STATISTICAL QUALITY CONTROL, A LOSS MINIZATION APPROACH

BY

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DECLARATION

I hereby declare that this project was carried out my me **RHODA OSAGIEDE** with matriculation number **PSC1809361**, I have not copied the work of any other author(s), all texts used have been dully cited and acknowledged.

Rhoda Osagiede **Date:**

CERTIFICATION

This is to certify that the project was carried out by **RHODA OSAGIEDE** with Mat No: **PSC1809361** in the Department of Statistics, Faculty of Physical Science and University of Benin in partial fulfillment for the requirement for the award of the Bachelor Sciences (BSC) Degree in Department of Statistics.

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ABSTRACT

Statistical quality characteristics of the components are determined process can be selected for manufacture of the components. In mass production, products are assembled using parts or components manufactured or processed on different process or machines. This requires complete interchangeability of parts while assembling them, on the other hand, there will always be variations in the quality characteristics [length, weight, thickness etc.] because of the inherit variability introduced by the machines, tools, raw materials and operators. The presence of unavoidable variation and the necessity of interchangeability require that some limits be specified for the variation for any quality characteristic.

A statistical control and loss minimization approach is a way of using statistical methods to minimize the total cost of quality including the cost of the defects and cost of controlling the process. this approach involves setting quality standards, collecting data on product or services characteristics, analyzing the data to detect problems and then correcting these problems. This approach also helps organization achieve the highest possible quality at the lowest possible cost.

The need of Statistical quality control in product and services cannot be over emphasized, as it helps in the improvement of manufacturing process to meet customer satisfaction.

DEDICATION

I dedicate this work to God Almighty for his unfailing mercy and for being my source, my sustainers who has been with me all through this journey.

Special dedication to my parent Mr. Osagiede and Mrs. Esther Osagiede for their love and great support in achieving this degree.

Furthermore, I want to dedicate this work to my project supervisor, Dr. S.A. Osagie for his continual support and impart of knowledge which brought about the success of this work.

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My gratitude goes to God Almighty for his love, strength, wisdom and has been merciful towards me, through the course and beyond.

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I am indeed forever grateful to my parents, Mr. and Mrs. Osagiede for their love and continuous support and prayers, I love you both and also acknowledge my siblings for their constant support and prayers, I am truly grateful.

TABLE OF CONTENT

CHAPTER ONE

1.0. INTRODUCTION

There has always been a great deal of attention for the quantity of manufactured products and services, in the current world of continually, increasing global completion and high demand for quality product and services. It is imperative for all manufacturing and serving companies to improve the quality of their products and services in the most economical way which allow for full customer satisfaction.

1.1 Background of the study

Quality of design are the result of deliberate engineering and management decisions which the manufacturer or supplier are intending to offer to the end users.

Quality control emphasizes testing of products to uncover defects and reporting of such defects to management to make the decision to allow or release products, whereas Quality Assurance (QA) is a way of preventing defects in manufactured products and avoiding problems when solutions or services to end users.

1.2 Significance of Satisfactorily Quality Control

1. It helps to show the importance of statistical techniques.
2. It shows the relevance of statistical quality has to play in the production of finished goods.
3. It shows the capacity of conformation to specification of the process.
4. It helps in the improvement in manufacturing process to meet customer satisfaction.

1.3 Objective of the Study

The objective of the study is drawn from the need to have quality control in product of Nigeria industries.

1.4 Scope of the Study

This study endeavors to give insight to the importance of statistical control process in manufacturing products while focusing on data collect by a firm on cost and expenditure.

1.5. Definitions of Terms

- Statistical Process Control (SPC): Statistical Process Control is a statistical tool that involves inspecting a random sample of the output from a process and deciding whether the process is producing products with characteristics that fall within a predetermined range.
- Acceptance Sampling (AS): Acceptance Sampling Technique where a random sample is taken from a lot and upon the results of the sample taken, the lot will either be rejected or be accepted.
- Lower Control Limit (LCL): The lower control limit is the minimum acceptable variation from the mean for a process that is a state of control.
- Quality Control (QC): Quality is one of the main important aspects that are used to assess cost of efficiency of any produce.
- Range: It is the difference between the maximum and the minimum.
- Standard Deviation (SD): This measures the amount of data dispersion around the mean.
- Control Chart: This Chart serves to defect special cause of variation. This chart has control limit lines at the center, top and button levels, sample data are plotted in dots on the charts to evaluate process situation and trends.
- Histogram: This diagram graphically displays, a set of frequency data in bar graphs and enables evaluators to determine problems by checking the dispersion, center values and the nature of dispersion.

- Quality Assurance (QA): IT refers to the planned and systematic activities implemented in a quality system, so that quality requirements for a product or service are fulfilled.
- Process Capability Index (PCI): This is a measure of how well a process is able to meet the specifications or requirements of a product or service.
- Six Sigma (SS): This is a quality management method that aims to reduce defects in a process by improving its capability and reducing its variation.
- Process Capability (PC): This is a measure of how well a process is able to produce products or service that meet the specification or requirements. A higher process capability means the process is more capable of meeting the specification.
- Statistical Variation (SA): This is the difference between the actual output of a process and the target output.
- Root Cause Analysis (RCA): This is a method of identifying and eliminating the underlying causes of a problem, rather than just treating symptoms.
- Process Capability Studies (PCS): These are used to determine whether a process is capable of producing products or service that meet the specification.
- Upper Control Limit (UCL): the upper control limit helps to identify when process is out of control.
- Center Line (CL): The upper control limit helps to identify when a process is out of control.
- Average Run Length (ARL): This is a measure of how long it takes to detect a process shift on a control chart.
- Process Mean (PM): This is a measure of the average value of all the data points in a process.

CHAPTER TWO

LITERATURE REVIEW

2.0 Introduction

This chapter reviews literature related to quality control. It reviews the meaning and history of statistical quality control and also many authors in the field of quality control.

2.1. Literature Review

The history of Quality Control is fascinating, it really started to take off in the early 20th century, with the work of the people like Walter Shewart, and Edwards Deming etc. They developed statistical methods for quality control that are still used today.

Quality is the main focus of every company, owing the fact that it existence depends on the continuity of these quality products over time, and the ever competitive market should be enough reason to maintain the standards of products, that is where the need of quality management comes in.

According to Joseph M. Juran defined Statistical Control as satisfying customer's requirements to fitness for use. He believed that Quality Control was not just about meeting the needs of the customer. He also believed that Quality Control should be built into products from the beginning, rather than being inspected in after the fact. Juran was a Quality Control Pioneer who worked in the manufacturing industry for many years. He developed the concept of the "Juran Trilogy", which consists of the three components (i) Quality Planning (ii) Quality Control (iii) Quality Improvement. These three components work together to create a cycle of continuous improvement "Juran on Quality by Design".

Standard Organization (2009) defined Quality control as a procedure or set of procedures intended to ensure that a manufactured product or performed service adhere to a define set of a quality criteria or meet the requirement of the client or customer.

Harriet Black Nembhard (2001) defined Statistical Control as a stable random variation about a target value that is produced by some common cause.

Investor World (2009) defined Quality Control as a step taken to make sure a company's product or service are of sufficient quality also called Quality Assurance (QS).

Springer refers to Statistical Quality Control as the application of statistical method to monitor and evaluate system trend to determine whether changing Key Input Variable (KIV) setting is appropriate.

Feigenbaum (2002 - 2004) defined Quality Control as "an effective system for coordinating the quality maintenance and quality improvement effect of the various groups in an organization so as to enable production at the most economic level which allows for full customer satisfaction".

The Concise Oxford Dictionary defined Quality Control as a degree of excellence, the ISO standard 8402 defines it as the totality of features and characteristics of a product or service that bear on its ability to satisfy, or implied needs.

Defoe and Juran (2010) defines Quality as fitness of purpose.

Conferment to requirement. Croshy (1979) statistic quality control over view see statistical quality to use when a process is working correctly and when it is not rom this point of view, quality control begins wash product design and planning where attributes of the product or service are determined and specified and countries through product production or service operation until feedback from the final consumer is lopped backed through the institution, for product important. It is implied that departments, workers' processes for producing a quality product or service.

Crotsch and Davis (2010) defines Quality Control as a dynamic state associated with products, services, people, processes and environment that meet or exceeds expectation and helps produce superior value.

Shewhart (1931) argues that there are two aspects of quality, the objective concept of quality resulting in quantitative measureable physical characteristics, which are independent of the second which is subjective aspect of quality which has to do with what we think, feel or sense.

Shewhart recognizes that it is necessary to establish standards of quality in a quantitative (objective) manner.

Shewhart reported that bringing a process into a state of statistical control, where there is only change -cause (common cause) variation, and keeping it in control was needed to reduce waste and improve quality. www.ncbi-n/m.nih.gov, through the use of statistical techniques and tools, University of California, Berkeley.

Statistical Quality Control is a method for monitoring the performances of a process or system in order to detect and eliminate any sources of error variability, Carnegie Mellon University.

Statistical Quality Control is a process for analyzing data to improve the quality of products and service, University of Minnesota.

Statistical Quality Control is the application of statistical control of products and processes in order to achieve and maintain consistent quality, Wiley.

Statistical Quality Control is the application of statistical methods to the control of quality in products and processes, McGraw-Hill Education.

Statistical Quality Control is a methodology for assessing, analyzing and improving the quality of products and processes, Encyclopedia of Information.

Statistical Quality Control is a systematic approach to monitoring, evaluating and improving the quality of products and processes, Encyclopedia of Management.

Statistical Quality Control is a set of techniques used to measure, monitor, and improve the quality of products and processes, Encyclopedia of Statistical Sciences.

Statistical Quality Control is a set of statistical methods used to evaluate and improve the quality of products and process, Oxford Dictionary of Statistics.

Statistical Quality Control is a set of techniques and tools used to measure and improve the quality of products and processes, Cambridge Dictionary of Statistics.

Statistical Quality Control is a methodology that uses statistical methods to ensure the quality of products and processes, Oxford Journal of Statistical Education.

Statistical Quality Control is the application of statistical tools and techniques to ensure that products and processes conform to requirements and specification. Philip B.

Statistical Quality Control is the use of statistics to monitor, control and improve the quality of products and processes, Genichu Taguchi.

Statistical Quality Control is a systematic approach to achieving and maintaining a level of quality of by measuring and analyzing results, identifying problems, and implementing solution, National Institute of Standard and Technology.

"Statistical Quality Control is a method of monitoring and improving quality by using statistical techniques to evaluate and improve process, products, and service, American Society for Quality".

Statistical Quality Control is the application of statistical methods to the monitoring and control of processes in order to ensure the quality of products and services, Bureau of Labour Statistics.

Statistical Quality Control is a set of tools and techniques used to monitor and improve the quality of a process or product by collecting and analyzing data, International Organization for Standardization.

Statistical Quality Control is the application of statistical methods to the analysis, evaluation and improvement of processes and products, American Statistical Association.

Statistical Quality Control is the use of statistics to measure and monitor quality, as well as to identify problems and improve processes, Encyclopedia Britannica.

Statistical Quality Control is a discipline that involves the application of statistical tools to measure and improve quality, National Institute of Technology.

Statistical Quality Control is a methodology for monitoring and controlling the quality of a product or service.

According to Jurdan and Gryna (1988), Quality is perceived as "Fitness for use", different usages of a product will result in different requirements with regards to the product.

Crosby (1979), defined Quality as "conformance to requirement", a product that is produced in conformance with specifications that are popular with customers in ranked as high quality product.

Okumoka (1994), defined Quality Control as a technique of verification and maintenance of a desired level of customer's requirements and satisfaction by carefully planning and application of correction at places where deviations occur before they become frequent and noticeable as this could lead to material and labour wastage.

Thirkettle (1962), defined Quality Control as a method of estimating the quality of the who i.e from the quality of a sample taken from the whole.

Illersee (1994) described Statistical Quality Control as a simple application of the theory based on the normal curve. According to him, modern mass production techniques involves ensuring that each or other physical properties are within ideal standard. Illersee explains further that it is far more economical to try to check any fault in the production process at the earliest possible moment in theory, the product can be checked when it comes from the machine but at this stage, the damage may have been done.

Statistical quality control is a systematic approach to the monitoring and control of the quality of products and process, quality progress.

Statistical quality control is a set of tools, techniques and concepts that enables us to measure and monitor the action when needed, six sigma.

2.2. Quality Control Procedure for Variable Data

In all production process, we aim to monitor extent to which the products meet specifications. However, there are two problems of product quality, i.e

i. Excessive variability and target specification

ii. Duration from target specification.

In early phases of development production process, design experiments are mostly used to optimize the two quality characteristics, the method provided in quality control are on-line or in-process quality control procedures to monitor an on-going production process.

Extraction of samples of a certain size from an on-going production is the approach used on the line quality control, we then create line charts of emerges in those lines, or if sample fall outside pre-specified limits, then use declare the process to be one of control and take action to find the cause of the problem, control charts are methods of statistical process content (Harrison et al). they enable the content of variation rather than attempting to control each individual variation.

The Statistical Control Chart can be defined as statistical tool used to detect excessive process variability due to specific assignable causes that can be corrected. It serves to determine whether a process is in state of statistical control, that is expected based on the natural statistical variability of the process.

2.3 Establishing Control Charts

One could arbitrarily determine when to declare a process out of control charts, outside UCL range. It is a common practice to apply statistical principles to do so, the method for constructing the upper and lower control limits is a straight forward application of the principles described. Without going into details regarding the derivation of the formula, we know that the distribution of individual data points or measurement. Over the square root of n (the sample sign) i.e

It follows that approximately 95% of the sample means will fall within the limits. This interval is defined by

In practice, it is common place to replace the 1.96 with 3 (so that the interval will include approximately 99% of the single means) and to define the upper and lower control limit as plus and minus 3 sigma limits. i.e \pm 3, limits respectively.

Generally, the principles for establishing control limits just described applies to all control. For example, the standard deviation, we estimated the expected variability of the respective characteristics in sample of the size we are about to take. Those estimates are then use to establish the control limits on the charts.

2.4 Exponentially Weighed Moving Average Control Chart (EWMA)

An Exponentially Weighed Moving Average Control Chart is a control chart for variable data (data that is both quantitative and continuous in measurement, such as measured in dimension or time). It is used when it is desirable to detect out of control situation very quickly (i.e to know immediately when a process goes out of statistical control). EWMA chart have an in built mechanism for incorporating information from all previous sub groups and assigning weights to all the information collected with closest subgroup assigned a higher – weight, hence, the control/out of control decision is made from information of previous. Subgroup as well as the current subgroup because the EWMA chart uses information from all samples, it detects much smaller process shafts than a normal control could.

The major advantage of EWMA chart is that they detect out of control conditions more quickly than the X chart and this detection can be made by sing only one role in (the process being within or outside the control limits).

2.5 Uses of the EWMA Chart

should use small values (e.g 0.2) to detect small shifts and target values (between 0.2 and 0.4) for larger shifts. An EWMA with λ = 10 is an $\bar{X}$ chart.

As with other charts, EWMA charts are used to monitor processes or event over time. The horizontal areas of the EWMA chart are time based, so that the chart show a history of the process. Hence, one must have data that is time ordered. i.e entered in the sequence from which it was generated. If this is not the case, then the trends or shifts in the process may not be detected but instead attributed to random variation.

The Exponentially Weighed Moving average is defined as

$Zo = \lambda xi + [1 - \lambda]\ zi - 1$

Where: $o < \lambda \leq 1$ is a constant

$Zo = No$ [sometimes $Zo = xi$

The control limits of an EWMA control charts are:

$$\text{UCL} = xi + \frac{3\sigma^2}{\sqrt{n}}\sqrt{\frac{\lambda}{1-\lambda}}(1-\lambda)2i$$

CL = N

$$\text{UCL} = xi - \frac{3\sigma^2}{\sqrt{n}}\sqrt{\frac{\lambda}{1-\lambda}}(1-\lambda)2i$$

Where λ is the width of the control limits and L=3. The design parameters of EWMA charts are L and λ, the parameters can be chosen to give the desired ARL performance. In general, $0.05 \leq \lambda \leq 0.25$, works reasonably well (especially with the larger values of λ) L between 2.6 and 2.8 is useful when ≤ 0.1.

2.6. The Shewart Control Charts

The $\bar{X}$ Chart

If samples of sign n are taken on successive occasions and the values of the character X is taken on each data in each sample, then for each sample the mean of the observed value $\bar{X}$ is calculated. We denote the value of the ith sample mean by $\bar{xi}$ and thus the value of each $\bar{xi}$ are plotted on the chart. Also, the range of such sample is calculated and denoted by Ri multiplied by a constant (dependent in sample size) is then used as an estimate for $Ui = \frac{\alpha}{2}\sigma x$

This is then added to and subtracted from the grand mean $\bar{X}$ of all the samples to form a control limit of the $\bar{xi}$ chart. i.e

$\bar{X} \pm$ estimate of $Ui = \frac{\alpha}{2}\sigma^2$

This happen if we have m sample of size n, the estimate of estimated value of the population

$E(x)$ is given as:

$$E(x)\frac{1}{m}\sum_{i=1}^{m} Ni = \frac{x}{2}\bar{x}\frac{1}{nm}\sum_{i=1}^{m}\sum_{i=1}^{m} xij$$

2.6.1 Shewart Control Chart on S

This type of control charts is usually more tedious to calculate than the $\bar{X}$ chart. In this chart, we take a pooled estimate of the population variance from the sample.

i.e

$S^2 = \sum_{mi/2}^{mi} si^2$ for $i = 1, 2 \ldots k$

The S control chart is a modern approach to calculate the standard deviation of each sub-group and plot these standard deviation to monitor the process standard deviation σ.

Assume that there are m preliminary sample variables, each of size n and let S_i denote the standard deviation of the ith sample.

$$.\bar{S} = \frac{1}{m} \sum_{i=1}^{m} Si$$

2.7 Shewart $\overline{X}$ and R Chart

If the sample size is relatively small (say equal to or less than 10), we can use the range instead of the standard and deviation of the sample to construct control charts on $\bar{X}$ and the range R. the range of sample is simply the difference between the largest and smallest observation.

The mean of R is $d_2\sigma$, where the value of d_2 is also a function of the sample size in form which the range R was obtained. An estimator of σ is therefore R/d

The $\bar{X}$ and R control chart.

R_1, R_2 …….. Rm be the range of k samples, the average is $\frac{R= R_1+ R_2+\cdots+ R_m}{m}$

The estimate of $\sigma = \frac{\bar{R}}{d_2}$

UCL = x + $\frac{3}{d_2\sqrt{n}} \bar{R}$

Centre line = x

LCL = x - $\frac{3}{d_2\sqrt{n}} \bar{R}$

2.8. The R Chart

This chart controls the process variability since the sample range is related to the process standard deviation. The centre line of the R chart is the average range.

To compute the control limits, we need an estimate of the true but known standard deviation $W\,{}^{R}/_{\sigma}$ (assuming the items that we measure follows a normal distribution of the sample size, n).

Therefore, since $R = W\sigma$, the standard deviation of R is σ_r = $d_3\sigma$. But since the true σ is known, we may estimate σ by $\sigma R = X\frac{R}{d_2}$. As a result, the parameters of the R chart with the customary 3- σ control limit are:

Centre line = $\bar{R}$

UCL = $\bar{R} - 3\frac{n}{\sigma}R = \bar{R} - 3d_3\frac{\bar{R}}{d_2}$

LCL = $\bar{R}D_4$

2.9 Cumulative Sum (CUSUM) Control Chart

A very effective alternative for the Shewart control chart is the Cumulative Sum Control Chart (CUSUM). This chart has much better performance (in terms of ARL) for detecting small shifts than the Shewart charts, but it does not cause the in control ARL to drop significantly.

The CUSUM chart plots the cumulative sum of the deviations of the simple values from the target value. For example, If N_o is the target for the process mean, and $\bar{X}$, is the average of the ith sample, then the cumulative sum control chart is formed by plotting the quantity.

Ci = $\sum_{i=1}^{i}[\bar{X} - N_o]$

Against the sample number i

Let x_i be the itch observation of the process, if the process is in control there $x_i -$ $N\ (N_i\sigma)$.

Assuming σ is known, we accumulate deviation from target No above the target with statistic $C^+ and C^-$. This $C^+ and C^-$ are one sided upper and lower [CUSUM] respectively. The statistics are computed as follow

$C^+ = \max [o, x_i - [N_o + K] + {}^{C^+}_{i=1}$

. $C_i^- = \max [o, (No + k] - xi + {}^{C^-}_{i=1}$

CHAPTER 3

3.0 Introduction

Let's consider the diameter of a shaft with specifications 1"±0.004" which means that the nominal value 1"(B), the Lower Specification Limit (LSL), 0.96 and the Upper Specification Limit (USL) is1.04, let the diameter (x) follow a density of function $f(x)$ with a mean of N and the variance of σ^2, assume that the cost of reworking an oversized shaft is Cw and the cost of scrapping an undersized shaft is Cs. We will also assume that a shaft that is reworked will fall within the specification limit.

Again consider another example involving two types of resistors, the quality characteristic is the resistance that has a tolerance range from 950 to 1050 ohms. The nominal value is 100 ohms, the histograms of the resistances of 50 resistors if each types are given in fig. 1 and fig. 2. It can be seen that even the rough the ranges of the resistance of both types are within the tolerance range, the width of the range of type A resistor is much narrower than the width of the range of type B resistors. It is obvious that customers prefer type A resistor to type B, because a randomly selected type A resistor has a larger probability of being closer to the tiny holes than

a randomly selected type B resistor.

Studies have also shown that products with quality characteristics for study normal distribution result in less failures, lower warranty costs and higher customer satisfaction compared to product with quality characteristic following a uniform distribution, even though the range of this uniform distribution is within the tolerance range resulting in zero proportion of defectives. This is because the uniform distribution has a larger variance than a truncated normal distribution with the same range.

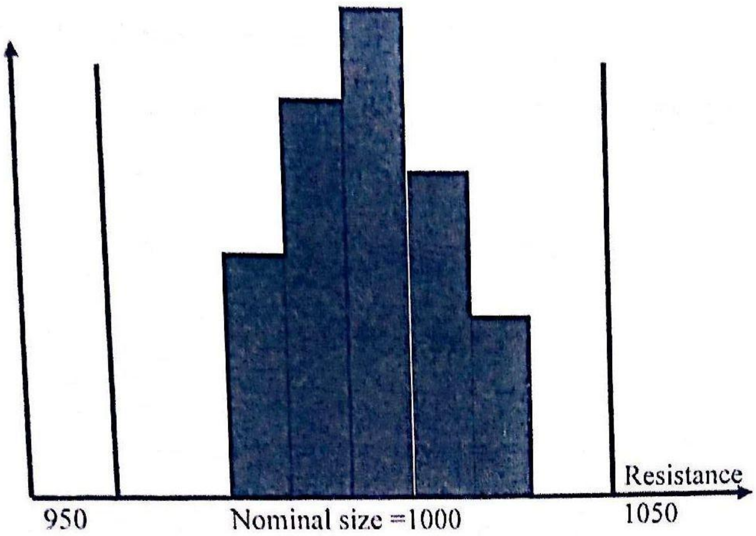


Figure 1: Histogram of resistance type A

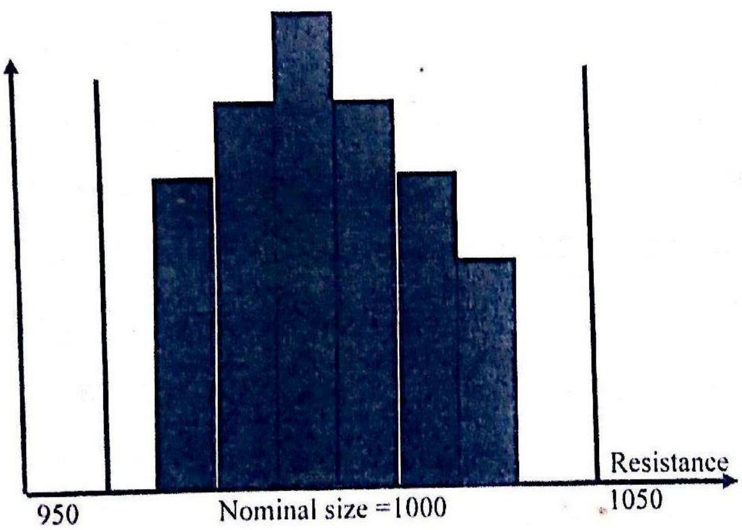


Figure 2: Histogram of resistance type B

3.1 Development of Loss Function

According to Taguchi, the cost of deviating from the target value is given by the loss function derived as follow

The loss function when the quality characteristics is X, its denoted by L(x) this can be written as;

$L(X) = L(X_o + X - X_o)$ (3.2)

Expanding the right – hand side using the Taylor series

$[F(X) = F(a + X\text{-}a) = F(a) + \frac{X-a}{!!} F(a) + \frac{X-a^2}{2!} F``(a) + \ldots\ldots]$

We obtain

$L(X) = L(X_o) + [X - X_o] L`(X_o) + [X\text{-}X_o]^2 L'(X_o) + \ldots\ldots\ldots$ (3.3)

Where L' (X_o) and L' (X_o) are the first and second derivate of L(X) respectively, evaluated at X_o

$L(x) = L'\left(\frac{Xo}{2}\right) X - Xo^2$

$= K'(X - Xo)^2, LSL \leq x \leq USL$ (3.4)

$E[L(X)] = E[X'(X - X_o]^2$, assuming the LSE and USL are contained with the range of X

$= K'E(X - \mu - X_o)^2 = K'E[X - \mu]^2 + 2(X - \mu)(\mu - X_o) + (\mu - X_o)^2$

Where N and x_o are constants as var $(X) = E(X - \mu)^2$

$E[L(X)] = K'[\text{var}(X) + 2(\mu - x_o)][E(X) - N) + (\mu - X_o)^2$

$= K'[\sigma^2 + (\mu - X_o)^2]$ (3.6)

Where $E(X) = \mu$ and $Var(X) = \sigma^2$

$E[(X - X_0)^2 = [\sigma^2 + (\mu - X_0)^2$

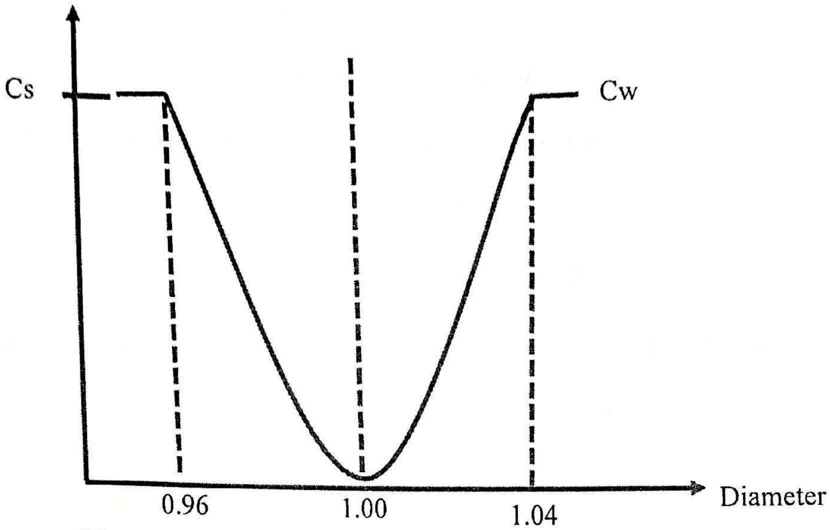


Figure 3

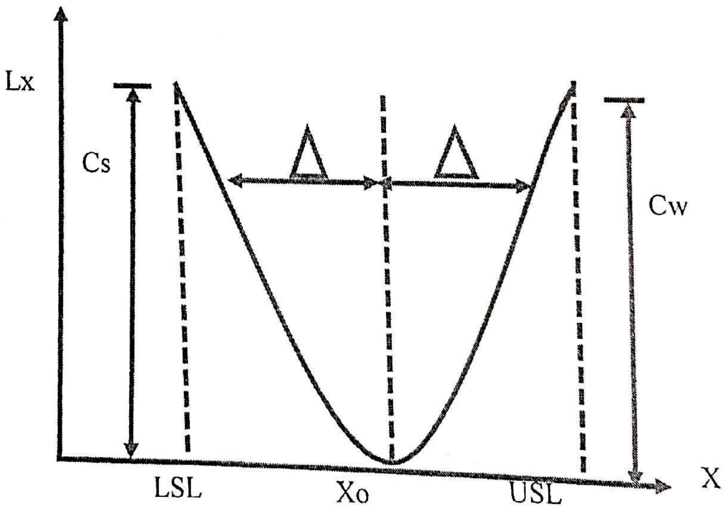


Figure 4

3.2 Loss Functions for Different Types of Quality Characteristics

3.2.1 Nominal – The – Better Type (N - Type)

3.2.2 Equal Tolerances on Both Sides of the Nominal size

Tolerances for these types of characteristics are specified as $B \pm \Delta$, where B is the nominal value and hence is the target and Δ is the allowance on either side of the nominal size (the tolerance is 2Δ). The lower specification limit and the upper specification limit are $B-\Delta$ and $B+\Delta$, respectively. Let the rejection costs incurred by the manufacture be Cs and Cw, when x is less than LSL and x is greater than USL, respectively.

When the costs Cs and Cw are equal, the loss function is

$L(X) = K' (X-X_o)^2$, $LSL \leq X \leq USL$ (3.8)

$= O$, Otherwise

$L [LSL] = K' [LSL - X_o]^2 = K'\Delta^2$

$= C$

Hence $K' = \frac{c}{\Delta^2} K' \frac{c}{\Delta^2}$ (3.9)

3.3.1 Loss When Regression Cost Are Equal

The expected value of the loss function defined in equation (3.8) is

$E [L(X) = K' S_{LCL}^{UCL} (X - Xo^2) f(X)\, dx$ (3.10)

$= k'v^2$

Where V^2 is called the mean – square deviation, its equal to V^2

$S_{LCL}^{UCL} (X - Xo^2) f(X)\, dx$ (3.12)

$= E[L(X^n)] = K'\sigma^{n2}$ (3.13)

$$\text{Where } \sigma^{n2} = \frac{1}{n}\sum_{i=A}(Xi - Xo)^2 \quad (3.14)$$

$$\text{As } \sigma^2 = E(X_i^2) - EXi^2 \text{ and } E(Xi) = N$$

$$\text{E}\left[\frac{1}{n}\sum_{i=1}^{n} Xi - Xo^2\right] = \frac{1}{n}\sum_{i=1}^{n}[\sigma^2 + u^2\Big) - (2xou + x_o^2]$$

$$= \frac{1}{n}(n\sigma^2 + nu^2 - 2Xonu + nX_o^2)$$

$$= [\sigma^2 + u^2 - 2Xou + x_o^2$$

$$= [\sigma^2 + [u - Xo]^2 \quad (3.15)$$

$$\text{E}[S^2 + (\bar{X} - Xo^2) = E\ [s^2 + \bar{X}^2 - 2\bar{X}Xo + X]$$

$$= E(S^2 + E(\bar{X}^2) - 2xoE(\bar{X}) + X_o^2)$$

$$\text{As var } \bar{X} = E(\bar{X}^2 - [E\bar{X}^2) \text{ and } S^2 = \sigma^2$$

$$\text{E}(S^2 + (\bar{X} - X_o^2) = \sigma^2 + \text{var }(\bar{X}) + E(X)]^2 - 2xoE(X) + X_o^2$$

$$\text{As } E(\bar{X}) = N \text{ and } var(\bar{X}) = \frac{\sigma^2}{n}$$

$$.E(s^2 + (\bar{X} - Xo)^2 = \sigma^2 + \frac{\sigma^2}{n} + u^2 - 2Xou + X_o^2$$

$$= \sigma^2 + [u - Xo^2 + \frac{\sigma^2}{n}] \quad (3.16)$$

When the costs Cw and Cs are not equal, the loss function is

$$\text{L}(x) = K'(X - Xo^2, LSL \leq X \leq Xo)$$

$$= K_2'(X - Xo)^2, Xo \leq X \leq X \leq USL$$

$$= \text{o, otherwise} \quad (3.17)$$

It is assumed that when $X = LSL$, the loss $L(X) = Cs$, and where $X = USL$ the associated loss $L(X) = Cw$. The resulting loss function is given.

Based upon these assumptions

$$\text{L(LSL)} = K'(LSL - Xo^2) = K'\Delta 2$$

$$= \text{Cs}$$

Hence $K' = \frac{Cs}{\Delta^2}$ (3.18)

Similarly, L(USL) = $K_2'[USL - Xo)^2 = K_2'\Delta^2$

= Cw

Hence, $K_2' = \frac{Cw}{\Delta^2}$ (3.19)

Now the expected value of the loss function defined in equation (3.17) is

$E[L(x) = K_1' S_{LSL}^{\infty}(X - Xo^2)f(X)dX + K_2' S_{xo}^{ux}(X - Xo^2)fX(dX)$.......(3.20)

$= K_1' V_1^2 K_2' V_2^2$ (3.21)

Where

$= V_1^2 = S_{LSL}^{xo}(X - Xo^2 f(X)dX)$ (3.22)

Is the part of the mean square deviation in the range from LSL to Xo and

$= V_2^2 = S_{LSL}^{X}(X - Xo^2 f(X)dX)$ (3.23)

Is the part of the mean square deviation in the range from xo USL. The estimate of the expected loss function in equation (3.21) is

$E[L(X^n)]K_1' V_1^2 + K_2' V^{n^2}$ (3.24)

Where

$. V^{n^2} = \frac{1}{n}\sum_{1=Ai} Xi - Xo^2$ (3.25)

Which estimate Equation (3.22)

$. V_2^2 = \frac{1}{n}\sum_{0=A2}(Xi - Xo^2)$ (3.26)

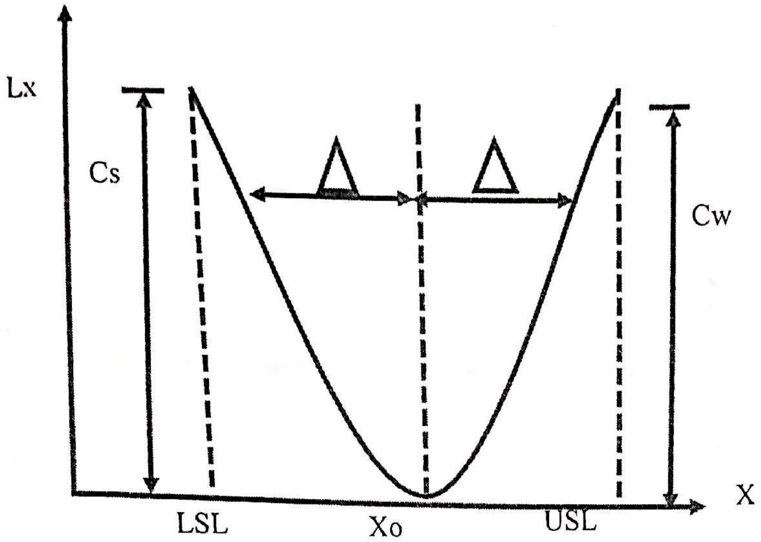


Figure 5

3.4. Unequal tolerance on Both sides of the Nominal Size

Tolerances are specified as $B^{+\Delta 2}_{-\Delta 1}$, hence the LSL is $B - \Delta 1$, the USL is $B + \Delta 2$ and the nominal size (which is the target value) is B.

$L(x) = ki\ (X - Xo^2), Xo - \Delta 1 \leq X \leq Xo$

$= K_2'(X - Xo^2), Xo \leq x \leq Xo + \Delta 2$

$= o$, otherwise (3.27)

As before, it is assumed that when $X = LSL$, the loss $L(X) = C$ and when $X = USL$, the associated loss $L(X) = Cw$. The resulting loss function is given in figure 6 base upon this assumption.

$L(LSL) = K_1'(LSL - Xo^2 = Ki\Delta^2 1)$

$= Cs$

Hence,

$K_1 = \frac{Cs}{\Delta^2}$ (3.28)

Similarly, $L(USL) = K_2[USL - Xo^2 = K_2^1 \Delta_2]$

$= Cw$

Hence $K_2' = \frac{Cw}{\Delta_2^2}$ (3.29)

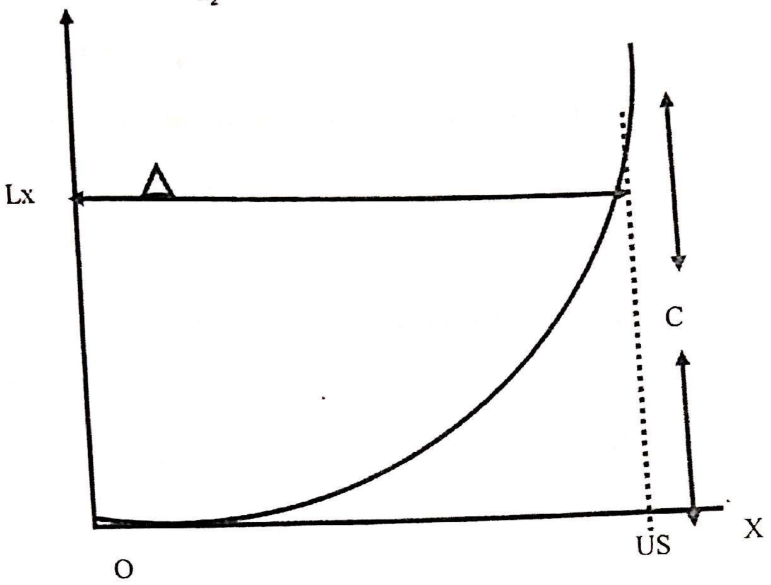


Figure 6

Loss when Tolerance are Unequal

3.5. Smaller – The – Better type (S Type)

Tolerances for this types of characteristics are specific as $x \leq \Delta$, where the upper characteristics is assumed that the quality characteristics x is non-negative. Let the rejection cost incurred by the manufacture be Cw when x is greater than USL. Some examples of this type of characteristics include impurity, shrinkage, noise level, flatness, surface roughness, roundness and wear. The implied target value xo is 0, hence the loss function is

$L(x) = K'^{X^2}, X \leq USL$

= o, otherwise (3.30)

As in the case of other quality characteristics is assumption the loss when $X = USL$ is Cw. The resulting loss function is given in figure 3.6

L(USL) = $K'\Delta^2$ = Cw

Hence, $K' \frac{Cw}{\Delta^2}$

The expected value of the loss function defined in eqn. 30 is

$$E[L(x) = K' S_0^{USL} X^2 f(X) dX \qquad (3.31)$$

$$K'V^2 \qquad (3.32)$$

Where V^2 is the mean – square deviation and is equal to

$$V^2 = S_0^{USL} X^2 f(X) dX$$

The estimate of the expected loss defined in Eqn (3.31 & 3.32)

$$E[L(x)] = K' V^{n^2}$$

Where

$$.V^{n^2} = 1/n \sum_{i=A} Xi^2 \qquad (3.35)$$

Where n is the sample size and A is the set containing all observations in the interval (O -Δ).

3.6. Larger – The – Better Type (L Type)

Tolerances for this type of characteristics are specified as X = Δ , where the lower specification limit is Δ. There is no upper specification limit for these characteristics. Let the rejection costs incurred by the manufacturer be Cw, when x is less than LSL. Some of the example of this type of characteristics are tensile strength, compressive strength, and miles per gallon. The implied target value X_o is ∞ and the loss function L(X) =K $(X – X_o)^2$ is equal to ∞ for all values of X. To eliminate this problem, the L-type characteristics transformed to a S-type characteristic using the transformation $Y\frac{1}{X}$, Now Y becomes an S-type characteristic with an upper specification limit $\frac{1}{\Delta}$ hence the loss function for Y is

$$L(Y) = K'Y^2, Y = \frac{1}{\Delta} \quad (3.36)$$

As in the cause of other quantitative characteristic, it is assumed that the loss function when x = Δ or when $Y = \frac{1}{\Delta}$ is Cw

$$K' = \frac{w}{\frac{1}{\Delta}} = Cw\Delta^2 \quad (3.37)$$

Now let its transform the variation Y back to the original variable X using the transformation $X = \frac{1}{Y}$

$$L(x) = \frac{X^1}{X_2}, x \geq \Delta$$

$$= o, \text{otherwise} \quad (3.38)$$

$$E[L(x) = K'S^{\infty}\frac{1}{X}f(X)dX \quad (3.39)$$

Where K′ the expected value of the loss function defined in equation 3.38

$$= K'\,V^2 \quad (3.40)$$

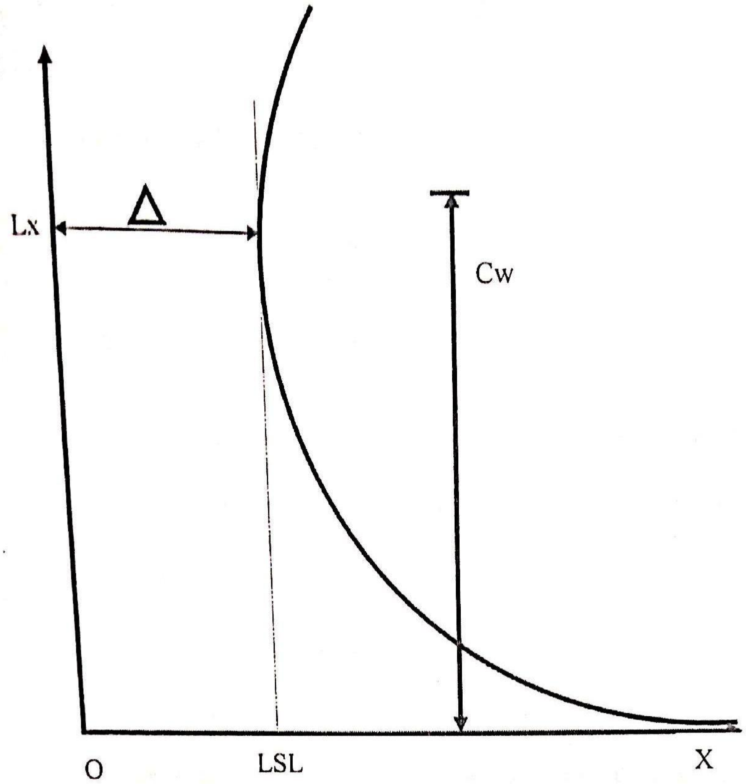


Figure 7
Loss for Larger – The – Better Type

Where V^2 is the mean – squared deviation

$$V^2 = S_{USL}^{\infty} \frac{1}{X^2} f(X)dx \quad (3.41)$$

The estimate of the expected loss define equation (3.39) and (3.40) is where

$$E(L(X^n) = K'V^{n^2} \quad (3.42)$$

$$.V^{n^2} = \frac{1}{n} \sum_{IEA} \frac{1}{X^2} \quad (3.43)$$

Where n is the sample size and Λ is the set containing all observations ≥ Δ, which is the LSL.

3.7. Robust Design Using Loss Function

The expected value of loss function

$$E[L(X) = K'[\sigma^2 + (N - Xo)^2]$$

Let x be the quality characteristic of the assembly with a mean N, variance σ^2, and target value xo in order to minimize the expected loss of X_1 we should

i. Move the mean of X_1 $u = Xo$

ii. Minimize the variance of $X_1\sigma^2$

Let us assume that the quality characteristic of the assembly X is to let us assume that the quality characteristic of the assembly X is a known function of the characteristics of the components of the assembly $X_1, X_2, X_3 \ldots\ldots Xk$

$$X = e\,(X_1\ X_2 \ldots\ldots Xk) \qquad (3.44)$$

$$X = e(X_1,\ X_2 \ldots\ldots\ldots Xk)$$

$$= e(N1 + \sum_{i=1}^{k} g(\underline{N})\,(X_1 - N_i) + \frac{1}{2}\,\frac{k}{z}\frac{k}{z}\frac{k}{z}$$

= e (N) => Taking the expected value of both side of eqn (3.45)

$$= E\,[e\,(X_1,\ X_2 \ldots\ldots\ldots Xk)]$$

$$= E\,[e(N)] + E\,[\sum_{i=1}^{k} gi\,(\underline{N})(Xi - N1) + \frac{1}{2}E\,[\sum_{j=i}^{k}\sum_{j=i}^{k} hij\,(\underline{N})\,(Xi - Ni)(Xj - Uj)]$$

$$= e(N) + \sum_{1=i}^{k} gi\,(\underline{N})[E\,(Xi) = E(Ni)] + \frac{1}{2}\sum_{i=j}^{k}\sum_{i=j}^{k} hij\,(N)E[CXi - Ni](Xi - Nj)] \qquad (3.46)$$

As gi (N), hij (N) and e (N) are constant

$E\,(Xi) = Nij$ so $E(Xi) - N = 0$

$E\,[C(Xi - Ni]\,(Xj - Nj) = E\,(Xi - Ni)^2$

$= \text{var}\,(Xi) = \sigma^2$

And where i = j

E $((Xi) - Ni\,(Xj - uj)$ = covariance of $(Xi,\ Xj)$

Combining these results

$$\sigma ij = cov\,(Xi, j) if\ Xj$$

$= \sigma ij\ if$ 0 = j

The covariance of Xi and Xj is 0, if Xi and Xi are independent

$$\text{N} = \text{e}(\underline{U}) + \tfrac{1}{2}\textstyle\sum_{i=1}^{k}\sum_{j=1}^{k} hij\ (\underline{N})\ \sigma ij \qquad (3.48)$$

The quantity of $N - Xo$ is called the bias and is be minimized (3.47)

$$(N - Xo) = e\ (\underline{U}) + \tfrac{1}{2}\textstyle\sum_{i=1}^{k}\sum_{j=1}^{k} hij\ (\underline{U})\ \sigma ij - Xo \qquad (3.48)$$

Now let us derive an expression for the variance of $xi\sigma^2$

$\sigma^2 = E\,[Cx - N]^2$

From eqns (3.45) and (3.47)

$$X - N = e(\underline{U}) + \sum_{i=1}^{k} gi(U)(Xi - Ni) + \frac{1}{2}\sum_{i=1}^{k}\sum_{j=i}^{k} hij\ (N)(Xi - Ni)(Xj - Nj)(Xj - Uj) - e(w) - \frac{1}{2}\sum_{i=1}^{k}\sum_{j=1}^{k} hij\ (u)\sigma ij$$

$$= \sum_{i=1}^{k} a.(\underline{U})(Xi - Ni) + \frac{1}{2}\sum_{i=1}^{k}\sum_{j=i}^{k} hij\,[cXi - No](Xj - uj) - \sigma.j \ldots\ldots\ldots (3.50)$$

$= X - N^2$ *is approximated by*

$$\sum_{i=i}^{k} -gi\ (N)(Xi - Ni^2)\,.\,hence\ the\ variance\ of\ X\ is$$

$$\sigma^2 = E[\sum_{i=1}^{k} gi\ [u](Xi - Ni^2) \ldots\ldots\ldots (3.51)$$

As

$$[\sum_{i=i}^{k} a, b.]^2 = \sum_{i==1}^{k} aibi \left[\sum_{i=1}^{k} aibi\right] = \sum_{i=1}^{k}\sum_{j=1}^{k} aibi\ ajbj = \sum_{i=1}^{k}\sum_{j=1}^{k} aiaj\ bibj$$

$$[\sum_{i=i}^{k} gi\ (\underline{N})(Xi - ui)]^2 = \sum_{i==1}^{k}\sum_{j=1}^{k} gi\ [\underline{u}]gi(\underline{u})(Xi - ui)(Xj - uj)$$

Hence 3.51 is

$$\sigma^2 = E[\sum_{i=1}^{k}\sum_{j=1}^{k} gi\ (\underline{N})gj(\underline{u})(Xi - Ni)(Xi - Uj)]$$

$$= \sum_{i=1}^{k}\sum_{j=1}^{k} gi\ (\underline{U})gi(U)\ E\ [(xi - Ui)(Xj - Uj)]$$

$$= \sum_{i=1}^{k}\sum_{j=1}^{k} gi\ (\underline{U})gi(\underline{U})\ \sigma ij]$$

Where

$$\sigma ij = cov\ (Xi, Xj), if\ i\ \neq j$$

$$\sigma^2, if\ i = j$$

Now the problem of robust design can be formulated as follows. Find the mean $U_{1,}N_2.............U_k$ that can be set as equal to the nominal sizes $B_i,..........,$ $B_k$so as to minimize

$$\sigma^2 = \sum_{i=1}^{k}\sum_{j=1}^{k} gi\ (\underline{N})\sigma ij\ \ given\ in\ eqn\ (3.52)$$

Subject to

$$e[U] + \frac{1}{2}\sum_{i=1}^{k}\sum_{j=1}^{k} hij\ (\underline{U})\sigma ij = xo\ \ given\ in\ eqn\ (3.49)\quad [3.53]$$

Some Recent Developments in Robust Design

Robust design improves the quality of a product by adjusting the means of the component characteristics so that variance of the assembly characteristic is minimized and the mean of the assembly characteristic is equal to its target value. The reduction in variance is equivalent to decreasing the sensitivity of the assembly reduction in variance is equivalent to decreasing the sensitivity of the assembly characteristic to the noise or uncontrollable factors in the design a quality size of the components is determined. Since Taguchi initial work in this area, many researchers have explained his contribution or provides a very good overview of his work. Efforts in the design stages of products have made a dramatize impact on the quality of these products. The design phase is divided into three parts namely:-

1. System Design
2. Parameter Design
3. Tolerance Design

System Design

In this step, the basic prototype of the product is developed to perform the required functions of the final product, and the materials, parts and manufacturing and assembly system are selected.

Parameter Design

The optimum means of the design parameters of the components are selected in this phase, so that the product characteristic is insensitive to the effect of noise factors. Robust design plays a major role in this step.

Tolerance Design

This step is carried out only if the variation of the product characteristic achieved in parameter design is not satisfaction. It optimum tolerances that minimize the total coast are determined. The optimization techniques used in this step include response surface methodology, integer, programming, non –linear programming and simulation.

3.9. Genichi Taguchi Loss Function

The use of statistical methods to optimize the design of products and processes so that they meet or exceed customer requirements, and so that the variation in the product of process is minimized. He emphasized the importance of meeting customer requirements and reducing variation. The only disadvantage of these function is that the results obtained are only relative and do not exactly indicate what parameter has the highest effect on the performance characteristic value.

Taguchi loss function is a mathematical model that shows the relationship between quality loss and the variation in process output. It is a way of minimizing the cost of poor quality by optimizing the process. The Taguchi loss function is based on the idea that there is always some variation in the output of a process. This variation can lead to quality loss, which is the difference between the actual output and the desired output. The goal of the loss function is to minimize these quality losses by minimizing the variation in the process. www.ncbi-n/m.nih.gov"www.sciencedirect.com".

Formula

$L = K (y-t)^2$

L = cost incurred as a quality derivate from the target

K = Quality loss coefficient

The following data were gotten from www.spcforexcel.com on the number of visitors that visit a particular site.

S/N	Visitors	Days
1	565	Wed
2	555	Thurs
3	390	Fri
4	296	Sat
5	387	Sun
6	603	Mon
7	612	Tues
8	614	Wed
9	584	Thurs
10	446	Fri
11	256	Sat
12	363	Sun
13	587	Mon
14	585	Tue
15	609	Wed
16	565	Thurs
17	451	Fri
18	286	Sat
19	378	Sun
20	678	Mon
21	610	Tue
22	650	Wed
23	600	Thurs
24	459	Fri
25	305	Sat
26	471	Sun
27	659	Mon
28	692	Tues
29	650	Wed
30	625	Thurs
31	452	Fri

CHAPTER 4

DATA ANALYSIS AND RESULT

4.1 Introduction

The analysis was ran using the R-chart, CUSSUM and the Taguchi Loss function.

[STB, LTB] method

Solution:

Solving with the R Chart

$.\bar{e} = \frac{Total\ No.of\ Visitors}{Total\ No.of\ Days}$

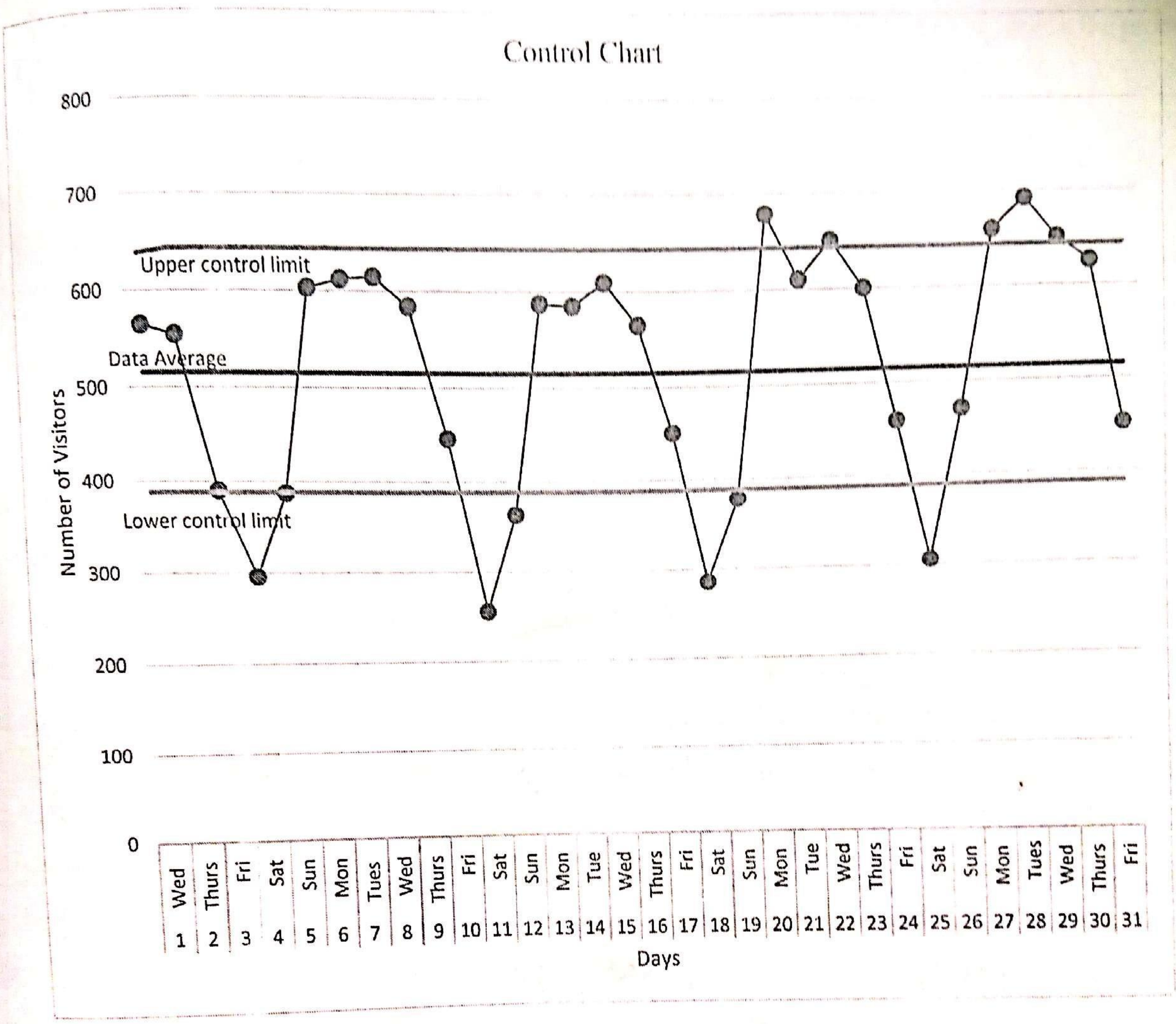
$\frac{15983}{31} = 516$

UCL = $\bar{C} + Z\sqrt{\bar{C}}$

UCL = 643

LCL = $\bar{C} - Z\sqrt{\bar{C}}$

LCL = 380


Control Chart
800
700
600
500
400
300
200
100
0
Upper control limit
Data Average
Lower control limit
Number of Visitors
Wed
Thurs
Fri
Sat
Sun
Mon
Tues
Wed
Thurs
Fri
Sat
Sun
Mon
Tue
Wed
Thurs
Fri
Sat
Sun
Mon
Tue
Wed
Thurs
Fri
Sat
Sun
Mon
Tues
Wed
Thurs
Fri
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31
Days

Conclusion:

The SPC is under control since the highest value falls within the UCL, hence the process has to be improved.

b. Solving with the CUSSUM Control Chart

S/N	Visitors xi	$xi - 600.5$	C^+	N^+	$600.5 - xi$	C^-	N^-
1	565	-37.5	0	0	37.5	37.5	1
2	555	-45.5	0	0	45.5	83	2
3	390	-210.5	0	0	210.5	293.5	3
4	296	-304.5	0	0	304.5	598	4
5	387	-213.5	0	0	213	811.5	5
6	603	2.5	0.25	0	-2.5	809	6
7	612	11.5	14	0	-11.5	797.5	7
8	614	14.5	27.5	0	-13.5	784	8
9	584	-16.5	-11	0	16.5	800.5	9
10	446	-1545	-143.5	0	154.5	955	10
11	256	-335.5	-4.79	0	344.5	1299.5	11
12	363	-237.5	-716.5	0	237.5	1537	12
13	387	-13.5	-730	0	13.5	15503	13
14	585	-15.5	-745.5	0	15.5	1506	14
15	609	8.5	-737	0	8.5	1557.5	15
16	565	-35.5	-772.5	0	-35.5	1593	16
17	451	-149.5	-922	0	149.5	1742.5	17
18	286	-314.5	-1236.5	0	314.5	2057	18
19	378	-222.5	-1459	0	222.5	2279.5	19
20	678	77.5	-1381.5	0	-77.5	2202	20
21	610	9.5	-1372	0	-9.5	2192.5	21
22	650	49.5	-1332.5	0	-49.5	2143	22
23	600	-0.5	-1323	0	0.5	2143.5	23
24	459	-141.5	-1464.5	0	141.5	228.5	24
25	305	-295.5	-1760	0	295.5	2580.5	25
26	471	-129.5	-1772.5	0	129.5	2710	26
27	659	58.5	-1714	0	-58.5	2651.5	27
28	692	91.5	-1622.5	0	-91.5	2560	28
29	650	49.5	157.3	0	-49.5	2510.5	29
30	625	24.5	15.405	0	-24.5	2486	30
31	452	-148.5	-1693	0	148.5	2634.5	31

Using $U_o = 600, \sigma = 1$

Shift size = 10 + 1 = 11

K = ½

H = 10

Conclusion: The N^+ and N^- in the above table indicate, the number of consecutive periods that the CUSSUM C_-^+ and C_1^1 has been non – zero. The CUSUM calculation show that the Upper CUSUM period 8 = 27.5. this is the first period at which $C_-^+ > H = 10$, we conclude that the process is out of control at that point.

c. The Taguchi Loss Function

Step: Find the range

Range = Highest value – Lowest value

= 692 – 256

= 436

Step 2: Find the upper and lower control limit

From the R – Chart

Upper limit = 643

Lower limit = 380.3

Step 3: Calculate the standard deviation of the process

Standard deviation = range divide constant (2.77); constant of range method, chi-square distribution used to evaluate the probability of a process.

Standard deviation = $\frac{436}{2.77}$ = 157.4 => 157

Step 4: Calculate the loss function using the Taguchi loss function

$L = (UCL - LCL) \text{ X } \frac{6}{S.D}$

$L = (463 - 380.3) \text{ X } \frac{6}{157}$

L = 263 X 0.038 = 8.97

Step 5: Find the loss – adjusted process capability index

Total loss = 5

Ration of loss = 136

$= {}^{5}/_{136} = 0.0368$

Step 6: Convert the loss adjusted process capability index to a Taguchi score

= subtract the loss adjusted used process index – 6

= 6 – 0.0368

= 5.9632 => 6

Conclusion: A score of 5.9632 means that the process has a high degree of risk, and is in need of improvement. A score of 5.9632 falls below the acceptable threshold of 6.0, which indicate that the process is not meeting specification.

d. The Smaller – The – Best – Method

Step 1: Find the tolerance => upper limit – lower limit

= 643 – 380.3

Tolerance = 263

Step 2: Find the STB score

STB = U – S.D

= 516 – 157

= 359

Conclusion: This means that the process is meeting the specification

e. The Larger – The – Best Method

Step 1: Find the mean

U – S.D

= 516 – 157

= 359

Conclusion: this means that the process is performing well and is meeting specification.

CHAPTER 5

SUMMARY

5.1 General Conclusion

This topic attempts to present a similarity among cases of quality loss function by employing the target – mean ratio and proposing a common formula for the cases. It shows that the target – mean ratio can take different values to represent the cases. These topic bring about uniformity with regards to the methodology among the cases of STB, LTB and Taguchi loss function. It leads to consistent results and because of this consistency, the results can be compared easily.

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