APPLICATION OF LINEAR ALGEBRA TO ARTIFICIAL INTELLIGENCE AND OTHER AREAS OF STUDY

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A PROJECT WORK SUBMITTED TO THE DEPARTMENT OF MATHEMATICS, FACULTY OF PHYSICAL SCIENCES, UNIVERSITY OF BENIN, BENIN CITY, IN PARTIAL FULFILLMENT OF THE REQUIREMENT FOR THE AWARD OF THE DEGREE OF BACHELOR OF SCIENCE (B.Sc.) HONOURS IN MATHEMATICS.

FEBRUARY 2025.

UNDERTAKING

This project was carried out by me, OSARHEAME DESIRE AYOMIDE with the MATRICULATION NUMBER PSC1909089. I have neither copied nor duplicated the work of any author(s). All work used have been duly cited and acknowledged.

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CERTIFICATION

This is to certify that this work, **APPLICATION OF LINEAR ALGEBRA TO ARTIFICIAL INTELLIGENCE AND OTHER AREAS OF STUDY** was carried out by **OSARHEAME DESIRE AYOMIDE PSC1909089** of the Department of Mathematics, Faculty of Physical Science, University of Benin, Benin City.

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Prof. C. I. Nkeki **Prof. D. Okuonghae**

(PROJECT SUPERVISOR) **(HEAD OF DEPARTMENT)**

DATE…………………………　　　　　　　　**DATE……………………………**

DEDICATION

I dedicate this work to Almighty God. Also, to my family and friends, and to everyone that supported during the course of this project work and my entire undergraduate program.

ACKNOWLEDGEMENT

Firstly, I offer my appreciation to the Almighty God for His blessings and guidance throughout this project journey and for blessing me with strength, resources and wisdom to carry out this project work successfully. Also, I thank my mother Mrs. Eunice Eigbochie and my father Mr. Anthony Osarheame for their unwavering support mentally, emotionally and financially during this project.

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Special recognition goes to my colleagues and friends whose support and companionship made my days at this institution truly meaningful (Anthony Onoseremen, Christopher Ulinfoh, and Divine Jamgbadi), I appreciate them all for their unrelenting support and love.

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TABLE OF CONTENT

ABSTRACT

This project work provides an overview on the application of linear algebra to artificial intelligence including natural language processing and machine learning. We discuss how linear algebra operations such as matrices, linear transformations,

eigen values and eigen vectors, are used to optimize AI models, analyze complex data structures and enable efficient computation. Beginning with an overview of fundamental concepts in linear algebra, such as vectors, matrices, and linear transformations, the study delves into specific applications of these concepts in AI.

One key area of focus is machine learning, where linear algebra forms the backbone of algorithms for tasks such as regression analysis, and principal component analysis for dimensionality reduction. This work also showcases the versatility of linear algebra by delving deep into the various reaches of linear algebra into many other fields and areas of study such as economics, physics and engineering.

CHAPTER ONE

INTRODUCTION

1.1. INTRODUCTION

Artificial intelligence is a branch of computer science that specifically deals with the development of intelligent computers and computer programs capable of carrying out tasks that usually necessitate human intelligence (Oxford Dictionary, Turing, 1950). The duties may encompass acquiring knowledge through experience, comprehending human language, identifying regularities, and formulating conclusions grounded in evidence. AI seeks to create machines that can emulate human behavior and adjust to unfamiliar circumstances. AI encompasses several applications such as Virtual Assistants (e.g., Google Voice, Apple Siri) and Recommendation Systems (e.g., Netflix, YouTube, Amazon). AI has emerged as a revolutionary technology that has extensive applications in many sectors including healthcare, finance, transportation, and entertainment. The core of artificial intelligence resides in its capacity to efficiently process and interpret large volumes of data, enabling it to make intelligent decisions. To enable AI to carry out tasks and make informed decisions, it is necessary to develop algorithms that provide instructions to the AI. These algorithms are referred to as AI algorithms. Russell and Norvig (2016) define AI algorithms as computational methods and methodologies employed in the field of Artificial Intelligence to facilitate autonomous learning, reasoning, and decision-making by computers. Therefore, we can assert that AI algorithms serve as the cognitive center of AI systems. AI algorithms encompass various types, such as Machine Learning Algorithms, Deep Learning Algorithms, Natural Language Processing (NLP) Algorithms, and Search Algorithms. One essential mathematical

Linear Algebra is a crucial science that plays a critical role in empowering AI algorithms. Linear Algebra empowers AI systems to derive significant insights and solve intricate issues by offering a robust foundation for representing, manipulating data, and generate precise forecasts. In this project, we will delve into the captivating. The utilization of Linear Algebra in artificial intelligence involves fundamental principles, methodologies, and instances that demonstrate its significant contribution to the operations of AI.

1.2. BACKGROUND OF THE STUDY

Linear algebra plays a crucial role in the field of artificial intelligence (AI). A study that has garnered much attention in recent years (Xu, 2021). Artificial Intelligence has helped the automation of complex activities, offering data-driven insights to improve decision-making and increase efficiency. In order to fully grasp the applications and influence of revolutionizing numerous sectors, it is crucial to have a deep understanding of the background of Linear Algebra.

Linear algebra is a fundamental mathematical framework that is widely used in the field of artificial intelligence. Linear algebra, which is based on the examination of vector spaces, linear transformations, and systems of linear equations, offers fundamental techniques for representing and resolving problems associated with AI.

Linear algebra plays a crucial role in AI by providing a foundation for representing and manipulating data. In the field of artificial intelligence, data is frequently represented as vectors, matrices, or tensors. Each element inside these structures corresponds to a certain aspect or attribute of the data. Linear algebra operations, such as vector addition, multiplication, and transformation, enable AI systems to efficiently handle and interpret large volumes of data.

Matrix operations are essential in multiple AI techniques, such as machine learning, deep learning, and computer vision. In machine learning algorithms such as linear regression and logistic regression, matrices are employed to represent datasets and model parameters. This allows the system to acquire knowledge from input data and provide predictions.

Deep learning involves the construction of neural networks, which consist of interconnected nodes organized into layers. Each connection between nodes is assigned a weight. Linear algebra operations, such as matrix multiplication and element-wise operations, are crucial for training and utilizing neural networks. These operations are used to transport signals across the network, update weights, and generate predictions.

Moreover, linear algebra enables the use of dimensionality reduction methods such as principal Component Analysis (PCA) and Singular Value Decomposition (SVD). These approaches are crucial for identifying significant characteristics from data with many dimensions and lowering the computing complexity in artificial intelligence jobs.

Linear algebra is employed in computer vision for several image processing tasks, including image transformation, filtering, and feature extraction. Matrices and tensors are mathematical representations used to depict images and their transformations. These representations allow artificial intelligence systems to identify patterns, objects, and faces within images.

The use of linear algebra principles and techniques into artificial intelligence has greatly advanced the creation of advanced AI systems that can effectively handle, analyze, and comprehend intricate data, as well as make intelligent decisions in diverse fields. Linear algebra plays an essential role in shaping the methodology and applications of AI as it continues to advance.

1.3. STATEMENT OF PROBLEM

In the ever-growing field of Data analytics, which involves the gathering of insight It is impossible to overstate the importance of using linear algebra, a branch of mathematics that focuses on the properties of linear equations, in the rapidly expanding fields of data analytics, which involves humans using data to gain insight, and artificial intelligence, which involves machines using data effectively to make predictions. Therefore, the goal of this project effort is to look at the following:

1. The efficient use of linear algebra to AI.

2. A variety of linear algebraic ideas and methods related to AI

3. Linear algebra plays a crucial role in artificial intelligence.

The study of mathematics is the foundation of artificial intelligence. Mathematical procedures related to vector algebra, a subset of linear algebra, are necessary for the storage and processing of data needed for machine learning. Writing many algorithms that call for mathematical approaches for numerical analysis is a necessary part of any Artificial Intelligence training. Another issue with linear algebra is the use of linear regression methods to model functions that support artificial intelligence.

1.4. AIM AND OBJECTIVES

The aim of this project work is to study the use of Linear Algebra in the field of Artificial intelligence. The objective is to consider data structures and algorithms in Linear Algebra that are applied in AI.

1.5. SCOPE OF THE STUDY

The area of Artificial Intelligence and Linear Algebra is vast and cannot be thoroughly explored within the short time allocated for this final year project. Therefore, this project work will focus only on examining the many applications of matrices in linear algebra as a data structure in the field of artificial intelligence. Linear Algebra is used in AI optimization models, including the Linear Regression technique, Neural Networks.

1.6. PRACTICAL USE OF ARTIFICIAL INTELLIGENCE

Artificial intelligence (AI) has become increasingly prevalent in numerous practical applications across various industries. Its ability to analyze vast amounts of data, make predictions, and automate complex tasks has revolutionized numerous sectors, enhancing efficiency, accuracy, and decision-making. AI has revolutionized various health, financial, transportation and many more sectors in the following ways:

i. Healthcare: AI is transforming the healthcare industry by improving diagnosis, treatment, and patient care. Machine Learning algorithms can analyze medical images (such as X-rays and Magnetic Resonance Imaging
(MRIs)) to detect diseases like cancer with high accuracy (Esteva et al., 2017). AI-powered chat-bots and virtual assistants are used to provide patient support and deliver personalized medical information (Gururajan et al., 2020). AI algorithms can analyze medical images, such as X-rays, MRI scans, and CT scans, with high accuracy. AI-powered image analysis can assist radiologists in detecting abnormalities, tumors, and other medical conditions early on, leading to quicker diagnoses and better treatment decisions. We also have AI Surgical Robots such as Da Vinci Surgical Robots which assists medical practitioners in carrying out surgeries.

ii. Finance: AI is extensively employed in the finance industry for fraud detection, risk assessment, and algorithmic trading. Machine Learning algorithms can detect fraudulent transactions by analyzing patterns and anomalies in large datasets (Dal Pozzolo *et al*., 2015). For example, PayPal, a Financial Technology company uses AI-driven systems to analyze transaction data to identify suspicious activities and protect users from fraudulent transactions. AI-powered robo-advisors provide automated financial advice and portfolio management (Ferrandiz and Carrasco, 2017). For example, SoFi, a financial technology company, uses AI-powered robo-advisors to offer personalized investment advice and financial planning to its users, making investment services accessible and cost effective.

iii. Transportation: AI is revolutionizing transportation systems with advancements in autonomous vehicles and traffic management. Self-driving cars use AI algorithms to perceive their surroundings, make decisions, and navigate safely (Bojarski *et al*., 2016). AI-based traffic management systems optimize traffic flow, reduce congestion, and improve transportation efficiency (Elbery *et al*., 2020). AI powered navigation systems collect data from people location also known as geographical data, and uses these data to predict the occurrence of traffic congestions. This helps the AI to predict optimal routes to take, in order to avoid traffic jams, an example is seen in Google maps AI.

iv. Retail: AI has transformed the retail industry with personalized recommendations, inventory management, and customer service. Recommendation systems utilize AI techniques to analyze customer preferences and behavior, providing personalized product recommendations (Schafer *et al*., 2007). AI-powered chat-bots assist customers in finding products, answering queries, and handling customer support (Maldonado and Weber, 2021). AI-powered recommendation engines analyze customer data, purchase history, and browsing behavior to provide personalized product recommendations. This enhances customer engagement and increases the likelihood of making relevant upsells and cross-sells. For example, Google search engines AI uses data collected from users to optimize user experience. You would notice that the more you search for things on google, the better it helps you in suggesting a new search. This is how Google AI trains on data gotten from search.

v. Manufacturing: AI is enhancing manufacturing processes through automation, predictive maintenance, and quality control. AI-powered robots and machines automate repetitive tasks, improving efficiency and productivity (Atzori *et al*., 2017). Predictive maintenance systems use AI algorithms to detect anomalies and predict equipment failures, reducing downtime and optimizing maintenance schedules (Li *et al*., 2020). Manufacturing process have been made easy through the use of AI robots, trained to perform manufacturing tasks. This have reduced the need for capital intensive labour and enhanced innovation.

The applications of AI are expanding in every sphere of life, as new developments and innovations emerge.

1.7. DEFINITION OF TERMINOLOGIES

In this study, Linear Algebra as a field in Mathematics will not be comprehensive enough without some Mathematical terms. For the purpose of research, some terms have been chosen and the meaning given to enable us carry out analysis. The Following are the definitions:

1. Vector: A vector represents an ordered collection of elements, typically arranged as either a column or row matrix. Examples of vectors include:

   I. A row vector: $A = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$

   II. A column vector: $A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

2. Scalar: A scalar denotes a single numerical value, often drawn from a field like real or complex numbers. Scalars are employed to scale vectors and matrices and usually denote directionless quantities. Examples of scalars comprise speed, length, distance, etc. For instance, a boy traveling at 5 meters per second exemplifies scalar quantity since it lacks direction. Scalar multiplication involves multiplying a scalar by a vector.

3. Linear Transformation: A linear transformation refers to a function that maps vectors from one vector space to another, maintaining vector addition and scalar multiplication properties. It's often represented through matrix multiplication. A linear transformation T is denoted as $T(x)=A \cdot x$, where A signifies the transformation matrix and x represents a vector.
4. Eigenvector: An eigenvector of a square matrix is a non-zero vector that undergoes only scaling by a scalar factor upon application of the matrix. The scalar factor is termed the eigenvalue associated with the eigenvector. An eigenvector v of a matrix A is such that $A \cdot v=\lambda \cdot v$, where λ represents the associated eigenvalue.
5. Eigenvalue: An eigenvalue of a square matrix is a scalar representing the factor by which an eigenvector is scaled upon application of the matrix.
6. Dot Product: The dot product, also termed scalar or inner product, is an operation that yields a scalar by summing the products of corresponding elements of two vectors.
7. Cross Product: The cross product is an operation applicable to two vectors in three-dimensional space, resulting in a vector perpendicular to both input vectors. It finds frequent use in vector algebra and geometric computations.

CHAPTER TWO

LITERATURE REVIEW

2.1. INTRODUCTION

In recent years, the science of AI has undergone significant expansion and progress, leading to profound changes in several industries and reshaping human lifestyles and work practices. In light of the ongoing progress in AI, it is crucial to possess a thorough comprehension of the current corpus of knowledge and research to discern significant patterns, obstacles, and prospects. This literature review is to offer a thorough and all-encompassing summary of the present state of study regarding the utilization of Linear Algebra in the field of artificial intelligence.

The literature review aims to consolidate and examine pertinent scholarly articles, research papers, and publications to acquire a comprehensive understanding of the practical implementations, theoretical underpinnings, and new developments in the Mathematics underlying AI. Through an extensive analysis of several literary works, recurring patterns and ideas are uncovered, shedding light on areas of inquiry that require additional investigation in the field of artificial intelligence.

This review will examine several facets of AI, encompassing Machine Learning techniques, natural language processing, computer vision, robotics, and other related

areas. Additionally, it will explore the use of Linear Algebra in the field of Artificial Intelligence. Furthermore, I would like to explore the various applications of AI in sectors such as healthcare, finance, transportation, and manufacturing.

2.2. LITERATURE REVIEW

Firstly, let us contemplate the genesis of Linear Algebra. The beginnings of Linear Algebra may be traced back to ancient civilizations, where mathematicians and academics established the fundamental principles that formed the basis for its further growth over many centuries (Christensen, J., 2012). The formalization and systematic study of Linear Algebra as a distinct Mathematical subject commenced during the seventeenth (17th) and eighteenth (18th) century. Ancient civilizations such as Egypt, Babylon, and Greece have contributed to the development of Linear Algebra. The ancient Egyptians and Babylonians employed fundamental arithmetic and geometric methods to resolve systems of linear equations and geometric quandaries. Euclid, the Greek mathematician, presented the ideas of geometry in his work "Elements." In this work, he also provided techniques for solving systems of linear equations by utilizing geometric concepts. The 17th century saw the advent of coordinate systems, thanks to the contributions of René Descartes (1596-1650) and Pierre de Fermat (1607-1665). These systems proved to be a highly effective means of describing points and lines in space. This provided a solid foundation for the examination of vectors and vector spaces. Carl Friedrich Gauss (1777-1855), a renowned mathematician, made notable advancements in the field of linear algebra, specifically in the examination of linear systems and the utilization of the method of least squares for curve fitting. Gauss's contributions to Linear Algebra and least squares methodology provided the foundation for the advancement of statistical Regression and the application of Linear models for data fitting. The techniques he developed continue to be extensively employed in diverse disciplines, such as Physics, Engineering, Economics, and Data analysis. James Joseph Sylvester (1814-1897) and Arthur Cayley (1821-1895) made significant advancements in the mid-19th century in using matrices and determinants as tools for solving systems of linear equations.

2.2.1. THE EVOLUTION OF AI

Computer scientist John McCarthy is credited with coining the phrase "Artificial Intelligence" in the 1950s. During this era, researchers directed their attention towards symbolic AI, employing rule-based systems to carry out tasks such as theorem proving and chess-playing. In 1957, Frank Rosenblatt pioneered the development of the Perceptron algorithm, which is recognized as one of the earliest Machine Learning (ML) algorithms. The Perceptron technique enables machines to acquire knowledge from historical data. This facilitated the development of neural networks and the identification of patterns. The time span from the 1950s to the 1970s might be seen as the era when machine learning was first developed. During the 1980s, expert systems, which employed knowledge-based rules to imitate human behavior, became widely embraced in diverse domains including medicine and finance. This has resulted in the development of robots capable of doing human functions, a notion within the field of artificial intelligence referred to as Robotics. Between 1980 and 1990, researchers in the field of artificial intelligence began to investigate neural networks and related models. In addition, during the 2000s, artificial intelligence began to be incorporated into practical applications, including web search engines, recommendation systems, and speech recognition technology. Machine learning algorithms have become more resilient, resulting in substantial enhancements in artificial intelligence capabilities. AI has experienced increased recognition in recent years, being featured in numerous academic research papers and embraced by various sectors. Let us now examine many authors and their contributions to the field of artificial intelligence.

2.2.2. ARTIFICIAL INTELLIGENCE DEFINITIONS AND CONCEPTS

John McCarthy, the individual credited with coining the term AI in 1955, provided a definition of AI as the field of study and application that involves the creation of intelligent devices, particularly computer programs capable of exhibiting intelligence. According to John McCarthy's definition, AI focuses on the creation of intelligent devices and computer systems. AI is centered around the development and creation of systems that exhibit intelligent behavior. This explanation is essential because McCarthy is credited with coining the term "Artificial Intelligence."

Elaine Rich (1983), a renowned AI researcher, defines AI as the field that focuses on enabling computers to perform tasks that humans now excel at. According to Elaine Rich's definition, AI focuses on instructing computers to perform tasks that humans excel at. The main objective of this project is to establish a comparison between AI and human capabilities, with the ultimate goal of equipping computers with the same level of proficiency as humans in performing these jobs. This definition illustrates that AI involves the development of computers that can be employed in domains where human abilities are highly regarded. Russell and Norvig (2021) provided a definition of AI as a field within computer science that seeks to develop intelligent machines with the ability to imitate human cognitive functions, including learning, reasoning, problem-solving, and decision-making. This definition provides a thorough and all-encompassing summary of the goals and aims of AI. The statement highlights the interdisciplinary aspect of AI, connecting it with computer science and the goal of creating computers that can imitate human cognitive capacities. AI is the field of study and application that focuses on creating computers capable of

exhibiting human-like abilities such as reasoning, learning, perception, and communication. These intelligent robots are designed to do a wide range of jobs with minimal human involvement. Nilsson's work in 1988. According to Nilsson, AI is the creation of intelligent computers that possess the abilities to reason, learn, perceive their environment, and communicate. AI seeks to replicate human cognitive processes and comprehension, enabling machines to engage in human-like interactions with people. AI refers to the capacity of a system to accurately comprehend external information, acquire knowledge from this information, and utilize this knowledge to accomplish particular objectives and activities by means of adaptable adjustment (Haugeland, 1985). This description highlights the data-centric aspect of AI and its ability to acquire knowledge from external and past data, which is referred to as Machine Learning. It also discusses the adaptable nature of AI systems, suggesting that they may readily react to new information and tasks. According to the definition given in Warwick (2019), Artificial Intelligence refers to the field of science and engineering that focuses on creating intelligent agents. These agents have the ability to observe their surroundings, make decisions, and perform actions in order to accomplish specific objectives. Warwick's definition highlights the significance of intelligent agents in the field of artificial intelligence. These agents function as intelligent systems that engage with their environment, make

decisions, and take actions in order to achieve specific objectives. This demonstrates the role of AI in developing autonomous agents capable of functioning and resolving issues independently. These definitions collectively demonstrate that AI is an expansive and complex domain. The definitions of AI focus on the development of intelligent systems, encompassing various disciplines, and its potential to revolutionize diverse industries. The various perspectives also demonstrated that AI is continuously advancing and possesses the capability to revolutionize technology and reshape human-machine interactions in the future. Subsequently, the subsequent section presents an overview of the concepts in AI and the many definitions supplied by multiple writers.

According to Mitchel (1997), Machine Learning is an area of Artificial Intelligence. Machine learning is the field of computer science that focuses on developing algorithms that allow systems to enhance their performance on a particular job by learning from experience. This definition concisely encapsulates the fundamental nature of the notion. Machine Learning involves the creation of computer algorithms that enhance the performance of computers on particular tasks by utilizing experience. The utilization of uncomplicated language renders the concept readily comprehensible, especially for neophytes in the subject. Nielsen (2015) provides a compelling definition of Neural Networks in the field of AI. Artificial intelligence

refers to a collection of algorithms that are influenced by the human brain and have the ability to identify patterns and acquire knowledge from data. The structure is composed of interconnected nodes, also known as neurons, which are organized in layers. Natural Language Processing (NLP) is a branch of AI that specifically deals with the interaction between computers and human language. It allows machines to comprehend, interpret, and produce human language. The reference is from the book "Manning & Schütze" published in 1999. Natural Language Processing (NLP) is essential in a wide range of applications, including speech recognition, language translation, sentiment analysis, chatbots, and information extraction. NLP techniques can extract useful insights, enhance communication between humans and machines, and improve automation in numerous industries by processing and analyzing large volumes of textual data. These definitions are not comprehensive enough to include all the topics in AI. There are further concepts that are not included in this project's definitions.

2.3. APPLICATIONS OF LINEAR ALGEBRA IN AI

Multiple writers have recorded the utilization of Linear Algebra in the field of Artificial Intelligence. The book "Deep Learning" authored by Ian Goodfellow, Yoshua Bengio, and Aaron Courville in 2015 provides a comprehensive exploration of deep learning, a specific branch of artificial intelligence that strongly relies on the principles and applications of linear algebra. The authors conducted a study on the design of neural networks, specifically focusing on the utilization of Matrix operations for both forward and backward propagation during the training process. The book demonstrates how Linear Algebra facilitates efficient computing in intricate neural network models. In his 2017 research paper titled "Attention Is All You Need," Vaswani introduces the Transformer architecture, which is a significant advancement in the field of Natural Language Processing. The author explained the application of self-attention mechanisms in efficiently processing sequential data through the use of Matrix multiplications. The author illustrates how the utilization of Linear Algebra enables the Transformer model to effectively manage extensive connections between elements in language problems, hence transforming machine translation and other NLP tasks. In addition, Krizhevsky (2012) presents the AlexNet architecture in groundbreaking research titled "ImageNet Classification with Deep Convolutional Neural Networks," which played a crucial role in the advancement of deep learning.

The authors described the utilization of Linear Algebraic operations, such as convolutions and Matrix multiplications, by Convolutional Neural Networks (CNNs) to extract hierarchical characteristics from images. The success of AlexNet showcased the efficacy of Linear Algebra in facilitating CNNs to attain cutting-edge image classification performance. Professor Gilbert Strang's course "Linear Algebra

for Machine Learning" offers a thorough introduction to the application of Linear Algebra in Machine Learning (ML). Strang demonstrates the essential role of matrices and vector spaces in several machine learning techniques, including linear regression, principal component analysis (PCA), and Singular Value Decomposition (SVD). The course perfectly shows the fundamental role of Linear Algebra in Machine Learning approaches.

CHAPTER THREE

METHODOLOGY

3.1. INTRODUCTION.

This chapter tries to examine the intricate relationship between Linear Algebra and AI. We aim to demonstrate the integration of mathematical concepts with actual artificial intelligence tools. By comprehending the profound mathematical correlations, we can discern the transformative impact of AI on various industries and the dynamics of human-machine interaction.

This chapter also has a focus on providing a thorough explanation and examination of crucial Linear Algebra tools that are indispensable in the field of Artificial Intelligence. The text provides examples of how Linear Algebra tools are applied in practical AI techniques. These Linear Algebra tools serve as the fundamental basis for the functioning of AI. In addition, the integration of mathematical principles and practical applications is employed to elucidate the significance of Linear Algebra in facilitating the rapid advancement of AI.

As we progress through this chapter, practical examples involving the application of Linear Algebra in Artificial Intelligence (AI) will be considered. Furthermore, Algorithm such as Principal Component Analysis for Dimensionality Reduction and Linear Regression Analysis for Machine Learning Prediction is considered.

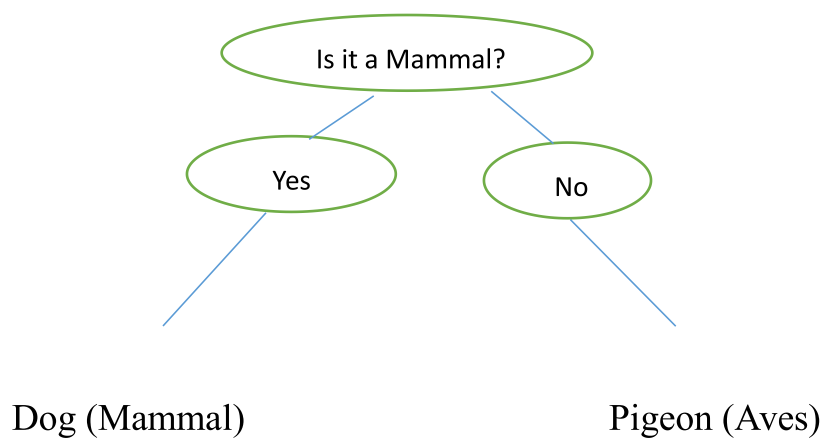
3.2. DATA STRUCTURES IN AI

Data structures in AI are structured forms that are used to store, organize, and represent data for the purpose of AI algorithms and models. These structures offer a methodical approach to handle and modify many forms of information, enabling effective data processing, analysis, and decision-making in AI systems. Linear Algebra ideas have a smooth transition into different data structures, and they have a crucial impact on how AI interacts with and handles data. These notions manifest as data structures, with an emphasis on Matrices and Vectors represented as arrays or lists, Scalars represented as individual elements, and Orthogonal Matrices represented as unit vectors. These structures also facilitate the effective storage and manipulation of AI-related data. Some examples of AI data structures include:

1. **List:** A list is a sequentially arranged assemblage of components. Every element has the flexibility to be assigned any data type, including numeric values, character sequences, or even nested lists. In the field of Natural Language Processing (NLP), a phrase can be represented as a sequence of words using a list, such as ["The", "book", "is", "on", "the", "table"]. Lists are utilized for purposes such as storing word sequences in natural language processing, managing feature vectors in machine learning, and describing action sequences in reinforcement learning.
2. **Arrays:** An array is a data structure that contains components of the same data type and is arranged in a certain sequence. An array can be used in image processing to represent the pixel intensities of a grayscale image. For example, [145, 186, 201, ..., 72] represents the intensities of the pixels. Arrays are extensively utilized in numerical computations, picture and signal processing, and feature storage in machine learning. Efficient vectorized operations are crucial for optimizing the performance of AI algorithms.
3. **Individual Elements:** The smallest units of data within AI data structures, such as arrays, matrices, or other types of structured data, are referred to as elements. Each individual element represents distinct qualities, features, or values linked to data points, allowing algorithms to carry out precise

calculations and changes. An instance of a singular element is the numerical value 145 within the List [145, 186, 201, ..., 72].

4. **Trees:** Trees are hierarchical data structures that are essential in numerous AI applications, enabling efficient organizing and retrieval of data. Trees are essential for jobs that entail intricate data structures and decision-making processes since they offer a means to depict linkages, hierarchies, and decisions. A tree is a data structure composed of nodes that are linked together by edges. Each node in the tree holds data and can have any number of child nodes. Below is an illustration of a tree that is utilized for the classification of animals:



There are various types of trees in AI, some of which include: Decision Trees, Game Trees, Parsing Tress, etc.

5. **Hash Tables:** In AI, hash tables are fundamental data structures that enable effective data storage and retrieval. Hash tables work especially well in

situations when having quick access to data via a unique key is essential. These structures improve the speed and effectiveness of data processing and are essential to many AI applications. Below is an illustration of a hash table that uses student IDs as keys to store student information:

Student ID	Name	Age	Grade
15284	Sunday Richard	18	C
23694	John Doe	17	A

The Hash table is very efficient in information retrieval, AI search engines use hash tables to quickly locate relevant documents based on keywords.

3.3. RELATIONSHIP BETWEEN AI DATA STRUCTURES AND LINEAR ALGEBRA

The relationship between AI data structures and Linear Algebra is crucial in modern AI applications. Linear Algebra provides the mathematical foundation for understanding and manipulating data, while AI data structures offer organized frameworks for efficiently storing and processing that data. There are several essential links connecting these two domains:

1. Matrices as Arrays: Matrices, central to Linear Algebra, align with two-dimensional arrays in AI data structures. These two-dimensional arrays can be visualized as organized grids of values, with rows and columns corresponding to data points and attributes. Arrays make it easier for AI algorithms to perform operations like addition, multiplication, and data extraction.

2. **Vectors as Lists:** Vectors, a fundamental concept in Linear Algebra, correspond with one-dimensional lists in AI data structures. These lists capture the essence of ordered sets of values, representing attributes or features associated with data points. They provide a flexible structure to store this information, enabling AI algorithms to process sequences, perform elementwise operations, and facilitate computations like dot products.

3. **Scalars as Individual Elements:** Scalars, fundamental in Linear Algebra, align with individual elements within AI data structures, such as Arrays and Lists. These elements serve as building blocks for storing and processing data, allowing AI algorithms to manipulate specific values and tailor their behavior according to scalar parameters.

4. **Matrices and Hash Tables:** Matrices and Hash Tables have a spectacular link. The keys of the Hash Table can be represented as pivotal elements of a matrix, while the attributes of each key can be represented as the rows of the matrix.

Retrieval of information from the Matrix would require a search based on the Pivotal elements.

3.4. AI ALGORITHMS

AI algorithms are a set of methodologies and techniques that enable machines to learn, reason, and make decisions autonomously. They form the backbone of AI applications, enabling systems to process data, recognize patterns, and generate insights. Some of these include:

1. **Machine learning algorithms:** Machine learning algorithms are divided into three categories: Supervised learning, Unsupervised learning, and Reinforcement learning. Supervised learning involves machines learning from labeled training data to make predictions or decisions. Applications of Supervised learning algorithm can be seen in Classification (categorizing data into classes) and Regression (predicting continuous values) tasks. Examples include decision trees, support vector machines (SVM), and neural networks. Unsupervised learning focuses on finding patterns and structures in unlabeled data. Its application involves Clustering (grouping similar data points), Dimensionality reduction (reducing features while retaining information), and anomaly detection. Examples include k-means clustering and Principal Component Analysis (PCA). Reinforcement learning focuses on training agents to take actions in an environment to maximize cumulative rewards, with applications in game playing, robotics control, and autonomous driving.

2. Natural Language Processing (NLP) algorithms are specialized techniques designed to enable computers to understand, interpret, and generate human language. These algorithms bridge the gap between human communication and machine understanding, enabling various applications such as text analysis, sentiment recognition, and language translation. Key NLP algorithms include tokenization, which breaks down a text into individual words or subword units, Part of Speech (POS) Tagging, which assigns grammatical tags to each word in a sentence, and Named Entity Recognition (NER), which identifies named entities within a text for tasks such as information extraction, sentiment analysis, and question answering.

3.5. RELATIONSHIP BETWEEN LINEAR ALGEBRA AND AI ALGORITHMS

Linear Algebra is a mathematical framework that enables AI algorithms to process data, make predictions, and learn patterns. It is a symbiotic relationship between Linear Algebra and AI algorithms, as it provides the mathematical framework for processing data, making predictions, and learning patterns. Some ways in which linear algebra and AI Algorithms operate in tandem include:

1. **Linear Regression, Matrix and Machine Learning:** Linear regression is a statistical technique that models the relationship between a dependent

variable and one or more independent variables by fitting a linear equation. The link between Machine Learning and Linear Algebra lies in model fitting, which estimates the parameters of a mathematical model to align it closely with observed data. Model fitting iteratively adjusts model parameters to minimize the difference between predicted and actual outcomes, leading to models that accurately describe the data. Linear regression is employed to create predictive models. Machines, uses this estimated model to make predictions. For example, if collated data on commodity prices and commodity demands of houses is available, a mathematical model can help machines predict commodity demand given house prices.

$$Demand_i = \alpha + \beta \text{Price}_i \qquad i = 1,2, \ldots, N \qquad (3.1)$$

Equation (3.1) above, is known as a Linear Regression and is a system of N equations, where N is the total numbers of observed data. α *and* β are the only unknowns in the System of Equations. Thus, with N >2, the Equations will have infinite solutions for α *and* β. Model fitting aims to find the value of α *and* β that best describes the relationship between Demand and Prices. The Least Squares Method is a fundamental method for finding this value, which involves setting up the System of Equations as a System of Matrices and applying Matrix inversion, multiplication, and transpose operations.

$$\beta = (X^T X)^{-1}(X^T Y) \tag{3.2}$$

where β is the coefficient matrix, X is the independent variable matrix, in this case the prices matrix and Y is the dependent variable matrix, in this case the demand matrix.

2. **Eigenvalues, Eigenvectors and Principal Component Analysis (PCA):**

The relationship between Matrix and Machine Learning is demonstrated by Equation (3.2). Principal Component Analysis (PCA) is another important concept in AI, which uses Linear Algebra concepts to extract essential information from complex datasets. PCA uses eigenvectors and eigenvalues to project high-dimensional data onto lower-dimensional subspaces, reducing complexity while retaining important information. It aims to find the directions along which the data varies most, determined by the eigenvectors and eigenvalues of the data's covariance matrix. For a dataset with N data points and m features, the steps for PCA include:

- **i.** Transform the data table into a data matrix
- **ii.** **Mean-Center the Data**: Subtract the mean of each feature from the corresponding data points.

iii. **Calculate the Covariance Matrix**: Calculate the covariance matrix **C** of the mean-centered data: $\boldsymbol{C} = (1/\boldsymbol{N})\ \boldsymbol{X}^{\boldsymbol{T}}\boldsymbol{X}$. Where **X** is an **N x m** matrix containing the mean-centered data points in its rows.

iv. Calculate Eigenvalues and Eigenvectors: Calculate the eigenvalues (λ) and eigenvectors (v) of the covariance matrix **C**. These eigenvectors represent the principal components.

v. **Sort Eigenvalues**: Sort the eigenvalues in descending order. The corresponding eigenvectors are the principal components.

vi. **Choose Principal Components**: Select the top **k** eigenvectors based on the explained variance. These eigenvectors form a new matrix **W**.

vii. **Transform Data**: Multiply the mean-centered data matrix **X** by the matrix **W** to get the transformed data matrix **Y**:

viii. Y = X W

These steps can be easily carried out using mathematical programming tools and languages like MatLab, Python, and R.

3.6. LINEAR REGRESSION FOR MACHINE LEARNING PREDICTION

Given the following data table, where each row represents an individual and the columns represent "Age" in years and "Income" in Naira:

Table 3.3: Dataset of Age and Income

	Age	Income
Felix	30	50000
Sandra	25	60000
Steven	40	75000
George	35	55000
Pamela	28	58000

Given the data table shown in Table 3.3. Taking the Income variable as the dependent variable and the Age variable as the independent variable, Y and X respectively, a Linear Regression model can be developed to train an AI to predict Income values using the Ages of individuals as shown below:

$$Income_i = \alpha + \beta Age_i \qquad i = 1,2, \ldots, 5 \qquad (3.4)$$

The above can be placed in Matrix form, using the given data as shown below:

Let Income Matrix = $\boldsymbol{Y} = \begin{bmatrix} 50000 \\ 60000 \\ 75000 \\ 55000 \\ 58000 \end{bmatrix}$

Let the Constants and Age Matrix = $\boldsymbol{X} = \begin{bmatrix} 1 & 30 \\ 1 & 25 \\ 1 & 40 \\ 1 & 35 \\ 1 & 28 \end{bmatrix}$

Thus, the model can be written as:

$$\begin{bmatrix} 50000 \\ 60000 \\ 75000 \\ 55000 \\ 58000 \end{bmatrix} = \alpha + \beta \begin{bmatrix} 1 & 30 \\ 1 & 25 \\ 1 & 40 \\ 1 & 35 \\ 1 & 28 \end{bmatrix}$$

The model can be solved for α and β using the Least Squares Estimation technique as shown:

$$\boldsymbol{\beta} = (X^T X)^{-1} X^T Y$$

$$X^T = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 30 & 25 & 40 & 35 & 28 \end{bmatrix}$$

$$X^T X = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 30 & 25 & 40 & 35 & 28 \end{bmatrix} \times \begin{bmatrix} 1 & 30 \\ 1 & 25 \\ 1 & 40 \\ 1 & 35 \\ 1 & 28 \end{bmatrix} = \begin{bmatrix} 5 & 158 \\ 158 & 5134 \end{bmatrix}$$

$$(X^T X)^{-1} = \begin{bmatrix} 7.272 & -0.224 \\ -0.224 & 0.0071 \end{bmatrix}$$

$$X^T Y = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 30 & 25 & 40 & 35 & 28 \end{bmatrix} \times \begin{bmatrix} 50000 \\ 60000 \\ 75000 \\ 55000 \\ 58000 \end{bmatrix} = \begin{bmatrix} 298000 \\ 9549000 \end{bmatrix}$$

Hence, $$\boldsymbol{\beta} = \begin{bmatrix} \beta_{\boldsymbol{o}} \\ \beta_1 \end{bmatrix} = \begin{bmatrix} 7.272 & -0.224 \\ -0.224 & 0.0071 \end{bmatrix} \times \begin{bmatrix} 298000 \\ 9549000 \end{bmatrix} = \begin{bmatrix} 28080 \\ 1405.9 \end{bmatrix}$$

The estimated model becomes:

$$Income_i = 28080 + 1405.9 \times Age_i \tag{3.5}$$

3.7. AI PREDICTION

You can use the obtained model to predict "Income" values for new "Age" values.

For instance, a young man of thirty-seven (37) years, would be predicted by an AI using the model to have estimated an income of:

$$\boldsymbol{Income = 28080 + 1405.9 \times 37 = 80098.3\ Naira}$$

This procedure demonstrates how Matrix operations are used to estimate a Linear Regression model using Least Squares. By formulating the data as Matrices and vectors, the calculation of coefficients becomes a straightforward Matrix multiplication and inversion process. This approach efficiently handles multiple variables and allows for easy extension to more complex Regression models.

3.8. PRINCIPAL COMPONENT ANALYSIS FOR DIMENSIONALITY REDUCTION

Principal component analysis can be applied on the data above to reduce the data to a single column data, without loss of data characteristics as shown below:

i. Transform the data table into a data Matrix

$$\text{Data matrix of age and income} = \begin{bmatrix} 30 & 50000 \\ 25 & 60000 \\ 40 & 75000 \\ 35 & 55000 \\ 28 & 58000 \end{bmatrix}$$

ii. Mean Centering: Calculate the mean of each column ("Age" and "Income") and subtract them from the respective columns to center the data around zero.

Mean (Age) = (30 + 25 + 40 + 35 + 28) / 5 = 31.6

Mean (Income) = (50000 + 60000 + 75000 + 55000 + 58000) / 5 = 59600

$$\boldsymbol{W} = \begin{bmatrix} 30 & 50000 \\ 25 & 60000 \\ 40 & 75000 \\ 35 & 55000 \\ 28 & 58000 \end{bmatrix} - \begin{bmatrix} 31.6 & 59600 \\ 31.6 & 59600 \\ 31.6 & 59600 \\ 31.6 & 59600 \\ 31.6 & 59600 \end{bmatrix} = \begin{bmatrix} -1.6 & -9600 \\ -6.6 & 400 \\ 8.4 & 15400 \\ 3.4 & -4600 \\ -3.6 & -1600 \end{bmatrix}$$

iii. Compute the covariance Matrix

$$\boldsymbol{C} = \begin{bmatrix} Var(Age) & Cov(Age, Income) \\ Cov(Age, Income) & Var(Income) \end{bmatrix}$$

Let X = Age and let Y = Income

Var(Age) = 29.04

Var(Income) = 2.7×10^{7}

Cov(Age, Income) = 55920

Hence: $C = \begin{bmatrix} 29.04 & 55920 \\ 55920 & 2.7 \times 10^7 \end{bmatrix}$

iv. Calculate the eigenvalues and eigenvectors of the covariance matrix: The Eigenvalues and Eigenvectors for Matrix **C** was computed using MATLAB pre built-in eig() function and the output given as:

Eigenvalues: $\lambda_1 \approx 2.701 \times 10^7$, $\lambda_2 \approx 36.4$

Eigenvector 1: [0.0002, 1]

Eigenvector 2: [0.999, -0.0002]

The Eigenvectors are known as the Principal Components, thus there are two Principal Components.

v. Choose Principal Component with the highest Eigenvalue i.e. $\begin{bmatrix} 0.0002 \\ 1 \end{bmatrix}$

vi. Transform data by multiplying the mean centered data with the chosen Principal Component.

$$New\ Data\ Matrix = \begin{bmatrix} -1.6 & -9600 \\ -6.6 & 400 \\ 8.4 & 15400 \\ 3.4 & -4600 \\ -3.6 & -1600 \end{bmatrix} \times \begin{bmatrix} 0.0002 \\ 1 \end{bmatrix} = \begin{bmatrix} -9601.6002 \\ -6200.6002 \\ 15408.4002 \\ -4596.5998 \\ -1603.6002 \end{bmatrix}$$

The transformed data is now a single-column matrix, representing the reduced dimensional data along the first Principal Component. This showcases how PCA simplifies the data while retaining the most significant information for analysis and visualization.

CHAPTER FOUR

FURTHER APPLICATIONS OF LINEAR ALGEBRA

4.1. INTRODUCTION

Linear algebra, once mastered, unlocks a treasure trove of problem-solving potential across various disciplines. This fundamental part of mathematics has far-reaching implications, influencing fields as diverse as computer science, physics, biology, economics and even social sciences. In this chapter, we embark on an exciting journey to explore the multifaceted applications of linear algebra.

4.2. CIVIL ENGINEERING AND LINEAR ALGEBRA

Linear algebra plays a vital role in many fields of engineering which further proves its versatility and efficiency. Some major departments that require the functions of linear algebra include civil, mechanical, computer, electrical, aerospace, and chemical engineering.

In civil engineering, one of the many ways in which linear algebra is used among others such as structural analysis is in traffic flow. Traffic flow is the study of interactions between vehicles, drivers, and infrastructure (including highways, signage, and traffic control devices), with the aim of understanding and developing an optimal road network with efficient movement of traffic and minimal traffic congestion problems. Mathematical theory of traffic flow and traffic equilibrium analysis was first introduced by Frank Knite in the 1920's and was refined into Wardrop's first and second principles of equilibrium. Current traffic models use a mixture of empirical and theoretical techniques. These models are then developed into traffic forecast, to take account of proposed local or major changes, such as increased vehicle use, changes in land use or changes in mode of transport and to identify areas of congestion where the network needs to be adjusted.

EXAMPLE:

A system of linear equations was used to analyze the flow of traffic for a network of four one-way streets in Kumasi, Ghana. The variables represent the flow of the traffic between the four intersections in the network. The data was obtained by counting the number of vehicles that travelled around the four one-way streets between the hours of 6 am to 10 pm, and 2 pm to 6 pm during the mid-week peak traffic hours. The arrows in the diagram indicate the direction of flow of traffic in and out of the network that is measured in terms of number of vehicles per hour (vph). The diagram in Figure 4.1 below describes the four one-way streets in Kumasi under study in the model:

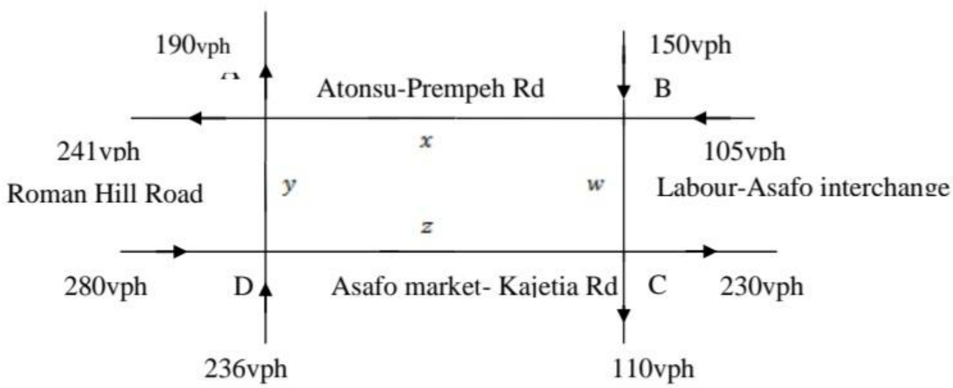


Fig 4.1.

Model Assumptions: the following assumptions were made to ensure the smooth flow of the traffic:

I) Vehicles entering each intersection should always be equal to the number of vehicles leaving the intersection.

II) The streets must all be one-way with the arrows indicating the direction of traffic flow.

The system of equations for the model was formulated as follows:

At intersection A: traffic in = x + y, traffic out = 241 + 190, thus, x + y = 431.

At intersection B: traffic in = 150 + 105, traffic out x + w, thus x + w = 255

At intersection C: traffic in = z + w, traffic out= 230 +110, thus, z + w = 340

At intersection D: traffic in = 280 + 236, Traffic out = y + z, thus, y + z = 516

The constraints were written as a system of linear equations as follows:

$$x + y = 431$$

$$x + w = 255$$

$$z + w = 340$$

$$y + z = 516 \tag{4.1}$$

We then used the Gauss-Jordan elimination method to solve the system of equations. The augmented matrix and reduced row-echelon form of the above system are as follows:

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 431 \\ 1 & 0 & 0 & 1 & 255 \\ 0 & 0 & 1 & 1 & 340 \\ 0 & 1 & 1 & 0 & 516 \end{bmatrix} \text{ Row operations } \begin{bmatrix} 1 & 0 & 0 & 1 & 255 \\ 0 & 1 & 0 & -1 & 176 \\ 0 & 0 & 1 & 1 & 340 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The system of equations that corresponds to this reduced row-echelon form is:

$$x + w = 255$$

$$y – w = 176$$

$$z + w = 340 \qquad (4.2)$$

Expressing each leading variable in terms of the remaining variable, we have:

$$x = -w + 255$$

$$y = w + 176$$

$$z = -w + 340 \qquad (4.3)$$

If we take a construction limit on w to be 100 vph, then the values of x. y, and z will be:

$$x = -100 + 255 = 155 \text{ vph}$$

$$y = 100 + 176 = 276 \text{ vph}$$

$$z = -100 + 340 = 240 \text{ vph} \qquad (4.4)$$

Due to the nature of the model, a driver has a certain amount of choice at the intersection. Therefore, many traffic flows are possible and as such, the system of the modelling equations has many solutions. Considering this, it is optimal to have as little traffic flow along z as possible. The flows can therefore be controlled along the various branches using traffic lights. According to the model, the third equation in the system shows that z will be a minimum when w is as large as possible, if it does not exceed 340. The largest value that can be assumed for w without causing negative values for x or y is 255. Thus, the smallest value for w is -255 + 340 = 85. Any road work on Asafo Interchange to Roman Hill Road should allow for traffic volume of at least 85 vph. Therefore, to keep the traffic flowing 240 vph must be routed between D and C, 155 vph between A and B, and 276 vph between A and D respectively.

4.4. ECONOMICS AND LINEAR ALGEBRA

One of the major ways which linear algebra can be applied to economics is through Input-Output economic models. Input-Output economics is a quantitative framework used to analyze the complex relationships between different sectors of an economy. Developed by Nobel Prize winner in the year 1973, Wassily Leontief¸ this approach examines how the output of one sector serves as the input of another, providing a comprehensive understanding of the economy's structural interdependencies. By mapping these interactions, input-output economics enables policymakers, researchers, and businesses to forecast economic trends, evaluate policy impacts and optimize resource allocations.

Roughly speaking, an economic system in this model consists of several industries, each of which produces a product and each of which uses some of the production of the other industries.

EXAMPLE:

A primitive society has three basic needs: food, shelter, and clothing. There are thus three industries in the society—the farming, housing, and garment industries—that produce these commodities. Each of these industries consumes a certain proportion of the total output of each commodity according to the following table:

		OUTPUT		
		Farming	**Housing**	**Garment**
CONSUMPTION	**Farming**	40%	20%	30%
	Housing	20%	60%	40%
	Garment	40%	20%	30%

Table 4.2.

Find the annual prices that each industry must charge for its income to equal its expenditures.

SOLUTION: Let P_1, P_2, and P_3 be the prices charged per year by the farming, housing, and garment industries, respectively, for their total output. To see how these prices are determined, consider the farming industry. It receives P_1 for its production in any year. But it consumes products from all these industries in the following amounts (from row 1 of the table): 40% of the food, 20% of the housing, and 30% of the clothing. Hence, the expenditures of the farming industry are

$$0.4P_1 + 0.2P_2 + 0.3P_3 = P_1 \quad (4.5)$$

A similar analysis of the other two industries leads to the following system of equations:

$$0.4P_1 + 0.2P_2 + 0.3P_3 = P_1$$

$$0.2P_1 + 0.6P_2 + 0.4P_3 = P_2$$

$$0.4P_1 + 0.2P_2 + 0.3P_3 = P_3 \quad (4.6)$$

This has the matrix form EP = P, where

$$E = \begin{bmatrix} 0.4 & 0.2 & 0.3 \\ 0.2 & 0.6 & 0.4 \\ 0.4 & 0.2 & 0.3 \end{bmatrix} \quad \text{and } P = \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix}$$

The equations can be written as the homogeneous system

$$(I - E)P = 0 \tag{4.7}$$

where I is the 3 × 3 identity matrix, and the solutions are

$$P = \begin{bmatrix} 2t \\ 3t \\ 2t \end{bmatrix} \tag{4.8}$$

where t is a parameter. Thus, the pricing must be such that the total output of the farming industry has the same value as the total output of the garment industry, whereas the total value of the housing industry must be 3/2 as much.

4.5. PHYSICS AND LINEAR ALGEBRA

In physics, one of the ways linear algebras can be applied is through electrical networks. Current flow in a simple electrical network can be described by a system of linear equations. A voltage source such as a battery forces a current of resistor (such as a lightbulb or motor), some of the voltage is “used up”; by Ohm’s law, this “voltage drop” across a resistor is given by

$$V = IR \tag{4.9}$$

where the voltage V is measured in volts, the resistance R in ohms (denoted by Ω), and the current flow I in amperes (amps, for short). Current flow in a loop is governed by KIRCHHOFF'S VOLTAGE LAW which states that the algebraic sum of the IR voltage drops in one direction around a loop equals the algebraic sum of the voltage sources in the same direction around the loop.

EXAMPLE

The network in Figure 4.3. below contains three closed loops. The currents flowing in loops 1, 2, and 3 are denoted by I_1, I_2 and I_3, respectively.

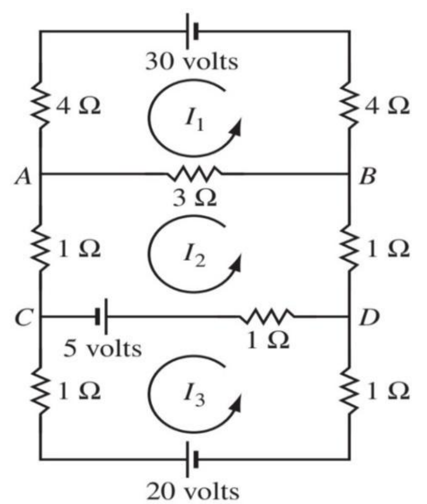


Fig 4.3.

Determine the loop currents in the network in Figure 4.3.

For loop 1, the current I_1 flows through three resistors, and the sum of the IR voltage drops is:

$$4I_1 + 4I_1 + 3I_1 = (4 + 4 + 3)\ I_1 = 11\ I_1 \qquad (4.10)$$

Current from loop 2 also flows in part of loop 1, through the short branch between A and B. The associated IR drop there is $3I_2$ volts. However, the current direction for the branch AB in loop 1 is opposite to that chosen for the flow in loop 2, so the algebraic sum of all IR drops for loop 1 is $11I_1 - 3I_2$. Since the voltage in loop 1 is +30 volts, Kirchhoff's voltage law implies that:

$$11I_1 - 3I_2 = 30 \qquad (4.11)$$

The equation for loop 2 is

$$-3I_1 + 6I_2 - I_3 = 5 \qquad (4.12)$$

The term $3I_1$ comes from the flow of the loop 1 current through the branch AB (with a negative voltage drop because the current flow there is opposite to the flow in loop 2). The term $6I_2$ is the sum of all resistances in loop 2, multiplied by the loop current. The term $-I_3 = -1.\ I_3$ comes from the loop 3 current flowing through the 1-ohm resistor in branch CD in the direction opposite to the flow in loop 2. The loop 3 equation is

$$-I_2 + 3I_3 = 25 \qquad (4.13)$$

Note that the 5-volt battery in branch CD is counted as part of both loop 2 and loop 3, but it is 5 volts for loop 3 because of the direction chosen for the current in loop 3. The 20-volt battery is negative for the same reason. The loop currents are found by solving the system

$$11I_1 - 3I_2 \qquad = 30$$

$$-3I_1 + 6I_2 - I_3 = 5$$

$$-I_2 + 3I_3 = 25 \tag{4.14}$$

Row operations on the augmented matrix yield the following: $I_1 = 3$ amps, $I_2 = 1$ amp, and $I_3 = -8$ amps. The negative value of I_3 indicates that the actual current in loop 3 flows in the direction opposite to that shown in Figure 4.3.

It is instructive to look at system (4.14) as a vector equation:

$$I_1 \begin{bmatrix} 11 \\ -3 \\ 0 \end{bmatrix} + I_2 \begin{bmatrix} -3 \\ 6 \\ -1 \end{bmatrix} + I_3 \begin{bmatrix} 0 \\ -1 \\ 3 \end{bmatrix} = \begin{bmatrix} 30 \\ 5 \\ -25 \end{bmatrix} \tag{4.15}$$

The first entry of each vector concerns the first loop, and similarly for the second and third entries. The first resistor vector lists the resistance in the various loops through which current I_1 flows. A resistance is written negatively when I_1 flows against the flow direction in another loop. Now, remember that $V = IR$ from equation (4.9)

where $R = [r_1 \quad r_2 \quad r_3]$ and $I = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$

this provides a matrix version of Ohm's law. If all loop currents are chosen in the same direction (say, counterclockwise), then all entries off the main diagonal of R will be negative. The matrix equation $V = IR$ makes the linearity of this model easy to see immediately. For instance, if the voltage vector is doubled, then the current vector must double.

$$IR = \begin{bmatrix} 30 \\ 0 \\ 0 \end{bmatrix}, \qquad IR = \begin{bmatrix} 0 \\ 5 \\ 0 \end{bmatrix}, \text{ and } \qquad IR = \begin{bmatrix} 0 \\ 0 \\ -25 \end{bmatrix}$$

Each equation here corresponds to the circuit with only one voltage source (the other sources being replaced by wires that close each loop). The model for current flow is linear precisely because Ohm's law and Kirchhoff's law are linear. The voltage drops across a resistor is proportional to the current flowing through it (Ohm), and the sum of the voltage drops in a loop equals the sum of the voltage sources in the loop (Kirchhoff). Loop currents in a network can be used to determine the current in any branch of the network. If only one loop current passes through a branch, the branch current equals the loop current. If more than one loop current passes through a branch, the branch current is the algebraic sum of the loop currents in the branch (Kirchhoff's current law). For instance, the current in branch AB is $I_1 - I_2 = 3 - 1 = 2$ amps, in the direction of I_1. The current in branch CD is $I_2 - I_3 = 9$ amps.

CHAPTER FIVE

RECOMMENDATION AND CONCLUSION

5.1. SUMMARY

Linear algebra is a foundational mathematical discipline in artificial intelligence, particularly in machine learning. It provides the mathematical framework for algorithms like linear regression and principal component analysis. In linear regression, linear algebra enables the optimization of model weights by minimizing last squares error using matrix operations. Similarly, PCA relies on linear algebra to reduce dimensionality by decomposing covariance matrices into eigenvectors and eigenvalues. These techniques are crucial in AI processes, including predictive modelling, data visualization and feature extraction. Linear algebra is also used to model and options mise traffic flow, reducing congestion and improving safety. It is also used to Annalise, and design circuits, ensuring efficient and safe transmission of electric current. In economics, linear algebra is used in input output analysis to model the flow of goods and services between industries, informing economic policy decisions. Linear algebraic techniques such as matrix inversion and eigenvalue decomposition are employed to analyze the structure of economic systems and predict the impact of policy changes.

5.2. RECOMMENDATION

In this comprehensive study of the application of Linear Algebra in the realm of Artificial Intelligence (AI), profound symbiotic relationship between these two domains have been unveiled. As AI continues its rapid evolution, it becomes increasingly clear that Linear Algebra forms the Mathematical bedrock upon which AI algorithms, Data structures, and techniques are constructed. The findings and analyses in this study suggest several key recommendations:

1. **Enhanced Mathematical Proficiency**: It is paramount for aspiring AI practitioners and researchers to cultivate a strong foundation in Linear Algebra. Proficiency in concepts such as matrices, vectors, eigenvalues, and eigenvectors are essential. Institutions and educational programs should emphasize these topics within AI curricula.

2. **Algorithm Optimization**: AI algorithm designers should leverage the principles of Linear Algebra to optimize their algorithms. Matrix factorization, Singular Value Decomposition (SVD), and matrix-vector operations can significantly enhance the efficiency and performance of AI models.

3. **Data Representation**: Linear Algebra offers powerful tools for data representation and transformation. AI practitioners should explore techniques like Principal Component Analysis (PCA) to reduce data dimensionality while preserving critical information.

4. **Cross-disciplinary Collaboration**: Encourage interdisciplinary collaboration between mathematicians and AI researchers. Cross-pollination of ideas between these fields can lead to innovative AI solutions and more robust mathematical theories.

5. **Accessible Education**: Foster educational resources and initiatives that make

Linear Algebra and its AI applications accessible to a broader audience. Simplified explanations, practical examples, and interactive learning tools can bridge knowledge gaps.

6. **Continuous Research**: Encourage ongoing research into the development of novel Linear Algebra-based AI algorithms. Exploration of advanced topics like tensor algebra and non-linear transformations can open new avenues for AI advancement.

5.3. CONCLUSION

The profound integration of Linear Algebra and Artificial Intelligence is a testament to the enduring power of Mathematics in shaping the future of technology. This fusion of disciplines has already yielded remarkable advancements, such as self-driving cars, natural language processing, financial crime detection, etc., and has touched every facet of modern life.

From fundamental concepts such as matrices, vectors, and eigenvectors to advanced techniques like SVD and PCA, Linear Algebra's reach in AI is profound. Linear Algebra fuels machine learning algorithms, facilitates data reduction, and optimizes model performance. This synergy has revolutionized industries, such as healthcare finance, automobiles, etc., and redefined how humans interact with technology.

In this ever-evolving journey, we stand at the threshold of a future where AI, underpinned by the Mathematical elegance of Linear Algebra, will continue to redefine industries, enrich human experiences, and tackle some of the most complex challenges humanity faces.

To navigate this future successfully, we must recognize that Linear Algebra is not merely a tool; it is an enabler of innovation, a bridge between data and decisions, and a compass guiding us through the vast landscape of AI possibilities.

The path forward is clear: a future where the synergy of Linear Algebra and AI continues to illuminate human way, enabling humans to build a more informed, equitable, and interconnected world. The adventure has only just begun, and the possibilities are boundless.

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