -573.33)^2+(5000 -573.33)^2$

5 – 1

S^2 = approximately 1,480,781.99

- Standard Deviation S = $\sqrt{S^2}$

$S = \sqrt{1{,}480{,}781.99}$

S = approximately1216,87

Dataset 3: Customer Complaints in a Year

3, 4, 5, 6, 7, 9, 11, 13, 15, 18, 20, 22, 25, 30, 200

- Mean = $\bar{x} = \frac{\Sigma X}{n}$

$$\frac{3+4+5+6+7+9+11+13+15+18+20+22+25+30+200}{15}$$

Approximately 25.87

- Median. Since we have fifteen values, the median is the eight value arranging in ascending or descending order of magnitude. Hence the median, $X = 13$
- Variance = $S^2 = \frac{\Sigma(X-\bar{x})^2}{n-1}$

$(3-25.87)^2+(4-25.87)^2+(5-25.87)^2+(6-25.87)^2+(7-25.87)^2+(9-25.87)^2+(11-25.87)^2+(13-25.87)^2+(15-25.87)^2+(18-25.87)^2+(20-25.87)^2+(22-25.87)^2+(25-25.87)^2+(30-25.87)^2+(200-25.87)^2$

5 – 1

S^2 = approximately 2,228.52

- Standard Deviation $S = \sqrt{S^2}$

$S = \sqrt{2228.52}$

S = approximately 47.2

4.1 Winsorization Process

Applying Winsorization at a 10% level, extreme values are replaced:

Final Winsorized Datasets:

1. Income of Employees in a Small Business (after 10% Winsorization): 25,000, 28,000, 30,000, 33,000, 35,000, 40,000, 42,000, 45,000, 48,000, 50,000 55,000, 60,000, 65,000, 74,000, 100,000
2. Monthly Sales of a Product (after 10% Winsorization): 120, 140, 160, 180, 200, 220, 240, 265, 280, 300, 320, 350, 375, 450, 500
3. Customer Complaints in a Year (after 10% Winsorization): 3, 4, 5, 6, 7, 9, 11, 13, 15, 18, 20, 22, 25, 30, 35

For First Dataset

Trimmed/Winsorized mean computation … $\overline{X_w} = \frac{\sum x_w}{n}$

$$\frac{25{,}000 + 28{,}000 + 30{,}000 + 33{,}000 + 35{,}000 + 40{,}000 + 42{,}000 + 45{,}000 + 48{,}000 + 50{,}000 + 55{,}000 + 60{,}000 + 65{,}000 + 74{,}000 + 100{,}000}{15}$$

= 48,667

Winsorized variance calculation

$$S_w^2 = \frac{\sum(X_w - \bar{X}_w)^2}{n-1}$$

$S_w^2 =$

$(25,000-48,667)^2+(28,000-48,667)^2+(30,000-48,667)^2+(33,000-48,667)^2+(35,000-48,667)^2+(40,000-48,667)^2+(42,000-48,667)^2+(45,000-48,667)^2+(48,000-48,667)^2+(50,000-48,667)^2+(55,000-48,667)^2+(60,000-48,667)^2+(65,000-48,667)^2+(74,000-48,667)^2+(100,000-48,667)^2$

5 – 1

S_w^2 = approximately 415,125,992.93

Winsorized standard deviation calculation

$S_w = \sqrt{S_w^2}$

$S_w = \sqrt{415{,}125{,}992.93}$

S_w = approximately 20,374.64

For second Data set

120, 140, 160, 180, 200, 220, 240, 265, 280, 300, 320, 350, 375, 450, 500

- Mean = $\bar{x} = \frac{\Sigma X}{n}$

$$\frac{120+140+160+180+200+220+240+265+280+300+320+350+375+450+500}{15}$$

- Approximately 273.33

Variance = Variance = $S^2 = \frac{\Sigma(X-\bar{x})^2}{n-1}$

$(120-273.33)^2+(140-273.33)^2+(160-273.33)^2+(180-273.33)^2+(200-273.33)^2+(220-273.33)^2+(240-273.33)^2+(265-273.33)^2+(280-273.33)^2+(300-273.33)^2+(320-273.33)^2+(350-273.33)^2+(375-273.33)^2+(450-273.33)^2+(5000-273.33)^2$

5 – 1

S^2 = approximately 12,504.28

- Standard Deviation S = $\sqrt{S^2}$

S = $\sqrt{12{,}504.28}$

S = approximately 111.82

For the third Data set

3, 4, 5, 6, 7, 9, 11, 13, 15, 18, 20, 22, 25, 30, 35

- Mean = $\bar{x} = \frac{\Sigma X}{n}$

$$\frac{3+4+5+6+7+9+11+13+15+18+20+22+25+30+35}{15}$$

Approximately 14.87

- Variance = $S^2 = \frac{\Sigma(X-\bar{x})^2}{n-1}$

$(3-14.87)^2+(4-14.87)^2+(5-14.87)^2+(6-14.87)^2+(7-14.87)^2+(9-14.87)^2+(11-14.87)^2+(13-14.87)^2+(15-14.87)^2+(18-14.87)^2+(20-14.87)^2+(22-14.87)^2+(25-14.87)^2+(30-14.87)^2+(35-14.87)^2$

5 – 1

S^2 = approximately 101.05

- Standard Deviation S = $\sqrt{S^2}$

S = $\sqrt{101.05}$

S = approximately 10.05

In line with the aim and objective as stated in chapter one, below is a table that shows the comparison of the measures of central tendency and measures of dispersion both before and after the application of Winsorization

4.1 Comparative Analysis before and after Winsorization

Data Set	Measure	Before Winsorization	After Winsorization
Income of Employees in Naira	Mean	142,000	48,667
	Median	42,000	42,000
	Variance	461,336,072,500	415,125,992.93
	Standard Deviation	679,217.25	20,374.64
Monthly Sales of a	Mean	573.33	273.33

Product	Median	265	265
	Variance	1,480,781.99	12,504.28
	Standard Deviation	1,216.87	111.82
Customer Complain (in a year)	Mean	25.87	14.87
	Median	13	13
	Variance	2,228.52	100.05
	Standard Deviation	47.20	10.05

Researcher's compilation (2025)

Discussion of Findings

Dataset 1: Income of Employees in a Small Business

The analysis of the income data for employees in a small business showed significant outlier at 1,500,000, which deviated greatly from the rest of the dataset. After calculating the normal mean, we found it to be 142,000, which is heavily influenced by the outlier, rendering it not fully representative of the majority of salaries. The median however, was 42,000 reflecting a more accurate central tendency for the majority of data points, as it is not affected by extreme values.

The variance and standard deviation for this dataset were quite high, with the variance reaching approximately 461,336,072,500 and the standard deviation around 679,217.2499. This indicates considerable spread in the data, largely due to the outlier.

After applying Winsorization to the data, we adjusted the extreme values of 1,500,000 to 100,000, bringing it closer to the general range of salaries. Following this adjustment, the Winsorized mean decreased to 48,667, providing a more realistic estimate of the central tendency. The Winsorized variance dropped to approximately 415,125,992.93 and the standard deviation was reduced to 20,374.64, highlighting a substantial decrease in the spread of the data. These results indicate that Winsorization successfully reduced the impact of the outlier and brought the dataset in line with the expected salary range for the business.

Dataset 2: Monthly Sales of a Product

The monthly sales data presented a consistent trend of growth, but a noticeable spike at the end (5,000) units created a significant outlier. The normal mean was calculated as 573.33 which was heavily influenced by this spike, while the median of 265 better reflected the typical sales figures across the months.

The variance was calculated to be approximately 1,480,781.99, with a standard deviation of 1,216.87. These figures show considerable variability in the sales figures, largely driven by the 5,000 units sold in the last month. After applying Winsorization, the outlier was adjusted to 500, leading to a new mean of approximately 273.33. The variance decreased to 12,504.28, and the standard deviation dropped to 111.82, which are more consistent with the distribution of the other sales figures. These findings illustrate that Winsorization successfully mitigated the distortion caused by the extreme sales figure, offering a more reliable central tendency and spread for the data.

Dataset 3: Customer Complaints in a Year

The dataset on customer complaints showed a steady increase in complaints over the year, but there was an extreme spike at the end (200 complaints), which stood far apart from the rest of the data. The normal mean for this dataset was approximately 25.87, while the median, being more resistant to the influence of the outliers was 13. The variance and standard deviation were calculated to be approximately 2,228.52 and 47.2 respectively, which were relatively high, reflecting the presence of the extreme complaint value.

When the Winsorization process was applied, the extreme outlier of 200 complaints was adjusted down to 35, aligning more closely with the other values. This adjustment led to a new mean of approximately 14.87, a more representative figure for the central tendency of complaints. The variance decreased significantly to approximately 101.05 and the new standard deviation was reduced to around 10.05, demonstrating a much more consistent distribution of complaints once the outlier was addressed.

CHAPTER FIVE

SUMMARY AND CONCLUSIONS.

5.1 Summary

This study focused on the application of Winsorization to mitigate the effects of outliers on three different datasets. The datasets used in this research represented different scenarios that are common in business environments: employee income, monthly product sales, and customer complaints. These datasets were analyzed both before and after applying Winsorization, and various statistical measures were computed to demonstrate the impact of this technique.

The datasets analyzed were:

1. **Employee Income in a Small Business:** This dataset included 15 data points for the salaries of employees working in a small business. One extreme outlier (1,500,000 Naira) significantly distorted the mean and other statistical measures. Winsorization adjusted this outlier to 100,000 Naira, producing a much more realistic mean and reducing the variance and standard deviation considerably.

2. **Monthly Sales of a Product:** The second dataset contained 15 months of sales data. A spike of 5,000 units at the end of the series was identified as an outlier. The Winsorization process adjusted this value to 500 units, improving the statistical accuracy of the data by producing a more reasonable mean and reducing variability.

3. **Customer Complaints in a Year:** The third dataset represented the number of customer complaints over 15 months, with the final value being an outlier (200 complaints), After

Winsorization, this extreme value was adjusted to 35 complaints, resulting in a more representative dataset.

In each dataset, the analysis showed that Winsorization effectively reduced the impact of outliers, providing more reliable central tendencies (mean, median) and less variability (variance and standard deviation). This allowed for a more accurate understanding of the datasets without the distortion caused by extreme values.

5.2 Conclusions

The main findings from the study include:

1. **Effectiveness of Winsorization:** The Winsorization method proved effective in handling outliers by adjusting extreme values to the nearest nonoutlier values. This significantly improved the central tendency and spread of the data.

2. **Impact on Statistical Measures:** In all three datasets, the application of Winsorization resulted in lower means, variances, and standard deviations. The outliers had a substantial impact on these measures before Winsorization, but after applying the method, the statistical measures became more aligned with the majority of the data points.

3. **Data Integrity in Business Decision-Making:** This study highlights the importance of cleaning and preprocessing data in business analytics. The presence of outliers can lead to misleading conclusions and may distort the decision-making process. Winsorization offers a simple yet effective technique to address this issue.

4. **Generalizability of the Technique:** The Winsorization method is applicable to a wide range of datasets and can be used in various industries such as finance, sales, and customer service, where extreme values may skew results and lead to incorrect analysis.

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