METHOD OF SOLVING LINEAR PROGRAMMING PROBLEMS: A CASE STUDY OF MOUKA FOAM COMPANY (BENIN CITY).

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UNDERTAKING

This project work was carried out by EGBAGBE ESTHER UWEMHE with the matriculation number PSC2105458. I have not copied any work of any author. All works utilized have been appropriately cited and referenced.

SIGNATURE ________________________________

EGBAGBE ESTHER UWEMHE

CERTIFICATION

This is to certify that this project work titled Method of Solving Linear Programming Problems: A Case Study of Mouka Foam Company (Benin City) was carried out by EGBAGBE ESTHER UWEMHE with matriculation number PSC2105458 under the supervision of Dr. Mrs. R. U. Omoregie.

Dr. Mrs. R. U. OMOREGIE. DATE
Supervisor

Prof. D. U. OKUONGHAE DATE
Head of Department

DEDICATION

This project work is dedicated to God Almighty and to Mr. and Mrs. Egbagbe for their support financially and otherwise.

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ABSTRACT

Linear programming is one of the most effective techniques used in decision-making and optimization problems, especially in business and industrial applications. This project focuses on the use of a linear programming to determine the most efficient way of maximizing profit and minimizing cost. Mouka Foam Company, Benin City, was used as a case study to demonstrate how mathematical models can support better production and resource allocation decisions

The simplex method was applied to the formulated linear programming problem derived from the assumed but realistic data of Mouka Foam Company. The process involved defining the objective function, identifying the constraints, introducing slack variables, and systematically applying the simplex algorithm to reach an optimal solution. The entire computation was manually solved and verified to ensure the accuracy of results

The result of the analysis shows that the simplex method provided an optimal solution that maximizes profit while minimizing production cost under the given constraints. The findings prove that linear programming is a reliable and efficient mathematical tool for managerial decision-making, especially in production planning and cost optimization.

CHAPTER ONE

INTRODUCTION

1.1 History of Linear Programming

The emergence of linear programming (LP) is rooted in the interplay between mathematics, economics, and the growing need for systematic decision-making methods in the twentieth century. While optimization concepts had been discussed in economic theory for centuries, it was during the 1940s that LP took shape as a distinct and practical methodology. Its formal development was closely linked to World War II, when military planners needed reliable tools to allocate scarce resources-such as manpower, raw materials

George B. Dantzig is widely recognized as the founder of modern LP. In 1947, he introduced the simplex method, an algorithm that could solve complex optimization problems involving multiple constraints and objectives. His work drew on earlier mathematical investigations into linear inequalities, notably those by Soviet mathematician Leonid Kantorovich and American mathematician John von Neumann. After its initial military use, LP rapidly expanded into civilian applications. By the 1950s and 1960s, industries were adopting it for production scheduling, transportation routing, and financial portfolio optimization. The spread of computer technology during this era further accelerated its adoption, enabling practitioners to tackle problems with thousands of variables and constraints that would have been impractical to solve manually. Advancements continued in the 1980s with Narendra Karmarkar's interior-point method, which offered a new approach to solving large-scale LP problems more efficiently. Today, LP remains an essential tool across sectors such as manufacturing, logistics, energy, and finance. Its enduring value lies in its adaptability, computational strength, and ability to provide structured solutions in situations where resources are limited and choices must be optimized.

Today, linear programming remains one of the most powerful and widely used optimization tools in science, business, engineering, and economics. Software packages such as LINDO, MATLAB, and IBM CPLEX have made it even more accessible for industry and academic use. The enduring significance of LP is a testament to its simplicity, generality, and effectiveness in modeling real-life decision-making problems.

1.2 Concept of Linear Programming

Linear programming (LP) is one of the most widely used techniques in operation research and mathematics. It deals with the problem of optimizing an objective function that is subject to a series of linear inequality or inequality constraints. The term linear indicates that both the objective function and the constraints can be expressed as linear functions of the decisions variable while the word programming refers to systematic planning or decision making under limited resources,

The central philosophy of linear programming is to allocate scarce resources in the most efficient way possible. These resources may include raw materials, machine time, labor hours, capital, or space depending on the context of application. Because of this linear programming has found relevance in a wide range of disciplines such as economics, engineering, logistics, agriculture, health care, telecommunications, and production planning.

Most production managers base their decisions in the total input used in the production and the total input produced. This system of decision making always have a setback in that, it brings a reduction in the accuracy of forecasting for the future, such as price fluctuation and shortage of raw materials. The problem of decision making therefore brings about the application of linear programming model, which is now seen as a concept which all decision linear programming problems fit into the following functions

- The decision variables: The variable in a linear program are a set of qualities that need to be determined in order to solve the problem i.e. the problem is solved when the best values of the variables have been identified.
- The objective function: The objective of a linear programming problem will be to maximize or to minimize some numerical value. The value may be the expected net present value of a project or it may be the cost of a project. The objective function indicates how each variable contributes to the value to be optimized in solving problem.
- The constraints: The constraint is defined as the possible values that the variable of a linear programming problem may take. They typically represent resources constraints ,or the minimum or maximum level of some activity or condition
- The non-negativity constraints: The variable of linear programming must always take non negativity values i.e. they must be greater than or equal to zero. The non-negativity constraints are part of all linear programming formulation and are always included in a linear programming formulation

1.3 Definition of Linear Programming

Linear Programming (LP) is a branch of mathematical optimization that focuses on achieving the best outcome such as maximum profit or lowest cost in a mathematical model whose requirements are represented by linear relationships. In simpler terms, it involves optimizing a linear objective function, subject to linear constraints, which are typically in the form of equalities or inequalities. LP models are made up of three primary components: an objective function, constraints, and decision variables. The goal is to find the values of the decision variables that maximize or minimize the objective function while satisfying all the constraints.

The foundations of linear programming were laid by George B. Dantzig in 1947 with the development of the simplex method, which remains one of the most widely used algorithms for solving LP problems. Linear programming is applicable across many fields including economics, business, engineering, and military planning. It provides a systematic and efficient way to make optimal decisions under a given set of resource limitations. Its real-world applications range from minimizing transportation costs, scheduling workers, to optimizing production levels in factories.

In operations research, linear programming is a fundamental topic because it provides a framework through which practical decision-making problems can be modeled and solved effectively (Hillier & Lieberman, 2021).

Linear programming is the analysis of the problems in which a linear function of a number of variables is to be optimized (maximized or minimized) whose variables are subjected to a number of constraints in the mathematical linear inequalities. From the above definitions, it is clear that

(i) Linear programming is an optimization technique, where the underlying objectives is either to maximize the profit or minimize the loss

(ii) It deals with the problem of allocation of finite(limited)resources between different competing activities in the most optimal manner

(iii) Linear programming has been highly successful in solving the following types of problems, blending strategy formulation, marketing and distribution

(iv) Though linear programming has wide diverse application, yet all linear programming has the following properties in common

(a) The objective is always the same (profit maximization or cost minimization)

(b) Presence of constraints which limit the extent to which the objectives can be achieved

(c) Availability of alternatives i.e. different course of action to choose from

1.4 Importance of Linear Programming

1. Optimal Use of Resources

Linear programming (LP) helps organizations determine the best possible use of limited resources such as time, labor, materials, and capital. It ensures that these resources are allocated in the most efficient way to maximize output or minimize costs, thereby improving productivity and profitability.

2. Improves Decision-Making

Linear programming provides a structure mathematical approach to decision making. It supports managers and planners in evaluating various options and selecting the most beneficial course of action. This is especially useful in complex business environment where intuitive decision may not yield optimal result.

3. Facilitates Cost Minimization

A key objective of LP is to minimize operational costs while meeting all necessary constraints. It is widely used to minimize transportation costs, production expenses, and labor costs in industries, leading to more economical business operations.

4. Maximizes Profit

By optimizing the allocation of resources and production scheduling, LP helps organizations to maximize their profit. It ensures that businesses invest resources in the most profitable ventures within their operational limits.

5. Enables Quantitative Analysis of Business Problems

LP transforms qualitative business challenges into quantifiable models that can be solved using systematic approaches. This enhances the analytical rigor in problem-solving and strategic planning.

6. Used in Workforce Scheduling

Organizations use LP for employee scheduling to ensure that workforce requirements are met at minimum labor costs. It helps assign shifts and tasks based on availability and productivity levels, making operations smoother and more efficient.

7. Reduces Waste

By accurately modeling resource constraints and needs, LP helps minimize waste of raw materials and time. It ensures that only the necessary resources are used, contributing to sustainability and cost control.

8. Aids supply chain management

LP is critical in optimizing supply chain processes, including procurement, manufacturing, inventory management, and distribution. It ensures smooth flow of goods and information with minimized delays and costs.

9. Supports Multi-Objective Optimization

Advanced linear programming techniques can handle multiple objectives simultaneously, such as minimizing cost while maximizing quality. This flexibility makes LP adaptable to various real-world industrial applications.

10. Applicable Across Various Sectors

The usefulness of LP is not limited to one industry. It is widely applied in agriculture (crop planning), transportation (routing and logistics), finance (portfolio optimization), energy (load distribution), and healthcare (staff allocation), making it a universal problem-solving tool.

11. Helps in Risk Management

By modelling various constraints and scenarios, LP can help in identifying and managing risks associated with operational decisions. This is especially important in industries where errors can lead to financial or safety repercussions.

12. Promotes Scientific and Logical Thinking

LP instills a logical, step-by-step approach to tackling complex issues. It emphasizes the importance of constraints and objectives, fostering scientific problem-solving skills in students and professionals alike.

1.5 Limitations of Linear Programming

2. Assumption of Linearity

Linear programming assumes a linear relationship between variables in the objective function and constraints. However, many real-world problems exhibit non-linear characteristics, limiting the applicability of LP

1 Single Objective Focus

LP models are usually designed to optimize a single objective function, such as profit maximization or cost minimization. In practice, organizations often need to balance multiple objectives simultaneously

3. Certainty in parameters

LP assumes that all coefficients in the objective function and constraints are known with certainty This is rarely the case in real-world situations, where data is often uncertain or subject to change

4. Inflexibility with Integer Constraints

LP does not handle integer or binary variables well. Many decision problems require solutions in whole numbers, which calls for more complex methods such as Integer Linear Programming

5. No Consideration for Qualitative Factors

LP only considers quantitative factors and ignores qualitative aspects like employee morale, customer satisfaction, or brand reputation, which may be crucial in decision-making

6. Sensitivity to Data Changes

LP models can be very sensitive to changes in input data. A small variation in coefficients can significantly alter the optimal solution, making the model unreliable under data uncertainty

7. Ignores Human and Social Factors

LP models are designed based on mathematical logic, often overlooking human behavior, ethical considerations, and organizational culture, which are essential in real-world implementation

1.6 Scope of Study

This study focuses on the application of linear programming methods in the optimization of production activities at Mouka Foam Company, Benin City. Specifically, it seeks to examine how linear programming can be employed to improve production efficiency by determining the optimal allocation of resources such as labor, raw materials, and machine hours. The scope also includes identifying common production constraints and modeling them as linear inequalities. The study considers only linear relationships among variables, excluding nonlinear or stochastic elements. It focuses on maximizing profit and minimizing cost, which are central objectives in industrial operations.

The research is limited to linear programming models and techniques such as the graphical method and the simplex method. While other optimization techniques exist, they fall outside the scope of this study.

1.7 Objectives

The primary objective of this research is to explore and demonstrate methods of solving linear programming problems and apply them to real-life situations in a production environment. The specific objectives include:

1. it explains the concept and importance of linear programming in resource optimization.

2. It identifies different methods for solving LP problems, such as the graphical method and simplex method.

3. It models production constraints and objectives of Mouka Foam Company using LP.

4. It analyzes how LP can improve production planning and profitability.

5. It provides practical solutions and recommendations based on LP analysis for enhanced decision-making.

1.8 Basic Terms in Linear Programming

Understanding linear programming requires familiarity with some fundamental terms:

1. **Decision Variables** – Variables that represent choices available to the decision maker.

2. **Objective Function** – A linear function representing the goal of the optimization, such as maximizing profit or minimizing cost.

3. **Constraints** – Linear inequalities or equations that restrict the values that the decision variables can take.

4. **Feasible Region** – The set of all possible points that satisfy the constraints.

5. **Optimal Solution** – A solution that yields the best value (maximum or minimum) of the objective function within the feasible region.

6. **Slack Variable** – A variable added to a 'less than or equal to' constraint to convert it into an equation.

7. **Artificial Variable** – A variable introduced to find an initial feasible solution in the simplex method.

8. **Simplex Method** – An algorithm for solving LP problems with more than two decision variables.

1.9 Work Outline

This research is structured into several chapters to provide a systematic exploration of linear programming and its application in production optimization:

- Chapter One introduces the research topic, outlines its objectives, significance, and scope.
- Chapter Two reviews relevant literature on the history and development of linear programming, including key concepts and solution techniques.
- Chapter Three presents the research methodology, including data collection and the analytical techniques used.
- Chapter Four applies the linear programming models to the case of Mouka Foam Company, analyzing and interpreting the results.
- Chapter Five summarizes the findings, draws conclusions, and offers recommendations for further study or practical implementation

CHAPTER TWO

LITERATURE REVIEW

2.1.0 Empirical Review On Linear Programming

Over the years, many empirical studies have been conducted on the application of linear programming in various sectors such as manufacturing, agriculture, health care, energy and supply chain management. These studies provide evidence that linear programming is not a theoretical but also a practical tool for solving real world optimization problems. Several empirical studies have demonstrated the relevance of linear programming in solving real world problems, particularly in production, resources allocation, and decision making in industries

2.1.1 Manufacturing Sector

In the Nigeria manufacturing industry, several researchers have investigated the role of linear programming in improving production efficiency

i. **Ogunleye and Adeyemi (2020)** applied simplex method of linear programming to optimize production in a textile company in Lagos. They used simplex method to allocate fabric, labor hours, and machine time across three product lines, and this increased profits by 18% and reduced excess use of labor hours. This demonstrated that linear programming is a valuable tool in industries where multiple products compete for limited resources.

ii. **Adewumi (2018)** studied a cement production company in ogun state and applied linear programming to determine the optimal mix of raw materials required for clinker production. The findings revealed that linear programming was able to reduced raw materials wastage by 12% and lowered production cost while maintaining product quality

iii. A related study by **Okafor (2023)** focused on the application of linear programming in a foam manufacturing company in Anambra state. The study showed that by applying linear programming to mattress production planning. The company was able to identify the exact number of orthopedic, student, and regular mattresses to produce, given its constraints in labor, machine hours, and foam chemicals. The study concluded that linear programming provides a scientific basis for production decisions and helped the company achieve higher profitability.

2.1.2 Agricultural Sector

The agricultural sector has also benefited from the application of linear programming

i. **Eze and Okoro (2021)** conducted a study on crop planning in enugu state. Their model allocated farmland among maize, cassava, and rice cultivation under resources constraints such as land size, fertilizer, and capital. The findings revealed that linear programming increased farm output and income compared to traditional farming practices

ii. In another study, **Adeyemo (2019)** applied linear programming to fish farming in Ibadan. The objective was to determine th best combination of feeds to minimize cost while ensuring proper nutrients requirements for fish growth. Results showed that linear programming helped reduced feed cost by 15% without affecting the fish growth or quality. This study highlighted the flexibility of linear programming in optimizing farm management decision.

2.1.3 Health and Medical Applications

Linear programming has also found application in health care resource allocation.

i. **Akinola and Bello (2020)** studied a general hospital in Abuja where linear programming was applied to allocate doctors, nurses and available operating theater time across various departments. The study revealed that linear programming helped in reducing patients waiting time while improving the efficiency of resources used

ii. Similarly **Hussain et al. (2021)** in Pakistan applied linear programming in planning vaccination distribution during the COVID-19 pandemic. Their results showed that linear programming minimized vaccine wastage and ensured equitable distribution across different regions. This provides evidence of the importance of linear programming in managing scarce resources in health emergencies

2.1.4 Supply Chain and Transportation

Linear is frequently used in supply chain management, especially for minimizing transportation and logistics cost.

i. **Yusuf and Ibrahim (2022)** studied a food and beverage company in Kaduna and developed a transportation model using linear programming for distribution of products to warehouses and retailers. The finding indicated that transportation costs were reduced by 22% while still meeting all customers' demands

ii. **Olawale (2019)** examined the distribution of petroleum products across depots in Nigeria. The results showed that linear programming provided the most cost effective distribution plan, considering the limited number of trucks and fuel supply.

2.1.5 Energy and Power Systems

The energy and power system has also witnessed several empirical applications of linear programming

i. **Uche and Nwankwo (2020)** applied linear programming to optimize electricity generation across multiple power plants in Nigeria. Their findings showed that linear programming minimized electricity generation cost while meeting electricity demand.

The study also emphasized that linear programming could play a significant role in addressing Nigeria's persistent energy crisis

ii. Internationally, **Singh and Prasad (2021)** conducted an empirical study in india on the use of linear programming for scheduling hydroelectric power generation, whereby they used linear programming model for balancing water resources allocation with electricity demand. This resulted to an improved electricity generation efficiency and reduced energy shortages.

2.1.6 International Evidence

Outside Nigeria, many empirical works have confirmed the effectiveness of linear programming in different contexts

i. **Williams (2018)** studied linear programming applications in the automobile industry in the united states and found that linear programming models helped manufacturers determine the optimal production schedule for cars under resources constraints

ii. In china**, Zhang and Li (2020)** investigated the role of linear programming in reducing air pollution from steel production. Their study showed that linear programming models allowed companies to minimize emissions by 10% while keeping production levels profitable.

iii. **Sharma (2017)** applied linear programming to a pharmaceutical firm in India. The study revealed that the use of Linear Programming techniques in production scheduling improved resource allocation by 15% and significantly reduced idle time in the factory.

The reviewed studies consistently show that linear programming is a powerful tool for decision- making across multiple sectors. For both Nigeria and international studies, one clear conclusion emerges; linear programing provides an empirical foundation for optimal decision making wherever resources are limited and competing needs to be balanced

CHAPTER THREE

METHODOLOGY

This chapter discusses the methodology adopted in solving linear programming problems. It focuses on the mathematical techniques used to obtain optimal solutions in resources allocation, production planning, and related industrial applications, they include;

- The Simplex method
- The big M method
- The graphical method
- The interior point method
- The two phased method
- The revised method

3.1.1 The Simplex Method

The method most frequently used when solving linear programming is the simplex method, the first step in applying the simplex method is to clearly define the decision variables, the objective function, and the system of constraints. The objective function represents either a maximization (e.g. profit) or the minimization (e.g. cost) goal, while the constraints embody the resource limitations. The model is then expressed in a standard linear form suitable for simplex computation. The simplex method is especially useful for solving linear programming problems that involves more than two decision variables, since graphical methods cannot handle such cases. In practical applications, the method has proven to be highly efficient. On average, it requires only about two or three times the number of equality constraints in iterations before reaching the optimal solution. This efficiency explains why it remains one of the most widely applied techniques in operations research.

3.1.2 **Standard Form of the Simplex Method**

The linear programming (maximizing problem) is of the form:

Maximize (or Minimize) $Z = c_1x_1 + c_2x_2 + \dots + c_nx_n$

Subject to:

$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n \leq b_1$

$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n \leq b_2$

.

.

.

$a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n \leq b_m$

with non-negativity conditions:

$x_1 \geq 0, x_2 \geq 0 \dots x_n \geq 0: b_1 \geq 0, b_2 \geq 0 \dots b_m \leq 0$

Where:

- Z = **The objective function** to be optimized (maximized in the case of profit and minimized in the case of loss).
- c_j = **The contribution coefficient** of each decision variable x_j
- x_j = **The decision variables** whose values are to be determined.
- a_{ij} = **The resources consumption coefficient**, representing the amount of resource i *required* by decision variable j.

- b_i = **The right hand side constants**, representing the available amount of resource *i*.

This formulation captures the essence of linear programming by defining (a) an objective function, (b) a system of linear constraints , (c) non negativity conditions.

To put the problem in the standard form, all inequality constraints are changed into equalities

In this situation where the constraints are expressed with less than or equal to, we introduce a non-negative variable known as a SLACK variable. This slack variable is added to each inequality constraints in order to convert it into an equation

Maximize $Z = c_1x_1 + c_2x_2 + ... + c_nx_n$

Subject to:

$a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n \leq b_1$

$a_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n \leq b_2$

.

.

.

$a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n + S_m = b_m$

here ,S,(for i =1,2,…,m) are the slack variables, which must also be non-negative.

3.1.3 Construction of the Simplex Tableau

The simplex tableau is a tabular arrangement that simplifies calculations during iterations. It represents the objective function and constraints in a compact form, enabling systematic pivoting operations to improve the solution. The tableau clearly shows basic and non-basic variables, their coefficients, and slack or surplus values added to transform inequalities into equalities. Here is a general layout of a simplex tableau

SIMPLEX TABLEAU STRUCTURE

Basic variables	X_1	X_2	S_1	S_2	RHS(b)
S_1	a_{11}	a_{12}	1	0	b_1
S_2	a_{21}	a_{22}	0	1	b_2
Z(obj fn)	$-c_1$	$-c_2$	0	0	0

Explanation of columns:

- x_1,x_2: decision variables
- s_1,s_2: slack variables(added to convert inequalities into equalities)
- RHS(b): the right hand side constants from the constraints
- Z row: represent the objective function, with coefficient as negative for maximization problems

What simplex method does is that, it makes number of solution variable equal to number of constraints, the values of the solution variables are read at the end of each row.it is important to note that the slack variables are the starting solution. This solution may not be optimal but feasible.

3.1.4 Iterative Simplex Algorithm

The algorithm proceeds step by step:

1. Formulate the problem in standard form.

Convert the linear program so the objective is a maximization, all constraints are equalities (by adding slack/surplus/artificial variables), and all variables are non-negative. This standard form is required to build the initial tableau.

2. Construct the initial simplex tableau.

Create a table that lists the coefficients of all decision and auxiliary variables, the basic variables, and the right-hand side (RHS) constants. The bottom (objective) row contains the negative of the objective coefficients for a maximization problem. The initial basic feasible solution is read from the tableau (usually the slack variables).

3. Check the optimality condition.

Inspect the objective (Z) row entries corresponding to non-basic decision variables. If all these entries are non-negative (≥ 0 for maximization), the current tableau gives an optimal solution and the algorithm stops. If one or more entries are negative, improvement is still possible and you proceed to the next step.

4. Select the entering variable (pivot column).

Identify the non-basic variable with the most negative coefficient in the Z row ,this is the variable whose increase promises the greatest immediate improvement in the objective. Mark the corresponding column as the pivot column.

5. Apply the minimum ratio test to select the leaving variable (pivot row).

For every row whose entry in the pivot column is positive, compute the ratio RHS ÷ (pivot-column coefficient). Ignore rows with zero or negative entries in the pivot column (they cannot limit growth). The smallest positive ratio identifies the pivot row; the basic variable in that row will leave the basis. If no positive coefficients exist in the pivot column, the objective is unbounded in that direction.

6. Perform the pivot (row operations).

a. Divide the pivot row by the pivot element to make the pivot equal to 1.

b. For every other row (including the objective row), subtract an appropriate multiple of the new pivot row so that all other entries in the pivot column become 0. After these operations the entering variable becomes basic (its column is a unit vector) and feasibility is preserved.

7. Update the basis and record the new solution.

Replace the leaving variable with the entering variable in the basic-variable column. Read the current basic feasible solution from the RHS values these are the current values of the basic variables while all no basic variables are zero. Note the new objective value from the Z row RHS.

8. Test for special cases and guard against cycling.

•Degeneracy: If a basic variable becomes zero after a pivot, the tableau is degenerate; repeated zeros can slow progress.

•Cycling: Although rare, cycling can cause the algorithm to repeat tableaus indefinitely. Use anti-cycling rules (for example, Bland's rule) if necessary.

•Unboundedness and infeasibility: If the minimum ratio test finds no valid leaving row, the objective is unbounded; if the initial phase (e.g., with artificial variables) cannot remove artificial, the model is infeasible.

9. Repeat from step 3 until termination.

Continue iterating: check optimality, choose an entering variable, select a leaving variable, pivot, and update. The process terminates when the optimality test in step 3 is satisfied (no negative entries in the objective row for maximization), or earlier if a special case (unboundedness, infeasibility) is detected. In practice, this iterative procedure typically converges quickly for most problems encountered in applications.

a) Identify the entering variable(the non-basic variables with the most negative coefficient in the objective function row
b) Determine the leaving variable using the minimum ratio test
c) Perform pivot operations to update the tableau
d) Repeat until no negative coefficient remain in the objective function row, indicating an optimal solution

3.1.5 Evaluating Efficiency of the Method

The efficiency of the simplex method has been one of the most discussed aspects of linear programming. From a theoretical perspective, the simplex algorithm is known to have an exponential worst-case complexity. This means that in the most extreme and unlikely scenarios, the number of iterations required to reach an optimal solution could grow very rapidly with the size of the problem. Despite this theoretical limitation, practical experience across industries has consistently demonstrated that the method performs remarkably well on most real-world problems

One of the main reasons for its efficiency is that the simplex method exploits the geometry of linear programming problems. Instead of searching through every possible solution point, the algorithm moves along the edges of the feasible region, focusing only on the vertices (corner points) where optimal solutions are guaranteed to lie. In practice, this means that the simplex method typically finds the optimal solution after examining only a small fraction of the possible feasible solutions.

Another aspect that contributes to its efficiency is the structured nature of the tableau computations. Each iteration involves straightforward row operations identifying the entering variable, selecting the leaving variable, and performing pivoting. Modern computational tools have further optimized these calculations, enabling the simplex method to solve very large problems with thousands of constraints and decision variables within reasonable time limits.

The efficiency of the simplex method is also reflected in its robustness when applied to different problem types. Whether the model involves production planning, resource allocation, scheduling, or transportation, the simplex procedure can be adapted to handle these variations effectively. Its extensions, such as the dual simplex and revised simplex methods, further increase its computational speed and stability in large-scale applications.

However, efficiency must also be considered in light of potential complications. Cases of degeneracy can cause the algorithm to stall, producing repeated basic feasible solutions without improving the objective function. Similarly, although rare, cycling may slow down or even prevent convergence unless anti-cycling rules are applied. Despite these challenges, refinements of the method have made it a reliable and efficient tool for decades.

In summary, while the simplex algorithm is not guaranteed to be the fastest method in every theoretical sense, it has proven to be highly efficient in practice. The combination of its logical structure, adaptability, and strong computational support explains why it continues to be one of the most trusted and widely used optimization methods in both academic research and industry practice.

3.2.0 Advanced Extensions of the Simplex Method

3.2.1 The big M Technique

In linear programming, many real- world problems do not always present themselves in the neat form required by the simplex method. In some cases, certain constraints are expressed as inequalities that cannot be handled by slack variable alone. For example, when constraints are of the type "≥" or "=", simply adding slack variables does not produce a valid initial basic feasible solution. To overcome this difficulty, researchers developed what is known as the **Big-M technique**. This method modifies the linear programming problem so that it can be solved using the simplex procedure while maintaining feasibility and optimality.

The Big M technique introduces artificial variables into the problem. These artificial variables serve as temporary placeholders that make it possible to begin the simplex algorithm with a valid initial solution. For every "≥" or "=" type of constraint, an artificial variable is added to the equation to ensure equality and feasibility. However, since artificial variables are not part of the original problem, they must eventually be removed from the solution. This is achieved by assigning a very large penalty cost, represented by the symbol "M", to these artificial variables in the objective function.

The role of the penalty cost is crucial. By associating each artificial variable with ±M in the objective function (−M for maximization and +M for minimization problems), the simplex algorithm is guided to drive these artificial variables out of the solution as quickly as possible. In practical terms, the presence of M ensures that keeping artificial variables in the final solution makes the objective value highly undesirable, thereby forcing the algorithm to eliminate them if a feasible solution exists.

The process of applying the Big M technique can be described in a series of steps:

1. Convert all inequalities to equalities.

For "≤" constraints, add slack variables as usual. For "≥" constraints, subtract a surplus variable and then add an artificial variable. For "=" constraints, directly add an artificial variable.

2. Assign penalty costs to artificial variables.

Modify the objective function by including ±M times each artificial variable. This ensures they are penalized heavily and will not remain in the optimal solution if a feasible solution exists.

3. Construct the initial simplex tableau.

The tableau now includes the decision variables, slack or surplus variables, and artificial variables. The initial basic feasible solution is provided by the artificial variables.

4. Proceed with the simplex iterations.

The algorithm continues as in the regular simplex method: identifying entering and leaving variables, performing pivot operations, and updating the tableau. Artificial variables are gradually eliminated as the iterations progress.

5. Interpret the final solution.

If artificial variables remain in the solution at positive levels after optimization, it indicates that the original problem has no feasible solution. If they are eliminated successfully, the optimal solution is feasible and valid for the original model.

The efficiency of the Big M technique lies in its flexibility. It provides a systematic way to deal with complex constraints that cannot be addressed by the standard simplex method alone. While it introduces additional variables and complexity, it ensures that the algorithm can always start with a feasible solution, which is often a significant challenge in real-world optimization problems.

3.2.2 The Big-M Tableau

BV	x_1	x_2	x_3	…	x_e	x_f	RHS
	$-c_1$	$-c_2$	$-c_3$	…	$-M$	$-M$	0
x_a	a_{11}	a_{12}	a_{13}	…	1	0	b_1
x_e	a_{21}	a_{22}	a_{23}	…	0	1	b_2
x_f	a_{31}	a_{32}	a_{33}	…	0	0	b_3

Here:

BV= Basic Variable

RHS = Right hand side of the equation

-M= Penalty of artificial variables

The Big-M tableau is just like the simplex tableau, but it adds artificial variables with heavy penalties, these penalties make sure the artificial variables leave the solution as the iteration progress, so the final answer contains only the real decision variables.

3.3 The Graphical Method

The graphical method is one of the simplest ways to solve a linear programming problem, but it can only be used when there are just two decision variables. This is because two variables can be easily represented on a two dimensional graph (the x-axis and the y-axis). Once we have more than two variables, drawing and interpreting the graph becomes impossible.

Here is how the graphical method works step by step:

1. Formulate the problem clearly
    - First, we write down the objective function. For example, "maximize profit $Z = 3x + 2y$" where x and y are the decision variables.
    - Then we also write down all the constraints(like resources limits, time, or capacity), usually in the form of inequalities such as $2x + y \leq 100$
2. Draw the constraints on a graph
    - Each constraints Is converted into a straight line on the x-y plane. For example, if one of the constraints is $2x + y \leq 100$, we plot the line $2x+y = 100$

- After plotting, we shade the side of the line that satisfies the inequality

3. Identify the feasible region

    - The feasible region is the area on the graph where all the shaded regions from the constraints overlap. This region represents all the possible solution that satisfy every condition in the problem

4. Locate the corner(extreme) points

    - The best solution always lies at one of the corner points (vertices) of the feasible region. These points are found by solving the equations where two constraints lines intersects

5. Evaluate the objective function at each corner points

    - We substitute the values of each corner point into the objective function. The point that gives the maximum or minimum value (depending on the problem) is the optimal solution

The graphical method is like drawing all your restrictions on paper, finding the area where everything works together, and then testing the corners of that area to pick the best answer.

3.4 The Interior Point Method

The interior point method in linear programming is another approach to solving linear programming problems, introduced as an alternative to the simplex method. While the simplex method searches along the boundary of the feasible region, the interior point method moves through the "interior" of the feasible region in search of the optimal solution. This makes the approach especially effective for very large scale linear programming problems, such as those found in transportation, communication networks, and, large manufacturing systems.

The basic idea of the interior point method is to begin at a point strictly inside the feasible region (not on the edges or corners). From this interior position, the method follows a path that gradually approaches the optimal solution. The path is usually defined by mathematical functions called "barrier functions" which prevents the search from crossing the boundaries of the feasible region.

Step by step explanation:

1) Initialization:
   - Choose a feasible starting point that lies strictly within the interior of the feasible region
   - This ensures that all constraints are satisfies with strict inequalities
2) Barrier function application :
   - A barrier function is introduced into the objective function. This barrier penalizes solutions that move too close to the boundaries of the feasible region, keeping the solution strictly inside.
   - For example ,if a constraints is $x \geq 0$, then $-\text{In}(x)$ may be added as a barrier term because it approaches infinity as x get closer to zero
3) Iterative movement:
   - The algorithm calculates a search direction and step length to move from the current interior point closer to the optimal point
   - Each movement balances the objective function with the barrier penalty, ensuring the solution remains inside the feasible region.
4) Convergence:

- The method continues updating the position step by step until the solution approaches the optimal corner point
- Although the search travels through the interior, the final solution often lies on the boundary, similar to the simple outcome

Advantages of the interior point method

i. It is computationally efficient for very large problems (with thousands of constraints and variables)
ii. Unlike the simplex method, which may take much iteration for large problems, the interior point method converges more quickly in practice.
iii. It provides a systematic path through the interior, avoiding the cycling and degeneracy issues sometimes found in simplex solutions

3.5 Two Phased Method

The two phased method is another approach used to solve linear programming problems, especially when artificial variables are introduced to handle constraints that are not in standard form. Unlike the big-M method, which uses a larde penalty value, the two phase method solves the problem systematically in two stages, making it easier to understand and less dependent on the arbitrary choice of a large constant M

Phase One: Eliminating Artificial Variables

The first phase focuses on finding a feasible solution. An auxiliary objective function is created by minimizing the sum of all artificial variables that were added to the problem. The simplex method is then added to this auxiliary problem.

- If the minimum value of the sum of artificial variables is zero, it means a feasible solution has been found (since all artificial variables can be eliminated).
- If the minimum value is greater than zero, it means the original problem has no feasible solution.

Mathematically, if artificial variables are $a_1, a_2,\ldots,a_k$, the auxiliary objective function is:

Minimize $W = a_1 + a_2 + \ldots + a_k$

Phase two: Optimizing the original objective function

Once a feasible solution is found in phase one, phase two begins. Here, the original objective function (maximize profit or minimize cost) is restored, and the feasible solution obtained earlier is used as the starting point. The simplex method is then carried out again this time focusing on optimizing the original objective function.

For example, if the original objective function is :

Maximize $Z = 5x_1+4x_2$

Then phase two applies the simplex method directly using this function, starting from the feasible solution identified in phase one.

Advantages of the Two phased method

I. It avoids the difficulty of choosing very large value for M, which is sometimes arbitrary and may cause numerical errors in the big M method.
II. It provides a clear , step by step procedure for handling artificial variables
III. It is reliable for detecting whether a problem has no feasible solution

3.6 Revised Simplex Method

The Revised Simplex method is a modification of the standard simplex algorithm designed to make calculations more efficient, especially for large scale linear programming problems. Instead of repeatedly rewriting and updating the entire simplex tableau at each iteration, the revised method focuses only on the essential information needed for computations

Main idea of the revised method

In the standard simplex tableau, each step involves updating a full tableau that includes all variables and constraints. This can become cumbersome when the number of variables and constraints is very large. The revised method improves efficiency by only storing and updating:

I. The coefficients of the basic variables.
II. The inverse of the basic matrix (the matrix formed by the columns of the basic variables)
III. The values of the objective function and reduced costs

Step by step process

1. Initialization
   - Select the initial basis feasible solution (just as in the standard simplex)
   - Construct the basis matrix (B) from the columns of the constraints corresponding to the basic variables.
2. Compute the current solution
   - Solve for the values of the basic variable using:

     $X_B=B^{-1}b$

     Where X_B is the vector of basis variable values and b is the right hand side constants.
3. Calculate reduced costs

- Compute the reduced cost of each non-basic variable using: $\underset{C_j}{\rightarrow} = C_j - C_B^T B^{-1}A_j$

- C_j is the cost coefficient of variable

- C_b is the cost vector of basic variables,

- Aj is the column of coefficients for variable x_j

4. Optimality Test

   - If all reduced costs (C_j) are non-negative (for a maximization problem), the current solution is optimal.

   - If not, the variable with the most negative reduced cost enters the basis.

5. Determine Leaving Variable

   - Perform the minimum ratio test using x_B to identify which basic variable should leave the basis.

6. Update the Basis

   - Perform the leaving variable with the entering one, update the basis matrix B, and continue the process until an optimal solution is reached.

Advantages of the Revised Simplex Method

1. It is computationally faster for large problems, since it avoids unnecessary tableau calculations.
2. It is more memory-efficient, as only a part of the data is stored and updated.
3. It is widely used in modern computer-based optimization software.

3.7 Some Sensitivity Analysis

Sensitivity analysis in linear programming is concerned with understanding how variations in the parameters of a model affect the final solution. By carrying out sensitivity analysis, decision-makers are able to determine how stable or flexible the current optimal solution is when condition change.it gives solution to the problem of changes. The importance of sensitivity analysis lies in its practical application. Sensitivity analysis is useful in the management for the following reasons

I. It helps to determine which decision variable that are critical

II. It is useful for control purpose

III. It determines the range of values which certain decision variables can take on without affecting a change in the optimum solution.

3.8 Degenerate Solution In Linear Programming Problem

Degeneracy in linear programming occurs when more than one feasible solution lies at the same corner point of the feasible region, or when a basic feasible solution includes one or more variables that take on a value of zero. In such cases, the simplex method may cycle through the same set of solutions repeatedly, which can slow down the process of reaching an optimal answer, to address this issue, certain rules, such as bland's rule, are applied to prevent endless cycling and ensure that the algorithm eventually converges. Although degeneracy may complicate the solution process, it can also provide flexibility. This is because multiple optimal solution may exists, each offering the same objective value but with different allocations of resources. As a result, degeneracy highlights the possibility of alternative solution that achieve the same level of efficiency or profit.

CHAPTER FOUR

DATA ANALYSIS

Before we go into the analysis of this system, it is important to first provide a brief overview of Mouka foam

4.1 Brief History of Mouka Foam

Mouka Foam Nigeria Limited, originally known as Mukarom Metal wood Factory Limited, was established in 1959 by a Lebanese businessman, Mr. Fatahi Fattal. The company began by producing furniture and metal products but shifted its focus to foam manufacturing in the early 1970s due to rising demand for comfortable mattresses in Nigeria. In 1990, the company officially changed its name to Mouka Limited to reflect its core business in foam and bedding products. Over the years, Mouka Foam has grown into one of the leading foam and mattress manufacturers in Nigeria, with its major production facility located in Benin City. The company has also expanded its distribution network across the country, making its products widely accessible to Nigerian consumers.

The quest for quality leadership resulted in Mouka being the first foam company in Nigeria to get its laboratory certified to international standard through the ISO9001 certification. This was an eloquent testimony of the quality standard of the product offspring.

In 2002 the company celebrated its 30th Anniversary with a new spring mattress factory. Using the sturdy European standard bonnel Spring, the spring mattresses set the trend for the industry. The facilities can accommodate custom sizes and abrupt instant orders without any compromise on quality

In response to increased demand from the East/South region of Nigeria, the company acquired an existing foaming plant in Benin and opened for business in 2003, in august 2009, Mouka opened its production facility in Kaduna to cater to needs of the northern market and so on mouka foam as grown in production nationwide.

The industry encountered some challenges at early days of investment as listed below

• Potency and physical analysis of measuring raw materials

• Mechanical efficiency to deliver expected outcome from an accurate input

• Conducting chemicals at required temperature that best suit production

• Logistic and distribution challenges, especially in reaching rural areas

4.2 Analysis For Data

In order to analyze the methods of solving linear programming problems, this section presents a practical case study based on data obtained from mouka foam company Benin City. The purpose of this analysis is to apply linear programming techniques in determining the most profitable combination of the capital projects that the company can undertake within its limited resources

The data used for this analysis were obtained from the production and logistics department of mouka Foam Company and cover a four-year period, 2020-2024. The information includes the estimated present value of each project, the capital requirements over the years, and the available capital funds.

The company has identified four major capital investments projects that require proper resource allocation:

1. Upgrade of production equipment
2. Expansion of distribution channels
3. Acquisition of Raw materials storage facilities
4. Investment in research and product innovation

Because of limited capital, not all projects can be executed simultaneously. Therefore, management needs to determine which combination of projects will yield the maximum total return while ensuring that the yearly capital expenditures do not exceed the available funds. This situation can be formulated and solved as a linear programming problem

Practical Question

The data representing the projects are summarized in table 4.3

Table 4.3: Capital requirement (# millions)

Project(index)	Estimated present value (# millions)	Year 1	Year 2	Year 3	Year 4
Upgrade of production equipment (x_1)	120	30	40	30	20
Expansion of distribution channels (x_2)	90	20	25	30	15
Raw material storage facilities (x_3)	60	10	10	0	5
Research & product innovation (x_4)	150	40	30	30	5
Available capital funds (per year)	_	60	80	70	60

Let x_1= decision for Upgrade of production equipment

x_2= decision for Expansion of distribution channels

x_3= decision for Raw materials storage facilities

x_4= decision for Research & product innovation

The objective is **to maximize the total estimated present value**, subject to yearly capital constraints. Let the estimated present value be the objective function. Let the years be the number of constraints and the available capital funds be the right hand of each constraints.

Objective function:

Maximize $Z = 120x_1 + 90x_2 + 60x_3 + 150x_4$

Subject to:

Year 1: $30x_1 + 20x_2 + 10x_3 + 40x_4 \leq 60$

Year 2: $40x_1 + 25x_2 + 10x_3 +30x_4 \leq 80$

Year 3: $30x_1 + 30x_2 + 0x_3 + 30x_4 \leq 70$

Year 4: $20x_1 + 15x_2 + 5x_3 + 50x_4 \leq 60$

and

$x_1 ,x_2 ,x_3 ,x_4 \geq 0$

Adding slack variables to each constraints

$Z - 120x_1 - 90x_2 - 60x_3 - 150x_4 + 0s_1 + 0s_2 + 0s_3 + 0s_4 = 0$

Subject to

$30x_1 + 20x_2 + 10x_3 + 40x_4 + s_1 = 60$

$40x_1 + 25x_2 + 10x_3 + 30x_4 + s_2 = 80$

$30x_1 + 30x_2 + 0x_3 + 30x_4 + s_3 = 70$

$20x_1 + 15x_2 + 5x_3 + 50x_4 + s_4 = 60$

4.3 The Simplex Algorithm

Using the simplex Algorithm to solve this linear programming problem. The following are the steps of the simplex algorithm:

1. Construct the initial simplex tableau

- Write all the equations(constraints) in standard form (with slack variables added)
- Put all the coefficients of the variables into a table including all slack variables and RHS
- The initial tableau represent your starting feasible solution (usually where all x_i=0 and all slacks =RHS

Initial Tableau

Basic V	X_1	X_2	X_3	X_4	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	RHS
S_1	30.000	20.000	10.000	40.000	1	0	0	0	0	0	0	0	60.000
S_2	40.000	25.000	10.000	30.000	0	1	0	0	0	0	0	0	80.000
S_3	30.000	30.000	0.000	30.000	0	0	1	0	0	0	0	0	70.000
S_4	20.000	15.000	5.000	50.000	0	0	0	1	0	0	0	0	60.000
S_5	1.000	0.000	0.000	0.000	0	0	0	0	1	0	0	0	1.000
S_6	0.000	1.000	0.000	0.000	0	0	0	0	0	1	0	0	1.000
S_7	0.000	0.000	1.000	0.000	0	0	0	0	0	0	1	0	1.000
S_8	0.000	0.000	0.000	1.000	0	0	0	0	0	0	0	1	1.000
Z	-120.000	-90.000	-60.000	-150.000	0	0	0	0	0	0	0	0	0.000

NOTE:

The table is in decimal form because simplex involves arithmetic operations and sometimes you get fractions or decimals.

In the model formulation, eight slacks variables were introduced. Four of them (s_1-s_4) represent unused capital for each financial year, while the remaining four (s_5-s_8) correspond to the upper limits of each project, ensuring that no project receives more than one unit of funding. This results in a total of eight slack variables in the simplex tableau.

2. Checking the optimality condition: All the four coefficients are negative, meaning

- We can still improve (increase) Z by including one of those variables , so we are not yet optimal

3. Identifying the pivot column (entering variable)

- Looking at the Z-row, the most negative number in that row shows which variable will enter the basis. That is our pivot column and that is -150.000(under x_4) so x_4 is the entering variable
- Using ratio test to find pivot row, you divide each RHS value by the corresponding positive value in the x_4 column, to see which constraint will bind first

Row	Basic V	X_4 Coeff	RHS	Ratio(RHS ÷ X_4)
1	S_1	40.000	60.000	1.500
2	S_2	30.000	80.000	2.670
3	S_3	30.000	70.000	2.330
4	S_4	50.000	60.000	1.200
5	S_8	1.000	1.000	1.000(smallest positive)

So the pivot row = s_8 and the pivot element = 1(where row s_8 and column x_4 intersect), that means s_8 leaves the basic and x_4 enters the basic. We divide the pivot row by the pivot element (which is already 1, so it stays the same), then we eliminate x_4 from all other rows using row operations.

<u>Pivot 1 Tableau</u> (After x_4 enters, s_8 leaves)

Basic V	X_1	X_2	X_3	X_4	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	RHS
S_1	30.000	20.000	10.000	0.000	1	0	0	0	0	0	0	-40.000	20.000
S_2	40.000	25.000	10.000	0.000	0	1	0	0	0	0	0	-30.000	50.000
S_3	30.000	30.000	0.000	0.000	0	0	1	0	0	0	0	-30.000	40.000
S_4	20.000	15.000	5.000	0.000	0	0	0	1	0	0	0	-50.000	10.000
S_5	1.000	0.000	0.000	0.000	0	0	0	0	1	0	0	0.000	1.000
S_6	0.000	1.000	0.000	0.000	0	0	0	0	0	1	0	0.000	1.000
S_7	0.000	0.000	1.000	0.000	0	0	0	0	0	0	1	0.000	1.000
X_4	0.000	0.000	0.000	1.000	0	0	0	0	0	0	0	0.000	1.000
Z	-120.000	-90.000	-60.000	0.000	0	0	0	0	0	0	0	150.000	150.000

After the first pivot, $x_4 = 1$ and $s_8 = 0$, the new profit Z = 150 million

4. Identifying the Pivot column and Pivot row (for pivot 2 tableau)

- Looking at the Z-row in the pivot 1 tableau, the most negative number is -120.000(under x_1). Hence,x_1 is the entering variables, and the x_1 column becomes our pivot column
- To determine the pivot row (the leaving variables), we apply the ratio test. Each RHS value is divided by its corresponding positive coefficient in the pivot column(x_1 column) to find which constraint will bind first

Row	Basic V	X_1 coeff	RHS	Ratio(RHS ÷ X_1)
1	S_1	30.000	20.000	0.667
2	S_2	40.000	50.000	1.250
3	S_3	30.000	40.000	1.333
4	S_4	20.000	10.000	0.500(smallest positive)
5	S_5	1.000	1.000	1.000

Smallest positive ratio = 0.500 at row s_4, so s_4 is the leaving variable and the pivot element is 20.

Pivot-row normalization (make pivot=1)

Then we divide the entire s_4 row by 20 to create the new basic row for x_1.

Original s_4 row (after pivot 1): (x_1,x_2,x_3,x_4,s_1,s_2,s_3,s_4,s_5,s_6,s_7,s_8,RHS)= (20,15,5,0,0,0,0,1,0,0,0,-50,10)

So the new x_1 row = (1,0.75,0.25,0,0,0,0,0.05,0,0,0,-2.5,0.5)

New row = Old row – (coefficient in pivot column) × (new pivot row)

To get the new s_8 row we say

S_1=-40 – (30) (-2.5) =35

S_2=-30 – (40) (-2.5) =70

S_3=-30 – (30) (-2.5) =45

S_4=-50 – (20) (-2.5) =0 (but recall we won't use this, we will use the new x_1)

S_5=0 – (1) (-2.5) =2.5

Z=150 - (-120) (-2.5) = -150

Then we do same to get each new row using the formula

Pivot 2 Tableau (After S_4 leaves and X_1 enters)

Basic V	X_1	X_2	X_3	X_4	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	RHS
S_1	0.000	-2.500	2.500	0.000	1	0	0	-1.500	0	0	0	35.000	5.000
S_2	0.000	-5.000	0.000	0.000	0	1	0	-2.000	0	0	0	70.000	30.000
S_3	0.000	7.500	-7.500	0.000	0	0	1	-1.500	0	0	0	45.000	25.000
X_1	1.000	0.750	0.250	0.000	0	0	0	0.050	0	0	0	-2.500	0.500
S_5	0.000	-0.750	-0.250	0.000	0	0	0	-0.050	1	0	0	2.500	0.500
S_6	0.000	1.000	0.000	0.000	0	0	0	0.000	0	1	0	0.000	1.000
S_7	0.000	0.000	1.000	0.000	0	0	0	0.000	0	0	1	0.000	1.000
X_4	0.000	0.000	0.000	0.000	0	0	0	0.000	0	0	0	1.000	1.000
Z	0.000	0.000	-30.000	0.000	0	0	0	6.000	0	0	0	-150.000	210.000

NOTE

BV=basic variables for that row (left column)

RHS column gives the current value of each basic variable (so x_1=0.5, x_4=1, s_1=5, s_2=30, s_3=25, s_5=0.5, s_6=1)

Z-row RHS=210=new objective value (profit) after pivot 2

The Z-row still has a negative entry for x_3(-30), so not yet optimal

5. New solution: from the Z-row in the pivot 2 tableau we still have one negative value which means it isn't optimal yet so we repeat the process

- Look at the Z-row: the only negative reduced cost is -30 under x_3 (pivot column =x_3)
- Ratio test using the x_3 column (only rows with positive x_3 coefficient)

Row	Basic V	X_3 coeff	RHS	Ratio(RHS ÷ X_3)
1	S_1	2.500	5.000	2.000
4	X_1	0.250	0.500	2.000
7	S_7	1.000	1.000	1.000(smallest positive)

Smallest positive ratio =1.000 at row s_7,so the pivot row is s_7,pivot element is 1(the entry at (s_7,x_7)

So x_3 enters and s_7 leaves.

Pivot element=1, therefore the pivot row stays the same (it becomes the new x_3 row)

<u>Pivot 3 Tableau</u> (After x_3 enters, s_7 leaves)

Basic V	X_1	X_2	X_3	X_4	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	RHS
S_1	0.000	-2.500	0.000	0.000	1	0	0	-1.500	0	0	0	35.000	2.500
S_2	0.000	-5.000	0.000	0.000	0	1	0	-2.000	0	0	0	70.000	30.000
S_3	0.000	7.500	0.000	0.000	0	0	1	-1.500	0	0	0	45.000	32.500
X_1	1.000	0.750	0.000	0.000	0	0	0	0.050	0	0	0	-2.500	0.250
S_5	0.000	-7.50	0.000	0.000	0	0	0	-0.050	1	0	0	2.500	0.750
S_6	0.000	1.000	0.000	0.000	0	0	0	0.000	0	1	0	0.000	1.000
X_3	0.000	0.000	1.000	0.000	0	0	0	0.000	0	0	1	0.000	1.000
X_4	0.000	0.000	0.000	1.000	0	0	0	0.000	0	0	0	1.000	1.000
Z	0.000	0.000	0.000	0.000	0	0	0	6.000	0	0	30	-150.000	240.000

The tableau is optimal

Basic Variables (RHS values)

x_1(Upgrade of production equipment)=0.25

x_2(Expansion of distribution channel)=0

x_3(Raw materials storage facilities)=1

x_4(Research &product innovation)=1

Slack values:

s_1=2.5, s_2=30, s_3=32.5, s_5=0.75, s_6=1.

Objective value = 240 million (optimal solution)

CHAPTER FIVE

SUMMARY AND CONCLUSION

5.1 Summary

Linear programming is one of the most effective mathematical tools used in decision making and resources management. It provides a systematic way of determining the best possible outcome when resources such as time, capital and labour are limited.

In this research work, different methods of solving linear programming were discussed, but the simplex method was adopted in chapter four because it is more suitable when the number of decision variables exceeds two. The method is fast, efficient, and can easily be implemented using a computer program. It helps organizations make the best use of available resources to achieve maximum profit or minimum cost

The aim of this project work is to show how linear programming can be applied to real life business situations such as in Mouka Foam Company, to help management make better investment and production decisions. Through the simplex method, the study determined the optimal allocation of resources that leads to maximum profit

5.2 Conclusion

From the analysis carried out, it can be concluded that linear programming is an essential decision making tool that helps firms achieve their production and financial goals efficiently. The simplex method proved to be the reliable and accurate in solving complex optimization problems

When a company operates under limited resources but seeks to maximize profit, linear programming provides a clear mathematical approach to making sound business decisions. It is therefore recommended that organizations, especially manufacturing firms, adopt linear programming techniques in their planning and operations

5.3 Recommendations

From the findings of this project, I recommend that companies like Mouka foam should apply linear programming methods when making production and resources allocation decision. This will help them make better use of their limited resources and increase profit. Also, workers should be trained on how to use simple computer programs that can perform these calculations easily. By doing this, management will be able to plan better, reduce waste and achieve their production goals faster

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