MATHEMATICAL MODELS OF AUTOMATION SYSTEM AND IT'S APPLICATION

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CERTIFICATION

This is to certify that this project work titled: **MATHEMATICAL MODELLING OF AUTOMATION SYSTEM AND IT'S APPLICATION was carried out by IBOI JUDE** (PSC2003845)

Under the supervision of.

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DEDICATION

I dedicate this project work to God Almighty who is my protector and provider.

ACKNOWLEDGEMENT

I would like to express my gratitude to my project supervisor, PROF. D. E. A.OMOROGBE for his invaluable guidance, expertise, and throughout this project. My sincere thanks goes to University Of Benin And Faculty Of Physical Science And Department Of Mathematics for providing enabling environment and opportunity for me to complete this 4 years programmed, The knowledge and skill gained throughout my studies have been essential in shaping my understanding and approach to this work. I am deeply grateful to my parents and my elder brother, my younger sister for their constant love, support, and belief in me. Their sacrifices and encouragement have been my source of strength throughout this journey.

Finally, I thank God for His unwavering guidance and blessings, without which this achievement would not have been possible.

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ABSTRACT

This study focuses on the mathematical modeling of automation systems and their application in healthcare, transportation, and energy. Automation, first introduced by George Devol in the 1950s through his invention of the industrial robot Unimate, allows machines to perform tasks automatically without human effort. The aim of this study is to show how mathematical models can be used to represent and control automated processes. Mathematical models were developed for patient temperature control, traffic flow analysis, and solar energy management. The results show that modeling improves the efficiency, accuracy, and reliability of automation systems. Automation supported by mathematics reduces human error, saves time, and enhances decision making. This research proves that mathematical modeling is the foundation of automation technology. It also contributes to national growth through better healthcare, transportation, and energy systems. Overall, automation driven by mathematics supports innovation and technological advancement in society.

CHAPTER ONE

INTRODUCTION

1.1 Background of Study

An automation system (AS) can be viewed as the use of machines and computers to perform tasks with little or no human control. The process of work done by hand is called manual operation, while work done automatically by machines or computers is called automation. Examples include washing machines (which wash and rinse automatically), microwave ovens, smart lighting systems, printers, and photocopiers. Automation allows machines to perform repeated and precise actions without getting tired or making human errors. It replaces human effort with logical and mechanical control. According to Devol (1912–2011), automation is the automatic operation of machines guided by preset instructions to handle materials and perform tasks without continuous human control. Devol also invented the first industrial robot called Unimate in *1956*, which is seen as the beginning of modern robotic automation. Unimate, meaning "Universal Automation," was a programmable robotic arm used to handle heavy and dangerous factory tasks. Automation started in manufacturing but now covers a wide range of fields such as:

. Agriculture

. Transportation systems

. Manufacturing and robotics

. Process control (factories and industries)

. Business and service automation

. Healthcare and medical systems

. Energy systems

This study will focus on three key automation systems (Healthcare and Medical Systems, Transportation Systems and Energy Systems). Each area of application classified automation in a different way.

A. The classification of healthcare and medical systems are:

Diagnostic systems (used to detect diseases)

Monitoring systems (track patient health status)

Preventive systems (focus on preventing illness)

B. The classification of transportation systems are:

Land transportation (cars, trains, oil and gas transport)

Passenger transport systems (buses, airplanes, taxis)

Manual systems (walking, bicycles)

C. The classification of energy systems are:

Renewable energy systems (solar panels, wind turbines)

Electrical energy systems (power generation and transmission)

Industrial energy systems (factories, refineries)

The structure of healthcare and medical automation, transportation automation and energy automation system are given in the following figure below.

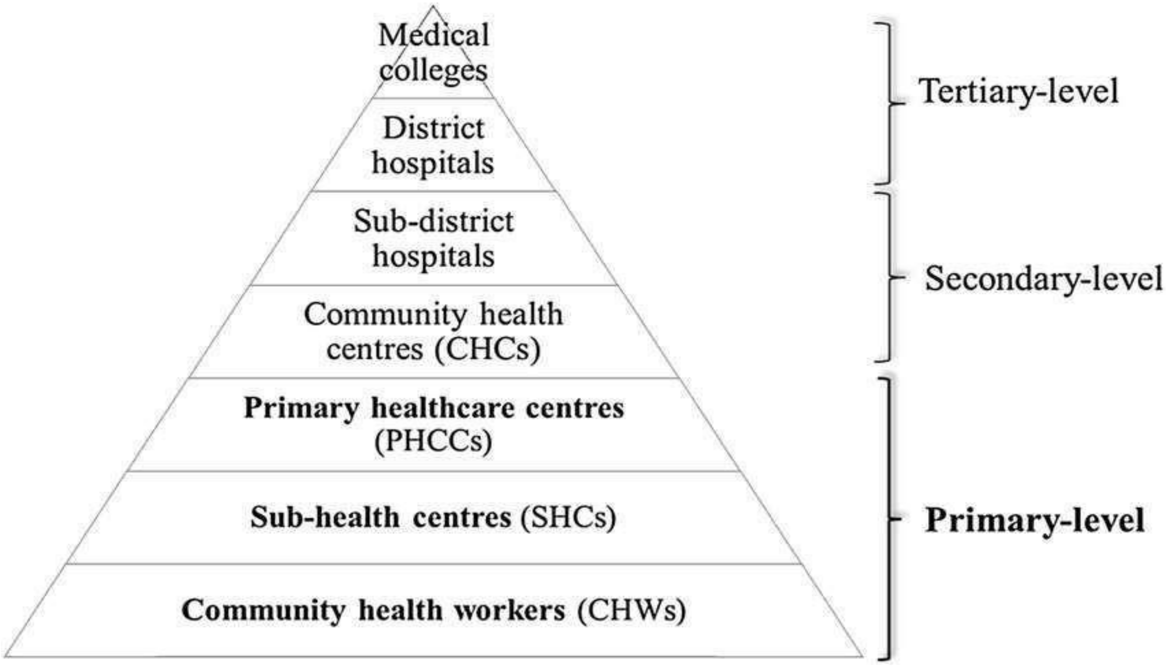


Figure 1.1 Structure of healthcare and medical automation system

Source: (Starfield 1994)

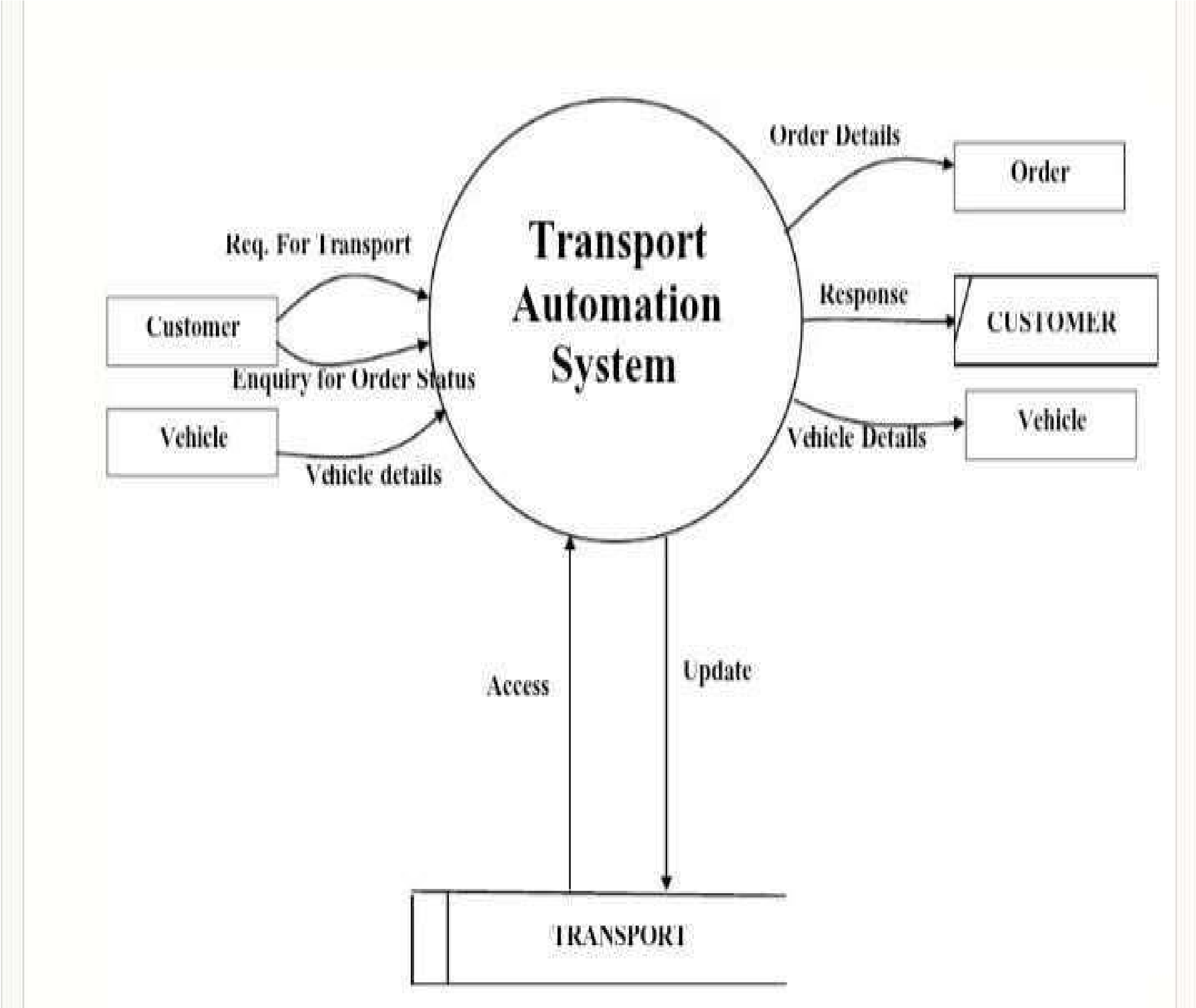


Figure 1.2 Structure of transportation automation system

Source: (May 1990)

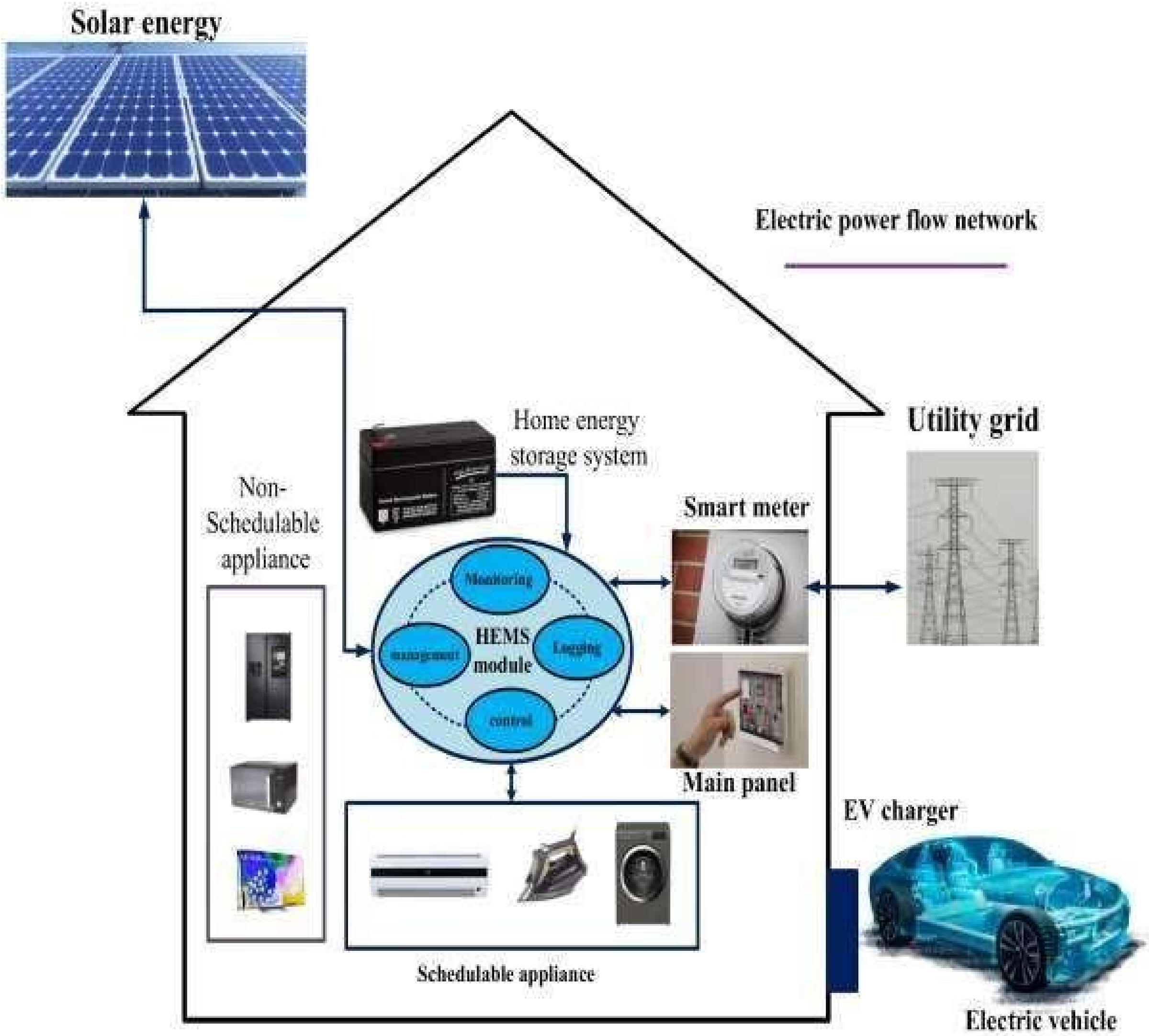


Figure 1.3 Structure of energy automation system

Source: (Ma & Wang 2021)

1.1.1 Mathematical View of Automation Systems

In an automation system, a control process is used to regulate machines and operations. There is no automation without a control system. A control system automatically manages or regulates a machine using input, process, and output, sometimes with feedback. A simple mathematical representation of an automation process can be written as:

1.1 $$Y(t) = f(X(t))$$ (Dorf & Bishop 2011)

Where:

$X(t)$ is the input (such as a sensor reading or user signal),

$Y(t)$ is the output (such as machine movement or temperature change),

f is the control rule or logic applied by the system.

For example, in a temperature control system, if the input temperature rises above a limit, the system automatically turns on a cooling fan. The relationship between input and output can be modelled simply as:

1.2 $$Y = k(X - X_0)$$

Where k is a proportional constant that determines how strongly the system responds. This kind of simple equation forms the basis of mathematical modelling in automation systems.

1.2 Statement of Problem

Many automation systems are designed and built without proper mathematical models. This leads to inefficiency, high energy use, and instability. Although automation replaces human effort, it still requires precise design to ensure that systems perform correctly and adapt to changes. The main problem is to develop mathematical models that accurately describe realworld processes. Such models can then be applied to improve performance, reduce errors, and enhance control and decision-making in automation systems.

1.3 Aim of Study

The aim of this study is to examine the mathematical modelling and simulation of automation systems and their applications in healthcare, transportation, and energy.

1.4 Objectives of Study

The objectives of the study are:

1. To examine the principles of mathematical modelling in automation systems.

2. To analyze how modelling is applied in industries such as healthcare, transport, and energy.

3. To review related literature on automation systems and control processes.

4. To represent selected automation systems using simple mathematical models.

1.5 Scope of Study

The scope of this study focuses on developing and explaining mathematical models that describe how automation systems work in three main areas healthcare and medical systems, transportation systems, and energy systems. Each model will show the relationship between input, process, and output, and how feedback ensures stability and accuracy.

1.6 Definition of Terms

1. AI, Artificial Intelligence: The intelligent method built into automation systems to improve decision-making and adaptability. Feedback Loop: A control mechanism that ensures automation systems remain stable and accurate.

2. Sensor: A device that converts a physical condition into an electrical signal.

3. Simulation: The use of computer based mathematical models to test and predict system performance.

4. Automation: The replacement of manual (human) tasks with machine based operations controlled by automation systems.

5. CPU, Central Processing Unit: The programmable logic unit that makes decisions and controls the automation process.

6. HMI, Human Machine Interface: A system that allows humans to communicate with and control machines.

1.7 Conclusion of Chapter

In this chapter, automation was defined as a self-regulating process that uses machines and computers to perform tasks without human effort. We looked at how automation began, its development, and how it is applied in various fields. Mathematical modelling helps in understanding and improving these systems by providing equations and logical relationships that describe how machines behave. Even though automation systems require effort, time, and cost to develop, they lead to better efficiency, reliability, and comfort. Mathematical modelling is therefore a key part of automation because it helps to predict, test, and improve system performance.

CHAPTER TWO

LITERATURE REVIEW

2.1 Introduction

This chapter reviews studies related to automation systems and mathematical modeling. It explains the meaning of automation, the different types of automation, and how mathematical models are used to represent real systems. Each model is described with its parameters and how it helps automation systems to function effectively. Automation and modeling work together to make machines operate automatically and more intelligently. By using mathematical models, engineers can study how systems behave, make predictions, and improve performance before real implementation.

2.2 Concept of Automation

Automation can be defined as the use of machines, control systems, and computers to operate processes automatically. According to Devol (1956), automation started with the invention of the Unimate robot, which was the first industrial robot used for repetitive factory work. Groover (2016) explained that automation combines mechanical devices, sensors, and computer logic to perform tasks without human effort. He identified three major types of automation systems:

1. Fixed Automation: Used in industries where production is repetitive, such as automobile assembly lines.

2. Programmable Automation: Used when the operation can be changed by reprogramming the machine, such as in CNC (Computer Numerical Control) machines.

3. Flexible Automation: Used in advanced systems that can quickly adapt to new tasks using

Artificial Intelligence (AI) and computer control.

Automation systems increase productivity, reduce errors, and improve safety in manufacturing, transportation, energy, and healthcare (Groover 2016).

2.3 Concept of Mathematical Modeling

Mathematical modeling is a method of representing a real system using mathematical symbols and relationships. Ogata (2010) defined a mathematical model as a set of equations that describe how a system behaves over time. Dorf &

Bishop (2011) explained that models help engineers to test and predict system behavior before physical design.

The general form of an automation system can be expressed as:

2.1 $Y(t) = f(X(t))$ (Ogata 2010) (Dorf & Bishop 2011)

$Y(t)$ is the system output, $X(t)$ is the input,

and

f *is* the control function that connects them.

Suppose the output Y of a system depends directly on the input X, it can be denoted as

2.2 $Y = kX$ (Nice 2011) (Ogata 210)

This shows that when the input increases, the output also increases in direct proportion.

Mathematical modeling is important because it allows systems to be tested, analyzed, and improved without using physical equipment.

2.4 Healthcare and Medical Automation Systems

Healthcare automation involves the use of sensors and computer Based machines to monitor and support patients. According to Trivedi and Singh (2017), mathematical models are used in medical systems to represent body functions such as heart rate, temperature, and blood pressure.

A simple healthcare model can be expressed as:

2.3 $$H(t) = H_0 + k(A - A_0)$$ (Trivedi & Singh 2017)

$H(t)$: current health condition,

H_0: normal or resting value, *A:* current activity

level, *A_0:* normal activity level,

k: constant that represents how sensitive the body responds to activity.

This model shows that when a person's activity level changes, the body responds automatically. Automation systems use this type of model to monitor patients and alert medical staff when there is a sudden change in condition (Trivedi & Singh 2017)

2.5 Transportation Automation Systems

Transportation automation deals with using machines and sensors to control the movement of vehicles and manage road systems. According to Greenshields (1935), traffic flow, speed, and vehicle density can be related by the model:

2.4 $$Q = V \text{ x } D$$ (Greenshields 1935) (May 1990)

Q: traffic flow (vehicles per hour),

V: average vehicle speed, and

D: vehicle density (vehicles per kilometer).

Suppose the traffic flow Q of a road depends on the speed V of vehicles and their density D, it can be denoted as $Q = V \text{ x } D$. This means that the flow of traffic increases when the speed of vehicles or the number of vehicles on the road increases. May (1990) explained that this model is used in traffic control systems and intelligent transportation to monitor traffic patterns and automatically adjust signals to reduce congestion.

2.6 Energy Automation Systems

Energy automation is used in power generation and distribution to control energy automatically. According to Dorf and Bishop (2011), energy systems are controlled using mathematical equations that show the balance between input and output power.

The basic model for energy balance is:

2.5 $$E(t) = P_{in}(t) - P_{out}(t)$$ (Dorf & Bishop 2011)

(Kothari & Nagrath 2014)

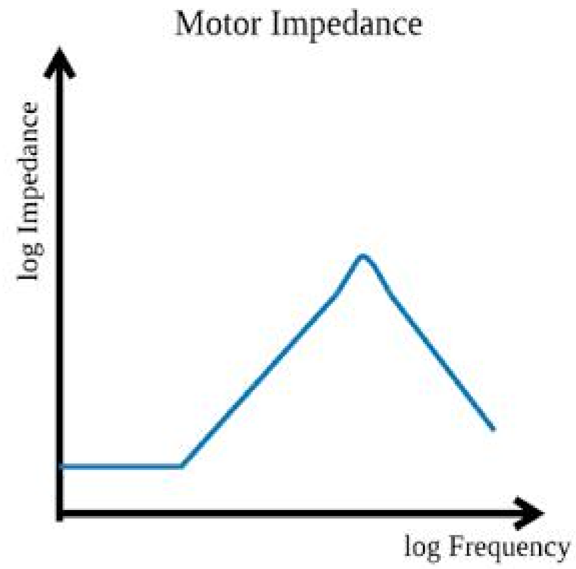
$E(t)$: energy remaining or stored, $P_{in}(t)$: input or

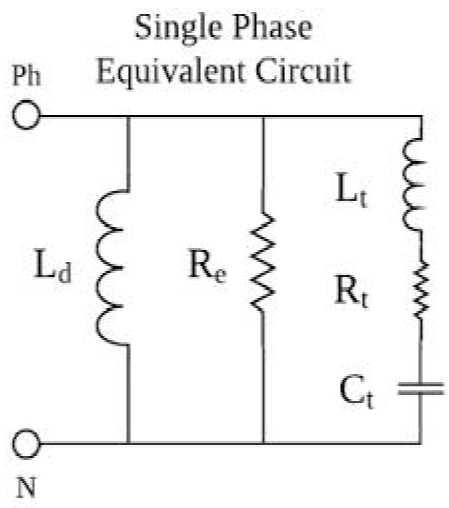
generated power, $P_{out}(t)$: output or used power.

Suppose an energy system generates power P_{in} and uses power P_{out}, the remaining energy can be represented as $E = P_{in} - P_{out}$ This helps the system to automatically balance energy flow and prevent overload. This model is very useful in renewable energy systems such as solar and wind plants, where power generation changes frequently. (Kothari, & Nagrath 2014).

2.7 Differential Equation Model

The dynamics of many systems whether they are mechanical, electrical, thermal, economic, biological and so on can be described in terms of differential equations. The differential equations can be obtained by the physical laws governing a particular system. This model is mostly in the time domain and in linear differential equations with constant coefficient. In this model, Newton's law is used for mechanical systems and Kirchhoff's law is used for electrical systems. Let's take an example of the water level control system using an automatic pumping machine. This system is an electrical system. Figure 1.4 and Figure 1.5 below is a diagram of motor impedance and single phase





equivalent circuit.

Figure 1.4 Motor Impedance **Figure 1.5 Single Phase Circuit**

Source: D. Ludois & A. Schroedermeier (2019), [Motor impedamce versus frequency plot and a suggested single phase equivalent circuit mode].

All the electrical elements are connected in parallel and series which can be resolved into series According to Kirchhoffs law, voltage across the RLC circuit (the single phase RIC circuit) is given by

$$V_R + V_L + V_C = 0 \quad (2.6)$$

$$Ri + L\frac{di}{dt} + \frac{1}{c}\int_0^t idt - V_i = 0 \quad (2.7)$$

$$V_i = R_i + L\frac{di}{dt} + \frac{1}{c}\int_0^t idt$$

$$V_i = R_i + L\frac{di}{dt} + V_c \quad (2.8)$$

Substitute the current passing through capacitor i = C $\frac{dV_c}{dt}$ in the above equation

⇨ $\mathbf{V_i} = RC\frac{dV_c}{dt} + LC\frac{d^2v_c}{dt^2} + v_c$ (2.9)

Divide through by LC

$$\frac{d^2v_c}{dt^2} + \frac{R}{L}\frac{dv_c}{dt} + \frac{1}{LC}V_C = \frac{1}{LC}V_i \quad (2.10)$$

This is a second order differential equation representing the electrical circuit of a single phase pumping machine. (Ogata, 2009).

2.8 Transfer Function

The transfer function of a linear time-invariant system is the ratio of the Laplace transform of the output Y(s) to the Laplace transform of the corresponding input X(s) with all initial conditions assumed to be zero.

Transfer Function = $\frac{Y(s)}{X(s)}$

The transfer function model of linear time invariant system is shown in figure 1.6

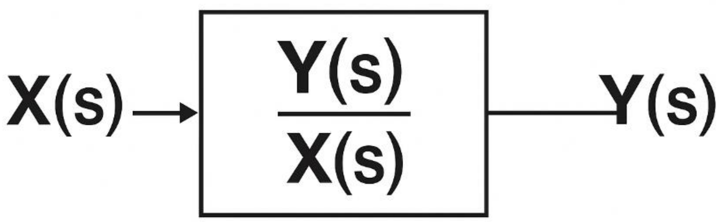


Figure 1.6 Transfer Function Of Open-Loop system

Source: Ogata (2009). [Mathematical modelling of control system].

In the last example of differential equation model, equation (2.10), we got the differential equation of an electrical system as

$$\frac{d^2V}{dt^2} + \frac{R}{L}\frac{dV_c}{dt} + \frac{1}{LC}v_c = \frac{1}{LC}V_i$$

Apply Laplace Transfer On Both Sides.

$$s^2V_c + \left(\frac{sR}{L}\right)V_c(s) + \left(\frac{1}{LC}\right)V_c(s) = \left(\frac{1}{LC}\right)V_i\,(s) \tag{2.11}$$

$$=> \{\, \boldsymbol{s^2} + \left(\boldsymbol{\frac{R}{L}}\right)\boldsymbol{s} + \boldsymbol{\frac{1}{LC}} \,\}\, \boldsymbol{V_c(s)} = \boldsymbol{\frac{1}{LC}V_i(s)}$$

$$\boldsymbol{\frac{V_i(s)}{V_i(s)}} = \frac{\boldsymbol{\frac{1}{LC}}}{\boldsymbol{s^2} + \left(\boldsymbol{\frac{R}{L}}\right)\boldsymbol{s} + \boldsymbol{\frac{1}{LC}}}$$

Where,

$\boldsymbol{V_c}$(s) is the Laplace transform of The output voltage V_c

$\boldsymbol{V_i}$(s) is the Laplace transform of the input voltage V_i

The Laplace transfer functions above are for open-loop systems. For a closed-loop control system, the Laplace transfer function is different because feedback is added to the input. When the output is fed back to the summing point for comparison with the input, it is necessary to convert the form of the output signal to that of the input signal. for example, in a water level control system, the output signal is usually the controlled water level. The output signal, which has the dimension of distance, must be converted to a current or voltage before it can be compared with the input signal. This conversion

is accomplished by the feedback element (sensor) whose transfer function is D(s), as shown in Figure 1.7 The role of the feedback element is to modify the output before it is compared with the input.

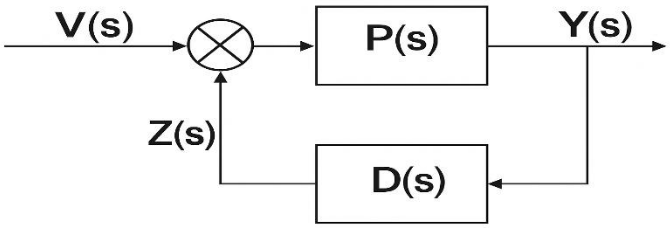


Figure 1.7 Closed-Loop Transfer Function Block Diagram

Source: Ogata (2009). [Mathematical modelling of control of control systems]

From figure 1.7 the transfer function relationship are:

P(s) = $\frac{Y(s)}{X(s)}$ (2.12)

D(s) = $\frac{C(s)}{Z(s)}$ (2.13)

And X (s) = V (s) - Z (s) (2.14)

Eliminating X(s) from these equation gives

Y(s) = P(s)[V(s) - Z(s)] (2.15)

Y(s) = P(s)[V(s) - $\frac{Y(S)}{D(s)}$]

$$Y(s) + \frac{P(s)Y(s)}{D(s)} = P(s)V(s)$$

$$Y(s)\left[1 + \frac{P(s)}{D(s)}\right] = P(s)V(s)$$

$$\frac{Y(s)}{V(s)} = \frac{P(s)}{1+\frac{P(s)}{D(s)}}$$

$$Y(s) = \frac{P(s)}{1+\frac{P(s)}{D(s)}}V(s)$$

Thus the output of the closed loop system clearly depends on both the closed loop transfer function and the nature of the input. (Ogata, 2009)

2.9 State Space Model

State space model (SSM) refers to a class of probabilistic graphical models (Koller and Friedman,2009) that describes the probabilistic dependence between the latent state variable and the observed measurement. The state or the measurement can be either continuous or discrete SSM provides a general framework for analyzing deterministic and stochastic dynamical systems that are measured or observed through a stochastic process. The SSM framework has been successfully applied in engineering, statistics, computer science and economics to solve a broad range of dynamical system problems (Zhe Chen,2013). The following basic terminology involved in state space modelling. The state of a dynamic system is the smallest set of variables (called state variables) such that knowledge of these variables at $t = t_0$, together with knowledge of the input for $t \geq t_0$ completely determines the behavior of the system for any time $t \geq t_0$. Note that the concept of state is by no means limited to physical systems. It is applicable to biological systems, economic systems, social systems, and other State Variables. The state variables of a dynamic system are the variables making up the smallest set of variables that determine the state of the dynamic system. If at least n variables $X_1, X_2, \ldots\ldots.. X_n$ are needed to completely describe the behavior of a dynamic system (so that once the input is given for $t \geq t_0$ and the initial state $t = t_0$ is specified, the future state of the system is completely determined), then such n variables are a set of state variables. State Vector. If n state variables are needed to completely describe the behavior of a given system, then these n state variables can be considered the n components of a vector x. Such a vector is called state vector. A state vector is thus a vector that determines uniquely the system state x(t) for any time $t \geq t_0$, state at $t = t_0$ is given and the input u(t) for $t \geq t_0$ is specified.

State Space. The n-dimensional space whose coordinate axes consist of the x1 axis, x2 axis, p, xn axis, where $X_1, X_2, \ldots\ldots, X_n$ are state variables, is called a state space. Any state can be represented by a point in the state space.

State Space Equations. In state space analysis we are concerned with three types of variables that are involved in the modelling of dynamic systems: input variables, output variables, and state variables. The state space representation for a given system is not unique, except that the number of state variables is the same for any of the different state space representations of the same system. The dynamic system must involve elements that memorize the values of the input for $t \geq t_0$. Since integrators in a continuous time control system serve as memory devices, the outputs of such integrators can be considered as the variables that define the internal state of the dynamic system. Thus the outputs of integrators serve as state variables. The number of state variables to completely define the dynamics of the system is equal to the number of integrators involved in the system. (Ogata,2009)

The state space model of Linear Time-Invariant (LTI) system can be represented as,

$$\overline{X} = AX + BU \qquad (2.16)$$

$$Y = CX + DU \qquad (2.17)$$

The equations (2.16) and (2.17) are Known as state equation and output equation respectively.

Where,

X and $\overline{X}$ are the state vector and the differential state vector respectively.

·U and Y are input vector and output vector respectively

- A is the system matrix.
- B and C are the input and the output matrices
- D is the feed-forward matrix

State Space Model from Differential Equation

Consider the series RLC circuit in figure 1.8 It has an input voltage, vi(t) and the current flowing through the circuit is i(t). There are two storage elements (inductor and capacitor) in this circuit. So, the number of the state variables is equal to two and these state variables are the current flowing through the inductor, and the voltage across the capacitor. From the circuit, the output voltage is equal to the voltage across the capacitor.

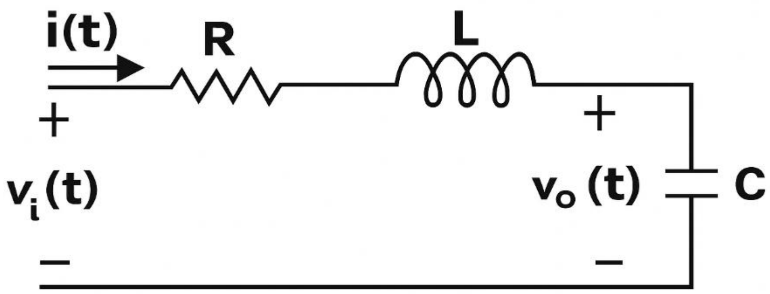


Figure 1.8 Series RLC Circuit

Source: [series RLC circuit]. Retrieved 5th December; 2022.

Inductor current = $i(t)$, Voltage across capacitor = $V_c(t)$

$$V_0(t) = V_c(t) \tag{2.18}$$

Apply Kirchhoff's voltage law around the circuit loop

$$V_i(t) = R_i(t) + L\frac{d_i(t)}{dt} + V_c(t) \tag{2.19}$$

$$\frac{d_i(t)}{dt} = -\frac{R_i(t)}{L} - \frac{V_c(t)}{L} + \frac{V_i(t)}{L}$$

Then, the voltage across the capacitor is

$$V_c(t) = \frac{1}{C}\int i(t)dt \tag{2.20}$$

Differentiate the above equation with respect to time

$$\frac{dV_c(t)}{dt} = \frac{i(t)}{c}$$

$$\text{State vector, X} = \begin{bmatrix} i(t) \\ V_{c(t)} \end{bmatrix} \quad (2.21)$$

Differentiate the vector

$$\overline{X} = \begin{bmatrix} \frac{d_i(t)}{dt} \\ \frac{dv_c(t)}{dt} \end{bmatrix} \quad (2.22)$$

Arrange the differential equation to standard form of state space model

$$\overline{X} = \begin{bmatrix} -\frac{R}{L} & -\frac{1}{L} \\ \frac{1}{C} & 0 \end{bmatrix} * \begin{bmatrix} i(t) \\ Vc(t) \end{bmatrix} + \begin{bmatrix} \frac{i}{L} \\ 0 \end{bmatrix} * [V_i(t)] \quad (2.23)$$

$$Y = [0 \quad 1] * \begin{bmatrix} i(t) \\ Vc(t) \end{bmatrix} \quad (2.24)$$

Where,

$$A = \begin{bmatrix} -\frac{R}{L} & -\frac{1}{L} \\ \frac{1}{C} & 0 \end{bmatrix}, \quad B = \begin{bmatrix} \frac{i}{L} \\ 0 \end{bmatrix}$$

$C = [0 \quad 1]$ and $D = [0]$

State space model of a system having transfer function of

$$\frac{Y(s)}{X(s)} = \frac{1}{s^2+s+1} \tag{2.25}$$

Rearrange the above equation

$$(S^2 + S + 1)Y(s) = X(s)$$

Apply inverse Laplace transform on both side

$$\frac{d^2y(t)}{dt^2} + \frac{dy(t)}{dt} + y(t) = X(t) \tag{2.26}$$

Let

$$Y(t) = X_{1,} \ \frac{dy(t)}{dt} = X_2 = \overline{X}_1 \text{ and } x(t) = x$$

Then, the state equation is

$$\overline{X}_2 = -X_1 - X_2 + X \tag{2.27}$$

The output equation is

$$Y(t) = Y = X_1 \tag{2.28}$$

$$\overline{X} = \begin{bmatrix} \overline{X}_1 \\ \overline{X}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} [X]$$

(2.29)

$$Y = [1 \quad 0] \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \tag{2.30}$$

2.10 Conclusion

This chapter reviewed automation and mathematical modeling in different areas such as Healthcare, Transportation, and Energy. It explained how automation reduces human effort and how mathematical models help represent system behavior. The equations $Y = kX$, $H(t) = H_0 + k(A - A_0)$, $Q = V x D$, *and* $E = P_{in} - P_{out}$ show how automation systems work through input and output relationships. These models help engineers to design systems that are more accurate, efficient, and intelligent. Furthermore, Differential Equation Model explains how changes occur in automation systems over time. It shows how variables such as temperature, speed, and energy can be expressed mathematically to understand and improve system performance.

CHAPTER THREE

METHODOLOGY / MODEL FORMULATION

3.1 Introduction

This chapter explains the method used to develop mathematical models for automation systems. The main goal of the methodology is to show how mathematical equations can represent reallife systems and how each parameter in the equation affects the performance of the system. According to Ogata (2010) the modeling process involves identifying the input, process, and output of a system and expressing their relationship mathematically. This chapter presents the general modeling approach and then explains the model formulation for healthcare, transportation, and energy systems.

3.2 Research Design

The research design used in this study is a descriptive review of the application of marhematical models to automation systems. The descriptive part explains the structure and behavior of automation systems, while the analytical part expresses them mathematically. Dorf & Bishop (2011) stated that in system analysis, mathematical modeling provides a way to predict the response of a system under different conditions. The design in this work focuses on representing automation systems as input–output models that can be tested and adjusted for accuracy.

3.3 Model Formulation Approach

To build any automation model, the first step is to identify the variables that affect the system.

Each automation system has:

Input (X): What enters or controls the system,

Process (f): The rule or controller that manages the input, and

Output (Y): The result or system response.

The general mathematical form of a control system can be written as:

3.1 $$Y = f(X)$$

According to Ogata (2010), this function shows that the output of any automated process depends on how the input is controlled by the system logic. Suppose the output Y of an automation system increases directly with the input X, it can be denoted as:

3.2 $$Y = kX$$ (Ogata 2010) (Nice 2011)

This relationship helps engineers understand how fast or slow a system reacts to a given input.

3.4 Healthcare Model Formulation

Healthcare automation systems use sensors and feedback controllers to monitor patients automatically. According to Trivedi, R. & Singh, A. (2017), biomedical devices such as heart monitors and ventilators are based on mathematical models that describe how the body responds to changes in condition. The general model can be expressed as:

3.3 $$T(t) = T_0 + k(U - U_0)$$ (Trivedi & Singh 2017)

$T(t)$: current temperature or patient condition,

T_0: normal or reference value,

U: current input (external or internal factor), U_0: normal input,

k: constant of sensitivity.

Suppose a patient's body temperature $T(t)$ changes with an external factor U, the relationship can be denoted as $T(t) = T_0 + k(U - U_0)$. This shows that as U increases, the temperature also increases proportionally. This type of modeling helps healthcare automation systems control and maintain stable conditions automatically.

3.5 Transportation Model Formulation

Transportation automation uses mathematical modeling to manage traffic flow and signal control. Greenshields (1935) introduced the relationship between vehicle speed, traffic flow, and vehicle density, this is expressed as:

3.4 $$Q = V \times D$$ (Greenshields 1935) (May 1990)

Q: traffic flow,

V: average vehicle speed, D:

vehicle density.

Suppose the flow of vehicles Q depends on the product of vehicle speed V and their density D, it can be denoted as $Q = V \times D$. This relationship helps traffic engineers understand how speed and congestion affect road performance. (Greenshields 1935 & May 1990)

May (1990) explained that automation systems apply this model in real time by adjusting signal lights and controlling vehicle entry to maintain smooth movement on highways.

3.6 Energy Model Formulation

Energy automation models are used to control power generation and distribution. According to Kothari & Nagrath (2014), energy balance is one of the most important models in power systems. It shows the relationship between generated power and consumed power.

This model can be expressed as:

3.5 $$E(t) = P_{in}(t) - P_{out}(t)$$ (Kothari & Nagrath 2014)

(Dorf & Bishop 2011)

$E(t)$: stored or remaining energy, $P_{in}(t)$: input power

(generated power), $P_{out}(t)$: output power (used power).

Suppose an energy system generates power $P_{in}(t)$ and delivers power $P_{out}(t)$, the difference $E = P_{in} - P_{out}$ represents the stored or unconsumed energy. Dorf & Bishop (2011) explained that this model helps in designing automation systems that prevent overload and maintain a balance between power supply and demand. This model is especially important in renewable energy automation where supply changes frequently, such as in solar or wind power systems.

3.7 Assumptions

In formulating these models, the following assumptions were made:

1. The systems operate under stable environmental conditions.
2. All sensors and feedback components provide accurate readings.
3. The system constants (k) remain steady for small changes in input.
4. Human control or interference is minimal.

These assumptions make the models simple and easy to analyze while still describing real system behavior.

3.8 Model Validation

Dorf & Bishop (2011) stated that validation of mathematical models involves comparing the model's response with actual system data. If both behave similarly, the model is considered valid. In this study, the models are based on standard control equations that are already proven effective in automation systems. Each model represents the core behavior of healthcare, transportation, and energy systems and can be used for simulation and further research.

3.9 Summary

This chapter explained how mathematical models were developed for different automation systems. Each model expresses the relationship between input, process, and output using basic equations. The models presented are simple but show how automation can be described mathematically. These models will be applied and illustrated further in the next chapter, titled "Illustration Examples and Results."

CHAPTER FOUR

ILLUSTRATION EXAMPLES AND RESULTS

4.1 Introduction

This chapter presents simple illustrations and results based on the mathematical models described in Chapters Two and Three. The aim is to show how these models can be used in automation systems such as healthcare, transportation, and energy. Each example uses assumed values to explain how the model works and how automation systems respond to changes in input.

4.2 Healthcare Automation System

Healthcare systems use sensors to monitor and control a patient's health automatically. As stated by Trivedi and Singh (2017), mathematical models help medical devices calculate patient changes in real time. According to Trivedi and Singh (2017) the general model is:

4.1 $$H(t) = H_0 + k(A - A_0)$$

$H(t)$ = new or current body temperature (°C)

H_0 = normal body temperature (°C)

A = patient activity level

A_0= normal or resting activity level

k = sensitivity constant of the body

4.2.1 Illustration Example

A patient recovering from malaria at the University of Benin Teaching Hospital (UBTH) is still connected to an automatic temperature sensor that records both the current body temperature and activity level. From observation, the patient's body temperature increases slightly whenever he or she move or walk around the ward. Given: H_0 = 36.5°C, A_0 = 2, A = 8, k = 0.25

Solution

The normal body temperature H_0 = 36.5°C

Normal activity level A_0 = 2 (sitting)

Current activity level A = 8 (light walking)

Constant k = 0.25 (this represent how sensitive the patient's body is to activity)

$H(t) = 36.5 + 0.25(8 - 2)$

$H(t) = 36.5 + 0.25(6)$

$H(t) = 36.5 + 1.5 = 38.0°C$

The patient's temperature increased from 36.5°C to 38.0°C when moving from rest to light walking. This means the automation system will automatically alert the nurse if the temperature goes above the normal threshold. This kind of model helps hospitals in Nigeria to monitor many patients at once using smart devices. It reduces stress for nurses and allows early detection of fever or relapse.

4.3 Transportation Automation System

Traffic systems use mathematical models to monitor and control vehicle movement. According to Greenshields (1935), traffic flow *Q* depends on vehicle speed *V* and density *D:*

4.2 $$Q = V \times D$$ (Greenshields 1935) & (May 1990)

where:

Q: traffic flow (vehicles per hour),

V: average vehicle speed,

D: number of vehicles per kilometer.

4.3.1 Illustration Example

The traffic condition at Ojota Bus Terminal, Lagos, during morning rush hours is being studied to improve signal timing through automation. At this time, vehicles move slowly because of high density on the main road. Given: V = 45km/h, D = 55veh/km

Solution

Q = 45 x 55

Q = 2475 vehicles/hour

About 2,475 vehicles pass through the road section every hour. If the sensor detects that vehicle density increases beyond 60 vehicles/km, the automation system will automatically extend the green light on the major road to reduce congestion. This example shows how simple modelling can help design smart traffic systems for busy roads in Nigeria.

4.4 Energy Automation System

Energy automation systems use sensors and control units to balance the power generated and the power used. As Kothari and Nagrath (2014) explained, the energy balance model is:

$$E(t) = P_{in}(t) - P_{out}(t) \quad (4.3)$$ (Kothari & Nagrath 2014)

(Dorf & Bishop 2011)

where:

$E(t)$: stored energy,

$P_{in}(t)$: input or generated power,

$P_{out}(t)$: output or consumed power.

4.4.1 Illustration Example

A solar energy automation system is installed in a secondary school in Benin City, Edo State. The solar panels generate power during the day and store extra energy in a battery that supplies power at night. I want to find how much energy remains in the battery after the school uses power for fans, lighting, and laboratory equipment. Given: $P_{in} = 400kW$, $P_{out} = 220kW$

Solution

$$E(t) = P_{in} - P_{out}$$

$$E(t) = 400 - 220$$

$$E(t) = 180kW$$

The stored energy in the battery is $180kW$, which can be used at night when there is no sunlight. If consumption rises later in the evening, the automation system will automatically balance the load by using part of this stored energy. This example shows how solar automation systems can help Nigerian schools reduce their dependence on the national grid.

4.5 Discussion of Results/Findings

This study investigate the application of mathematical model in automation systems is one of the most important scientific approaches that link mathematics with real-life technology. Through the examples and models examined in this work, The study reviewed that automation is not only about machines performing tasks automatically, but also about how mathematical models help machines to become intelligent, responsive, and reliable. Every automatic process depends on change and mathematical model are the tools that describe and control these changes over time. This study also shows the application of automation to healthcare, transportation and energy system for industrial and societal development. It is also reviewed that these study that all automated systems depend on a form of control, and that control can be represented as mathematical model that show input, output, and feedback relationships. This study epitomized that without a framework of mathematical model an efficient automation system would be a mirage. Various types of system were studied in this work. For example, in a healthcare monitoring system, the change in body temperature over time can be expressed as a mathematical model that helps the computer or machine to make decisions. Also transportation, traffic flow can be represented using a mathematical model that relates speed, density, and time. The same thing applies to energy systems where the balance between power input and power output can be modelled and controlled automatically. As a result mathematical modelling is to simplify real systems so that they can be studied, predicted, and improved.

This study also shows that automation systems can help Nigerian hospitals improve patient safety, reduce human error, and give faster response. From the transportation example, I discovered that small increases in average vehicle speed can cause large improvements in total traffic flow. This proves that automation in traffic light systems can help reduce congestion in Lagos and other cities, saving time and fuel for drivers. From the energy example, solar power systems can store extra energy and automatically balance consumption at night. This showed me that automation supports energy management, reduces waste, and helps schools, hospitals, and homes enjoy steady power supply without full dependence on the national grid.

From all these examples and results, from this study mathematical models are the pivots of automation systems. They help engineers and scientists to understand how systems change with time, and to control those changes using feedback and sensors. Also automation depends heavily on mathematical model because it allows systems to predict outcomes, make corrections, and remain stable even when the environment changes. Without mathematical models, automation would not be reliable or safe.

The use of mathematical model in automation systems has great importance for society and national development. In society, it helps improve the quality of life by reducing human stress and error. For example, automated hospital systems can save lives by monitoring patients continuously; automated traffic lights can prevent accidents and reduce congestion; and automated energy systems can provide electricity to rural areas through renewable sources like solar. All these make life easier, safer, and more productive for people.

In terms of national development, automation supported by mathematical modelling plays a key role in economic growth. It helps industries to produce faster and with fewer errors, which increases output and supports industrialization. It helps in education by training students to apply mathematical knowledge in solving real problems. It helps in technology advancement by encouraging innovation and local content creation. Automation also supports sustainable development by promoting clean energy and environmental protection. In general, countries that use automation grow faster because their systems work more efficiently, and mathematics especially mathematical model is what makes this possible.

Another major finding from this study is that automation also creates new opportunities for engineers, technicians, computer scientists, and mathematicians. In Nigeria, the use of automation can lead to new industries such as smart healthcare, intelligent transport systems, and renewable energy control systems. This will not only solve real problems but also create employment and support the government's vision for digital transformation. Mathematical model provide the analytical tools to design, test, and simulate these systems accurately before they are built in real life.

4.6 Conclusion

From the results and discussions presented in this chapter, it can be concluded that the application of mathematical model plays a vital role in the design and control of automation systems. Through the examples of healthcare, transportation, and energy systems, this study has shown that mathematical model help to describe how automated systems behave, predict future outcomes, and maintain stability during changes. Automation therefore depends on mathematics to function accurately and efficiently.

The findings revealed that automation systems built on mathematical models can greatly support human activities, improve productivity, and promote sustainable development. In Nigerian society, automation supported by mathematical modelling can lead to better healthcare delivery, smarter transportation management, and more reliable energy supply. This contributes directly to national growth, technological advancement, and economic development. In summary, It can be concluded that mathematical models provide the scientific foundation that allows automation systems to think, react, and adapt. They make automation more intelligent and useful for solving real-life problems in society.

CHAPTER FIVE

SUMMARY, CONCLUSION, AND RECOMMENDATIONS

5.1 Summary

This project focused on the mathematical modeling of automation systems and how automation is applied in different fields such as healthcare, transportation, and energy.

The study started in Chapter One with an introduction to automation, its background, objectives, and importance. Automation was explained as the process of using machines, computers, and control systems to perform tasks automatically without continuous human help.

Chapter Two reviewed past studies and explained key concepts such as automation, mathematical modeling, and how these models are applied.

Real authors such as Groover (2016), Ogata (2010), Dorf and Bishop (2011), and Trivedi and Singh (2017) were used to support the explanation.

Different models were discussed, including:

- The general control model $Y = kX$
- The healthcare model $H(t) = H_0 + k(A-A_0)$
- The transportation model $Q = V \times D$
- The energy model $E= P_{in} - P_{out}$

Each model was described in simple terms, showing its parameters and how it can represent real-life automation systems.

In Chapter Three, the methodology was explained. It showed how to build mathematical models by identifying system inputs, processes, and outputs. Each model was explained with its assumptions and parameters, without using numeric solving.

Chapter Four presented the illustration examples, results and findings. It gave simple calculations and showed how the models can work in practice. The results showed that:

. In healthcare, patient monitoring can be automated using sensors and equations.

. In transportation, traffic flow can be controlled by using speed and vehicle density data.

. In energy, automatic control can maintain balance between power supply and demand.

Overall, the study proved that mathematical models are important in automation systems because they help engineers predict, control, and improve performance before physical implementation.

5.2 Conclusion

Automation has made work faster, safer, and more accurate in all areas of human life. This study showed that by applying mathematical models, automation systems can be better understood and improved. The models developed in this work $Y = Kx$, $H(t) = H_0 + k(A - A_0)$, $Q = V \times D$, and $E = P_{in} - P_{out}$ all show how inputs affect outputs in control systems. These models also demonstrate that automation relies on feedback and mathematical relationships to perform correctly. In conclusion, mathematical modeling provides a foundation for building, testing, and refining automation systems in healthcare, transport, and energy. It reduces error, increases efficiency, and allows engineers to simulate system behavior before using physical equipment.

5.3 Recommendations

Based on my findings of this study, the following recommendations is as followed:

1. Encourage Model Based Design: Engineers and students should use mathematical modeling early in the design process to save time and cost before real life testing.

2. Improve Education and Training: More training should be given to students and professionals in automation and control systems, especially in modeling and simulation software.

3. Adopt Automation in Key Sectors: Government and industries should apply automation models in critical areas such as hospitals, transportation systems, and energy control to improve service delivery.

4. Promote Research in Automation: More research should be carried out on new models that can improve the accuracy and safety of automation systems, especially using Artificial Intelligence (AI) and data driven control.

5. Use Local Data for System Design: Future studies should collect real data from industries in Nigeria and other developing countries so that automation models can better reflect local conditions.

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