GRADUATION OF MORTALITY RATES USING DIFFERENCE EQUATION

BY

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BEING A PROJECT SUBMITTED TO THE DEPARTMENT OF STATISTICS IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE AWARD OF THE BACHELOR OF SCIENCE (B.Sc.) DEGREE IN STATISTICS, PHYSICAL SCIENCE, UNIVERSITY OF BENIN, BENIN CITY

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UNDERTAKING

This project work was carried out by **OSARENOMA ENDURANCE OSARO** , with the matriculation number PSC1607825. I have not plagiarized any work. All published work used in this project have been cited and referenced appropriately

______________________________ ______________________________

OSARENOMA ENDURANCE OSARO Signature & Date

DEDICATION

This project work is dedicated to almighty god, for his infinite mercy, love ,guidance and source of knowledge throughout my academic sojourn in the university of Benin. Also to my beloved parents Mrs. and Mrs. Osarenoma, who has been my mentor and source of inspiration and to my siblings for their love and encouragement.

CERTIFICATION

This is to certify that this project work was carried out by **OSARENOMA ENDURANCE OSARO**, in the department of Statistics of the Faculty of Physical Science, University of Benin.

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My family especially my parents Mrs. and Mrs. Osarenoma for their financial supports and also to my siblings and my friends for their love and support, God bless you all.

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ABSTRACT

This project work conducts a study on graduation of mortality rate using difference equation. The study illustrated the use of difference equation with an example solved manually and with the use of SPSS statistical software.

The study started by introducing graduation, smoothness, mortality rates and some measures of mortality. We also looked at the methods of graduation, types of graduation and their advantages and disadvantages. We also looked at the test of graduation, where we considered two test which are the sign test and the chi-square test.

Furthermore, the study looked at the meaning of difference equation, and we saw a few examples of difference equation. Methods of solving difference equation were also looked into

Finally, illustrations were used to show how graduation of mortality rates works and results showed how we can predict ones death by how old he/she lives using the provided datasets.

CHAPTER ONE

INTRODUCTION

1.2 BACKGROUND OF THE STUDY

Graduation is the process whereby “smooth mortality rates” are created from “crude mortality rates”.

Smoothness is an important part of graduation but another is the extrapolation of mortality rates to ages at which data may be unreliable or even non-existent.

Mortality rate, or death rate is a measure of the number of deaths (in general or due to a specific cause) in a particular population, scaled to the size of that population per unit time. Mortality rate is typically expressed in units of death rates per 1,000 individuals per year; thus, a mortality rate of 7.5 (out of 1,000) in a population of 1,000 would mean 7.5 deaths per year in that entire population or 0.75% out of the total. It is distinct from “morbidity” which is either the prevalence or incidence of a disease, and also from the incidence rate (the number of new appearing cases of the disease per unit time).

An important specific mortality rate measure is the “crude death rate”, which looks at mortality from all causes in a given time interval for a given population.

Some specific measures of mortality includes:

1. **Perinatal Mortality Rate :** The sum of fetal deaths (stillbirths) past 22 (or 28) completed weeks of pregnancy plus the number of deaths among live-born children up to 7 completed days of life, divided by the number of deaths.
2. **Maternal Mortality Rate :** Number of deaths of mother assigned to pregnancy related causes during a given time interval, divided by the same time interval.

3. **Infant Mortality Rate :** Number of deaths among children less than 1 year of age during a given time interval divided by the number of the live births during the same time interval.
4. **Child Mortality Rate (Under five Mortality Rate) :** Number of deaths of children less than 5 years old, divided by number of live births.
5. **Age Specific Mortality Rate (ASMR) :** The total number of deaths per year at a specific age, divided by the number of living of living persons at that age (e.g age 52 at last birthday).
6. **Cause Specific Death Rate :** Number of deaths assigned to a specific cause during a given time interval divided by the mid-interval population. Etc

1.2 METHODS OF GRADUATION

There are three main methods of Graduation

- Graphical Graduation
- Graduation by Reference to a Standard Table
- Graduation by Mathematical Formula

1.2.1 Graphical Graduation

This is the fitting of a smooth curve to the data by hand. Given the subjectivity of this process, it is generally used for small data sets. An attempt is made to meet the twin aims of the process smoothness and adherence to data.

1.2.3 Graduation by Reference To A Standard Table

If you have a mortality study with relatively little data and suspect that the lives under consideration are similar to those whose experience formed the basis of a standard table then this method can be used.

The standard table (based on a large number of lives) imposes a basic structure on our new graduation. The levels of mortality rates might well be different but the progression of rates should be fairly similar.

1.2.3 Graduation by Mathematical Formula

Problem but finding a formula which gives adequate adherence to data can be difficult. In some cases a single formula can be used for all ages but in other cases a blend of functions is used (which off course introduces additional problems).

Other related Graduation methods include use of "spline functions". These join together low order polynomials to achieve a smooth continuous function across the age range.

The choice of "pivotal ages" (known as Knots) can be difficult but the method is relatively easy to use with adequate computing facilities.

1.3 ADVANTAGES AND DISADVANTAGES OF GRADUATION

In this chapter we shall be considering the advantages and disadvantages of the methods of graduation

1.3.1 ADVANTAGES OF GRAPHICAL GRADUATION

1. Useful when you have very little data
2. Allows exercise of judgment and experience
3. The use of confidence intervals allows weighting towards certain ages where precision of estimates is greater

1.3.2 DISADVANTAGES OF GRAPHICAL GRADUATION

1. Requires skill to achieve a good balance between smoothness and adherence to data
2. Subjective, so may be biased
3. Difficult to obtain more than two significant figure
4. Not suitable for producing standard tables
5. Overlapping problems of graduation has been done in sections

1.3.3 ADVANTAGES OF STANDARD TABLE GRADUATION

1. Information in a more extensive graduation of similar data is used
2. Can be used with little data
3. Can incorporate prior expectations of the form of the curve
4. Helpful when there is little data in our new experience at extreme ages
5. Since the stranded table will be smooth, the graduated rates will also be smooth (as long as a simple formula has been used)

1.3.4 DISADVANTAGES OF STANDARD TABLE GRADUATION

1. It is not always obvious which standard table to use
2. Adherence to data can be a problem exhibited in the graduated rates
3. Features in the standard table will be exhibited in the graduated rates
4. Not appropriate for producing new standard tables, based on large amounts of data

1.3.6 ADVANTAGES OF PARAMETRIC GRADUATION

1. Smoothness is guaranteed provided that a reasonably low order formula is chosen
2. Suitable for large data sets (e.g standard tables)
3. It is not subjective
4. With computers and an appropriate estimation procedure (e.g maximum likelihood) we can assess the significance of the improvement in model fit as additional parameters are added.
5. If the same formula is fitted to different experiences, the parameter estimates can be compared possibly with intuitive interpretation

1.3.6 DISADVANTAGES OF PARAMETRIC GRADUATION

1. It is often difficult to fit a single curve to the entire age range which adequately models infant mortality, the accident hump and normal aging
2. Usually difficult to deal with small data sets. In particular, extrapolation of values can be unreliable when formulae are used.
3. Usual problems is there are heterogeneous data to analyze.

The above advantages and disadvantages was gotten from the school of mathematics and computer sciences Hariot Watt University Edinburgh, Scotland.

1.5. DIFFERENCE EQUATION

Difference Equation is a mathematical equality involving the difference between successive values of a function of a discrete variable. A discrete variable is a variable whose variable is obtained by counting.

For example, the discrete variable X may have the values $X = a, X = a + 1, X = a + 2, \ldots, X = a + n$

The function Y has the corresponding values $Y_0, Y_1, Y_2, \dots, Y_n$ from which the differences can be found. Any equation that relates the values of ΔY to each other or to X is a difference equation. In general, such an equation takes the form $y - a_i y_i - 1 = b_i$.

Difference equations are very similar to differential equations but they are defined in discrete domain. Example $x_n = x_{n-1} + 1$.

so when $x_0 = 0$

$$x_0 = x_{0-1} + 1 = x_0 = 0$$

$$x_0 = 0 + C \quad C = 0$$

$$x_2 = x_{2-1} + 1 \quad x_1 + 1 = x_2$$

n	0	1	2	3	4	5	6	7	. . .
Xn	0	1	2	3	4	5	6	7	. . .

TABLE 1.1

Another example is using the Fibonacci sequence: The Fibonacci sequence is such that each number is the sum of the two preceding ones, starting from 0 and 1.

The rule can be written as $Fn = Fn_{-1} + Fn_{-2}$

where;

Fn is term number "n"

Fn_{-1} is the previous term (n_{-1})

Fn_{-2} is the term before that (n_{-2})

$F_0 = 0 \qquad F_1 = 1$

The sequence therefore gives:

TABLE 1.2

F_0	F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8	F_9	F_{10}	F_{11}	F_{12}	F_{13}	F_{14}
0	1	1	2	3	5	8	13	21	34	55	89	144	233	377

Example: Let's find F_{13}

$F_{13} = F_{13-1} + F_{13-2}$

$F_{13} = F_{12} + F_{11}$

$F_{13} = 144 + 89$

$F_{13} = 233$

1.5 AIM AND OBJECTIVE OF THE STUDY

The aim of this study is to apply difference equation to graduate mortality rates

The objectives of the study are

- To help remove sampling errors that may be present in graphs.
- To estimate the parameter of a mortality rate
- To produce smooth rates.
- To obtain a smooth time path

1.6 SIGNIFICANCE OF THE STUDY

This study significant as it gives a precise way of using difference equation to define graduation of mortality rates. The study is important because it shows various measures of mortality rates, and also show the importance of Difference Equation.

1.7 SCOPE OF THE STUDY

The focus of this project is on the application of the difference equation to mortality rate. This study will also show the importance of smoothing of mortality rate.

1.8 ORGANIZATION OF THE STUDY

This study is structured into five chapters. In chapter one, we gave the background and objectives of the study. In chapter two, we reviewed existing work in graduation of mortality rate and difference equation. In chapter three we looked at the methodology behind graduation of mortality rate and difference equation. In chapter four we illustrate with some basic examples and the implementation in SPSS software. Finally chapter five, summarizes and concludes the study.

1.9 DEFINITION OF TERMS

This session shall give the definition of some basic terms used in this work

- **CRUDE MORTALITY RATE:** This is also known as "death rate", it is a measure of the number of deaths in a particular population, scaled to the size of that population, per unit of time. And it is typically expressed in units of deaths per 1,000 individuals per year.

- **SMOOTHNESS:** The smoothness of a function is a property measured by the number of continuous derivatives it has over some domain. At the very minimum, a function could be considered "smooth" if it is differentiable everywhere.

- **EXTRAPOLATION:** Extrapolation is a process of estimating the value beyond the distinct range of the given variable based on its relationship with another variable.

- **MORBIDITY:** Morbidity is another term for illness. It refers to the consequences and complications (other than death) that result from a disease.

- **SPLINE FUNCTIONS:** Spline functions are formed by joining polynomials at fixed points called knots.

- **DISCRETE VARIABLES:** Discrete variables are numeric variables that have a countable number of values between any two values. A discrete variable is always numeric. For example; the number of customer complaints or the number of flaws or defects.

CHAPTER TWO

LITERATURE REVIEW

2.1 INTRODUCTION

In this chapter we shall review the works done by previous authors as it relates to Graduation Mortality and Difference Equation.

2.2 GRADUATION MORTALITY

Gompertz (1825) started a revolution in graduation thinking when he developed a parametric function that was intended to be consistent with the fundamental characteristics of how mortality should behave as a function of age (a 'law of human mortality')

Brockett and Zhang (1986) present a new method based upon an information theoretical approach for graduation. This method allows us to obtain isotonicity, convexity, or any other of a variety of desired attributes of the graduated series. Also this technique can handle intervals of non-isotonicity quite simply as well.

In a study from Bangladesh, it was found that prematurity, respiratory illnesses and drowningwere the most common cause of child death. Child death was inversely associated with socio- economic status. It highlighted the social distribution of different diseases in community. Hanifiet (2014)

A study was done in India to know the determinants of neonatal mortality. Socioeconomic factors, father's occupation, rajput caste were important determinants. Authors concluded that neonatal mortality was affected by socio-economic factors, biological and community level determinants. Upadhyay (2012)

In a cross sectional study, under-5 mortality was found to be strongly associated with maternal and infant health programs. It was also strongly associated with the proportion of births attended by trained person. To raise the public health spending remains the main challenge in developing countries for improving the quality of maternal and child healthcare. McGuire (2006)

A study was conducted to describe the association between under-5 mortality and socioeconomic, political, and healthcare factors between rich and poor children. In this study, higher incomes were associated with lower under-5 mortality rates. Houweling (2005)

A study was done to determine the global mortality of under-5 children due to acute respiratory (ARI) infections. It showed that out of 15 million under-5 deaths, 4 million died of ARIs. Majority were infants and more than 90% deaths occurred in developing countries. Leowski (1986)

A study was done to determine the relationship between reduction in government health-care spending and child mortality in high and low income countries. Authors found that reductions in government healthcare spending was associated with a significant increase in child mortality. Maruthappu (2014)

In a review article, authors highlighted the importance and need of integrated management of maternal, child and neonatal health conditions. They stated that a firm political commitment and partnerships were required for interventions at large scale. Bhutta (2012)

A study was done to analyze cause of mortality in children under 5 years of age in Iran. Authors found out that 60% of mortality occurred in the rural areas and 52% were boys. Congenital anomalies and chromosomal anomalies were amongst the common cause of death. Rahbar (2013)

In a study from Kenya, a study was done to examine relationship of rainfall and temperature on under 5 malaria or anemia mortality. In this study, Malaria or anemia mortality was found to be associated with changes in local temperature and rainfall. It described the biological relationship as well. Sewe (2015)

A study was done to describe the trend of child mortality in Kenya. It was found that child mortality declined in Kenya in both rural and urban areas. Authors concluded that there was a need of extra emphasis on urban slums to reduce child mortality. Murage (2014)

A study was done in India to identify socially modifiable factors that affected child mortality in hospital admitted children. Authors found administrative issues and family related reasons were most common modifiable factors. Mahajan (2014)

In a study from Tanzania, authors found that more than one third of deaths in the population were due to birth asphyxia and prematurity related causes. Authors concluded that child survival in that area could be improved with better antenatal care, maternal and newborn care. Regular training on resuscitation skills would also be helpful. Mmbaga (2012)

2.3 DIFFERENCE EQUATION

Mickens (2001) this paper is an introduction to nonstandard finite difference methods, which are useful to construct differential equations. In his paper, he described exact finite difference scheme, also rules for constructing nonstandard scheme with its application.

Fukagata and Kesagi (2002) they developed highly energy conservative finite difference method for cylindrical system. They proved that when approximate interpolation schemes are used then energy conservation in discretized space is satisfied. This holds for both equally and unequally spaced mesh on cylindrical coordinate system but not on Cartesian coordinates.

Farjadpur and Roundy (2006) finite difference time domain method suffer from reduced accuracy due to discretization, for modeling discontinuous dielectric materials. They show that accuracy can be improved by using sub pixel smoothing, if it is properly designed. Also this scheme attains quadratic convergence. Zhong and Zhi (2006) In their research paper, They proposed numerical methods for solving non-linear Poisson-Boltzmann equation $\Delta\psi = \sinh \psi$, where ψ is the electrostatic potential. A monotone iterative method was given for semi-linear partial differential equation of elliptic type. The modified central finite difference scheme is introduce. Numerical solutions agree with solutions obtained by adaptive finite element method.

Dolicanin and Nicolic (2010) in their research paper, finite difference method is used to study of phenomenon in the theory of thin plates. FDM based on replacing differential equation into difference equation. This method can efficiently solve the problem of bending of thin plates. It is used to find solutions for the plate deflection, moments, stress, strain etc.

Islam and Alias (2010) Finite difference method is used to discretize a parabolic partial differential equation. They presented a mathematical simulation model using one dimensional parabolic equation. This model is regarding to moisture and temperature behavior of tropical herbs during dehyadration.here Jacobi, Gauss seidal and Red black Gauss seidal iterative methods are studied. It has proved that dehydration model is capable to simulate mass and temperature distribution through numerical methods approach. This mathematical simulation is time consuming and capable to reduce the risk of real experiments in actual process.

Mehra and Patel (2010) in their research paper three methods finite difference, spectral and Wavelet Galerkin Method (WGM) are compared. These methods are tested on Advection and Klein-Gordan equation.WGM gives better accuracy in comparison with other two methods.

Obaid and Ouifki (2013) They design a numerical method known as Nonstandard Finite difference method (NSFDM).This method is use to solve model of HIV represented by a nonlinear system of ordinary differential equations. The model describes the dynamic of HIV epidemic by partition of human population into susceptible and infectious subpopulations. They will use the forward difference approximations for the first derivative. The proposed schemes are unconditionally stable.

Mousa and Reda (2013) in their research paper the Adomian Decomposition Method (ADM) and the method of lines (MOL) are discussed to obtain solitary solutions of the Korteweg-de Vries equation (KdV). The numerical solutions by MOL are compared with the analytical solutions of Adomian Decomposition Method. In order to check the accuracy of the considered methods they have compared the obtained results with the exact ones. The observation is that the MOL is more effective and convenient than the Adomian Decomposition Method for solving such type of equations.

Gao and Zhang (2013) they developed a staggered-grid finite difference scheme with variable order accuracy for porous media. They use method on dispersion relation to determine the orders of accuracy. The variation of parameters gives validity of scheme. Proposed method can decrease computational cost without reducing accuracy.

Ghods and Mir (2014) In this article evaluation of finite difference method in modeling a rectangular thin plate structure is given. In big construction systems subjected to arbitrary load, including complex boundary condition solution by analytic method is impossible. Therefore differential equation are discretized by finite difference method .This method is relatively strong method for numerical solution of plate equation with different loading and support solutions conditions.

Pathak and Doctor (2014) they describe spline collocation, finite difference and finite element method for solving flow of electricity in a cable transmission lines. These methods are used to solve parabolic partial differential equation. Solution obtained from spline method is more accurate than finite difference and finite element method. Also spline implicit method has accurate results than spline explicit method.

2.4 SUMMARY

Due to the uniqueness of graduation of mortality rate and different equation, different authors, professionals, mathematicians and statisticians have taken the need to do various studies on them.

CHAPTER THREE

METHODOLOGY

3.1 INTRODUCTION

In this chapter we shall discuss graduation of mortality rates, difference equation and advantages and disadvantages of the methods of graduation. In section 3.2, the graduation of mortality rates is presented. Section 3.3 contains the difference equation and the process involved. The various advantages and disadvantages of the methods of graduation is presented in section 3.4 and section 3.5 summarizes the chapter.

3.2 GRADUATION OF MORTALITY RATES

One of the most important tasks in actuarial science is to describe the actual but unknown mortality pattern of a population. In order to achieve this, the actuary calculates from raw data the crude mortality rates, which usually form an irregular series. Because of this, it is common to revise the initial estimates with the aim of producing smoother estimates with a procedure called "graduation".

Graduation has two basic characteristics: smoothness and goodness of fit to the observed data. These two characteristics are in competition and in order to achieve one of them we have to sacrifice the other. Smoothness is usually measured by

$$S = \sum_{x=1}^{n-3} (\Delta^3 \ V_x)^2$$

while goodness of fit is given by

$$F = \sum_{x=1}^{n} wx(Ux - V_x)^2$$

where Vx are the graduated values, Ux are the initial values and Wx are weights.

Note: Both the formula for smoothness and goodness of fit was given by Takis Papaiannou and Athanasios Sachlas.

By postulating a model of mortality and then incorporating some actual data we will have obtained estimates of $\mu x \, or \, qx$ at successive ages. These estimates are called crude mortality rates and are subject to:

1. Random fluctuations: Since on statistical consideration any estimate is just a single sample from the sampling distribution of the estimator.
2. Irregularity: Since we have random sampling, the crude estimates of mortality will not proceed smoothly over the years of age.

Intuitively we expect a smooth progression of mortality rates given large enough samples; we do not expect mortality rates to move in discrete steps since the main factor influencing human mortality is the gradual aging process.

We prefer to use smoothly progressing rates in actuarial work, not just from a theoretical viewpoint, but because smooth rates of decrement imply a smooth progression of premium rates at ages rise.

3.2.1 A MEASURE OF SMOOTHNESS OF GRADUATION

The achievements of smoothness is one of the main purpose of graduation. Smoothness, however has never been defined except in terms of concepts which themselves defy definition and there is no accepted way of measuring it. The customary method of testing smoothness in the case of a series or of a function by examining successive orders of differences (called here after the classical method, Barnett

J,A 77,18,19) is generally recognized as unsatisfactory. If we require successive differences to be smooth, we have to judge their smoothness by their own differences.

In other to measure smoothness, there is evidently a need for 'symmetrical differential coefficients' (S.D.C's) which treat the two variables impartially and which do not assume large values merely because one variable is increasing rapidly with respect to the other or because a differential coefficient is increasing rapidly compared with one of the variables. As we have seen differentiation must not be carried out with respect to either of the variables x and y and this implies that successive S.D.C's shall be derived by differentiation with respect to another variable t, such that the differential dt is a symmetrical function of dx and dy. The S.D.C's must possess the following properties:

1. They must be unchanged in absolute value if x and y are interchanged.
2. They must indicate smoothness in the case of all functions intuitively regarded as being smooth (e.g they must indicate perfect smoothness).
3. S.D.C's of the second and higher orders must not automatically become infinite when any (ordinary) differential coefficient is zero.
4. They should have an easily appreciated geometry significance.

3.2.2 THEORITICAL GRADUATION

To construct a smooth series of death probabilities $\{Vx\}$ which is as closer as possible to the observed series $\{Ux\}$ and in addition they assume that the true but unknown underlying mortality pattern is

(i) Smooth

(ii) Increasing with age, i.e monotone

(iii) More steeply increasing in higher aged i.e convex.

(iv) The number of deaths in the graduated data equals the number of deaths in the observed date

(v) The total age of death in the graduated data equals the total age of death in the observed data.

By the term total age of death we mean the sum of the product of the number of deaths at every age by the corresponding age.

3.2.3 THE GRADUATION PROCESS

In the figure below, the dots represents the crude rates and the line represents the graduated rates. The graduated rates are then our estimates of the 'true underlying' rates of mortality.

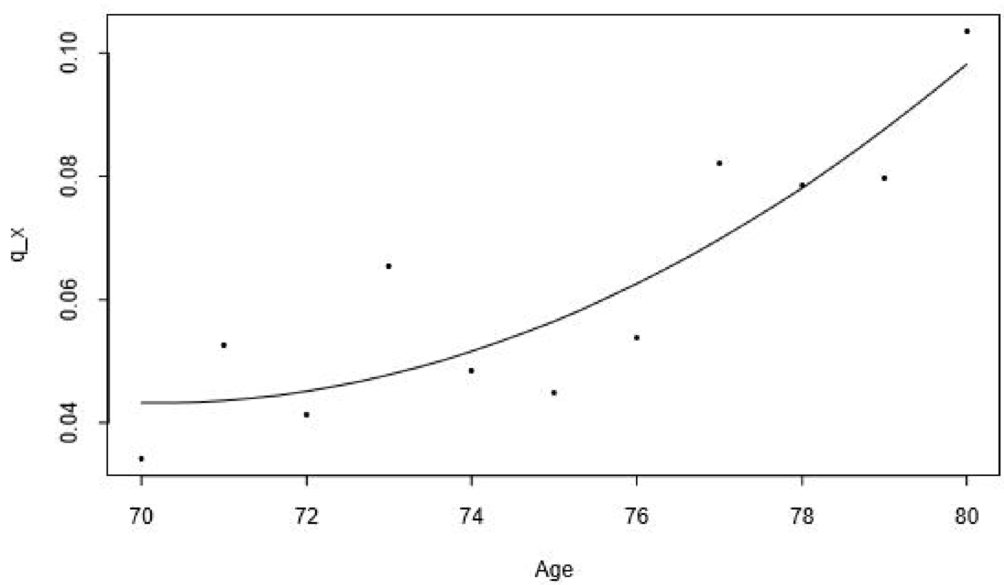


FIGURE 3.1 A graduation of $qx\ for\ 70 \leq x \leq 80$

It is important to note that the graduation process cannot remove non-random error or bias in our estimates which might arise from poor data collection methods or an inappropriate statistical model.

There are two basic steps in a graduation

1. The construction of a set of gradients rates or forces of mortality from a set of crude estimates. This is a curve fitting problem.
2. The testing of graduated estimates to determine whether they are

a. Accepting smooth and
b. An acceptable fit to our original data

An element of judgment and experience is needed in most graduation exercise.

3.3 TEST OF GRADUATION

A way to check if there is an over graduation or under graduation is the cumulative deviation test. It checks out the overall bias. It is primarily concerned with the overall graduation rates to tell if it is very high or very low. What we do is to look at an overall figure. We take an assumption say a z- statistic; which is observed death overall.

$$z = \frac{observed\ deaths - expected\ death}{S.D\ of\ deaths}$$

Graduation is measured by a number of tests and we will be looking at two of them.

3.4.1 X^2 TEST

This is a test for overall goodness of fit. There are two cases

1. We can test whether our data are consistent with a standard table with

$$x = \sum \frac{(\mathrm{D}x - \dot{\mathrm{D}}x)^2}{Dx} = \sum Z_x^2$$

Where Dx the actual numbers are observed and $\dot{\mathrm{D}}x$ are the expected number observed under the tested model. The higher the value of χ the greater are the difference between our observed data and our graduated values i.e we have a one-sided test whose null distribution is approximately

$$x = \sum_x Z_x^2 \sim \chi^2 N$$

Where N is the number of age groups.

2. However if a graduation method has made use of the D_x data in the construction of q_x or $\mu_{x+\frac{1}{2}}$ (as is usually the case) then the X^2 statistic will have fewer than N degrees of freedom. The reduction in the degrees of freedom will depend on the method of graduation.

THE SIGN TEST

Suppose that S denotes the number of positive Z_x and that there are n age groups. If there is no positive or negative bias in the $\{Z_x\}$ then each Z_x is equally likely to be positive or negative so $S \sim B\left(n, \frac{1}{2}\right)$. Since we wish to identify positive or negative bias, a two-sided test is required. Suppose we observe $S = s$. There are two cases;

- Small n: if $s > n/2$ then the significance probability is $2\,Pr\,(S \geq s)$ while if $s < n/2$ then the significance probability is $2\,Pr\,(S \leq s\,)$

- Large n: we can use the normal approximation with $E(S) = \frac{n}{2}$ $var = n \times \frac{1}{2} \times \frac{1}{2} = \frac{n}{4}$ or $S \sim N\left(\frac{n}{2}, \frac{n}{4}\right)$

3.4 DIFFERENCE EQUATION

Difference equation is a mathematical equally involving the differences between successive values of a function of a discrete variable. A discrete variable is one that is defined or of interest only for values that differ by some finite amount, usually a constant and often 1; for example, the discrete variable x may have values $x_0 = a_x$ $x_1 = a + 1, x_2 = a + 2, \ldots, x_n = a + n$

The function y has the corresponding values $y_0, y_1, y_2, \ldots, y_n$ from which the differences can be found:

$$\Delta y_0 = y_1 - y_0$$

$$\Delta y_1 = y_2 - y_1$$

$$\Delta y_n = y_{n+1} = y_n$$

Any equation that relates the values of Δy to each other or x_i is a difference equation. In general, such an equation takes the form

$$y_i - a_i y_{i-1} = b_i.$$

EXAMPLES OF DIFFERENCE EQUATION

1. $y(k) = ay(k-1) + f(k)$

2. $y(k) + \frac{1}{2}y(k-1) + \frac{1}{4}y(k-2) = f(k)$

3. $y(k) = \frac{1}{T}[f(k) - f(k-1)]$

We call these difference equation because we are representing the output of a discrete-time (discrete time, views values of variables as occurring at distinct, separate "points in time" or equivalently as being unchanged throughout each non-zero region of time, i.e time is viewed as a discrete variable.) invariant system which we normally denote as $y(k)$ as a kind of linear combination of time differences of other signals. So one of those signals is $y(k-1)$ which is the output at the previous time, while $f(k)$ is the input to the system.

We note that a difference equation is a simple relationship that relates the current output $[y(k)]$ with previous output $[y(k-1), y(k-2)]$ to input $[f(k)]$ and possibly previous input $[f(k-1)]$

$f(k)$ is the input with time k

$f(k-1)$ is the input at the previous time $k-1$

3.5 METHODS OF SOLVING DIFFERENCE EQUATION

3.5.1 TIME-SHIFTING

$y(k) = ay(k-1) + f(k)$ ……………..(*)

What we do here is to shift the equation around

Let $k = k + 2$

. $y(k+2) = ay(k+2-1) + f(k+2)$

$y(k+2) = ay(k+1) + f(k+2)$ …………..(**)

Equation (*) and (**) are the same, what we just did is to shift the time by letting k equals k+2

3.5.4 ADVANCED OPERATOR FORM

$y(k+N) + aN_{-1}\, y(k+N-1) + \ldots + a_1\, y(k+1) + a_0\, y(k) = bm\, f(k+M) + b_{M-1}\, f(k+M-1) + \ldots + b_0\, f(k)$

It is usually written from large shift $y(k+N)$ up to the smallest shift $y(k)$. The a_1, b_m are the scale factors or numbers that are the weighing coefficients of each of the signals. On the left we have a with the signals y and b with signals f.

3.5.5 DELAY OPERATOR FORM

If we take the advance operator form and replace $k+M$ with k so we kind of subtract capital N from everything, we can therefore write it in a different form and that is what we call the "delay operator form".

$y(k) + aN_{-1}\; y(k+1) + \ldots + a_1 y(k-N+1) + a_0 y(k-N) = b_N f(k) + b_{N-1} f(k-1) + \ldots + b_0 f(k-N)$.

This looks similar to the advance operator form in terms of one having all the y's on the left and all the output$f$'s on the right. But it is all being shifted. I can start from $y(k)$ and then I have $y(k-1), y(k-2)\; up\; to\; y(k-N)$. Same ything also happened on the right side of the equation; we have $f(k)\; to\; f(k-1), f(k-2), f(k-3)\; up\; to\; f(k-N)$.

In this form where we shift from $y(k)\; to\; y(k-1)\; up\; to\; y(k-N)$ is called the "delay operator form".

Example:

$$3y(k+3)+4y(k+2)+y(k)=9f(k+2)-3f(k+1)$$

Let's put this into the delay operator form, say we don't want to use $(k+3)$ $we\ would\ like\ to\ have\ \ y(k),\ y(k-1), etc.$ What we can do is to replace $k\ \ with\ k-3$

Let $k+k-3$

$$3y(k-3+3)+4y(k-3+2)+y(k-3)=9f(k-3+2)-3f(k-3+1)$$

$$3y(k)+4y(k-1)+y(k-3)=9f(k-1)-3f(k-2)$$

This is not quite a delay operator form yet because 3 is allocated to $y(k)$ at the beginning of the equation. So we divide both sides of the equation by 3.

We would therefore get

$$y(k)+\frac{4}{3}y(k-1)+\frac{1}{3}y(k-3)=3f(k-1)-f(k-2)$$

We can now introduce a_s and b_s N = 3

Note that the last value is the most negative value, we now have

$a_2=\frac{4}{3}$ $a_1=0$ (because we do not have $y(k-2)$) $a_0=\frac{1}{3}$

$b_3=0$ (because we do not have $f(k)$ term, $b_2=3,\ b_1=-1,\ b_0=0$ (because there is no $(k-3)$ term

3.5.4 THE Z-TRANSFORM METHOD

Given a first order difference equation

$$y(k+1) - 3y(k) =- 6 \qquad if\ y(0) = 1$$

In solving this by Z-Transform, we will take the z-transform of every term in the above equation.

$Ƶ\{y(k+1)\} - 3\,Ƶ\big(y(k)\big) = Ƶ(-6)$

Solving the z-transform of -6

$Ƶ(-6)$

$= -6\,Ƶ(1)$ →where 1 is the same as the unit step sequence (u(k))

$= -6\,Ƶ\big(u(k)\big)$

$= -6\ \frac{z}{z-1}$

$Ƶ(-6) = \frac{-6z}{z-1}$

Next is the z-transform of $y(k+1)$

To do this, we must recall the first shift

If $Ƶ(\big(f(k)\big) = F(z)$

First shift theorem

$\mathcal{Z}((f(k+1)) = zf(z) - zf(0)$

Then

$\mathcal{Z}((f(k+1)) = z \quad \mathcal{Z}(F(k)) - zf(0)$

i.e. $\mathcal{Z}((y(k+1)) = z \;\; \mathcal{Z}(y(k)) - zy(0)$ $\qquad Given\, y(0) = 1$

$\mathcal{Z}((y(k+1)) = z \;\; \mathcal{Z}(y(k)) - z$

Inputting into the given equation

$y(k+1) - 3y(k) =- 6$

We have

$z \;\mathcal{Z}(y(k)) - z\,\mathcal{Z}(y(k)) = \frac{-6z}{z-1}$

We have common terms which are $\mathcal{Z}(y(k))$

So we combine them together (left hand side)

$(z-3) \;\; \mathcal{Z}(y(k)) = \frac{-6z}{z-1} + z$

Combining the right hand side

$(\mathrm{z}-3) \;\; \mathcal{Z}(y(k)) = \frac{-6+z(z-1)}{z-1}$

$(z-3) \;\; \mathcal{Z}(y(k)) = \frac{z^2-7z}{z-1}$

Divide both sides by $z-3$

$$\mathcal{Z}\big(y(k)\big)=\frac{z^2-7z}{(z-1)(z-3)}$$

To find out $y(k)$ we need to take the inverse z-transform ($\mathcal{Z}$)

But first we factorize and divide by z first

$\mathcal{Z}(y(k))=\frac{z(z-7)}{(z-1)(z-3)}$ we cannot do partial fraction here because we have an improper fraction

We divide both sides by z

$$\frac{\mathcal{Z}\,(y(k))}{z}=\frac{z-7}{(z-1)(z-3)}$$

We can now apply partial fraction

$$\frac{\mathcal{Z}(y(k))}{z}=\frac{A}{z-1}+\frac{B}{z-3}$$

We then find A and B

First we find A

Since A is on top of $z-1$

We have

$A=\frac{z-7}{(z-1)(z-3)}$ put $z-1=0\;\therefore z=1$

We cover $(z-1)$ it is not useful here

$A = \frac{Z-7}{Z-3} = \frac{1-7}{1-3} = \frac{-6}{-2} = 3$ ∴A = 3

To get B

B is on top of $z - 3$

$B = \frac{z-7}{(z-1)(z-3)}$ put $z - 3 = 0$ $\therefore z = 3$

We cover $(z - 3)$ it is not useful here

$B = \frac{Z-7}{Z-1} = \frac{3-7}{3-1} = \frac{-4}{2} =- 2$ ∴B= -2

Putting them into the equation we have

$$\frac{\mathcal{Z}\ (y(k))}{z} = \frac{3}{z-1} + \frac{-2}{z-3}$$

We multiply the z with every term

We now get

$$\mathcal{Z}\ \big(y(k)\big) = \frac{3z}{z-1} - \frac{-2z}{z-3}$$

To now get $y(k)$ by using the inverse we have

$$y(k) = 3\ \mathcal{Z}^{-1}\left(\frac{z}{z-1}\right) - 2\ \mathcal{Z}^{-1}\ \left(\frac{z}{z-3}\right)$$

$\mathcal{Z}^{-1}\ \left(\frac{z}{z-1}\right) = uk$ $\quad\quad$ $\mathcal{Z}^{-1}\ \left(\frac{z}{z-3}\right) = 3^k$

The general expression for $y(k)$ is

$y(k) = 3\ u(k) - 2(3^k)$

3.6 SUMMARY

The achievement of smoothness is one of the main purposes of graduation. In graduation, we generally deal with discrete series of values. In the discrete case, the classical method essentially examines successive orders of differences of a function for equal interval of the argument.

Many problems in probability give rise to difference equations. Difference equations relate to differential equations as discrete mathematics relates to continuous mathematics.

Anyone who has made a study of differential equations will know that even supposedly elementary examples can be hard to solve. By contrast, elementary difference equation are relatively easy to deal with.

CHAPTER FOUR

DATA ANALYSIS, INTERPRETATION AND PRESENTATION OF RESULTS

4.1 INTRODUCTION

In this chapter we will look at the implementation of graduation of mortality rate, using difference equation. We would difference equation and relate it to graduation of mortality rate and present results to these solutions both manually and using the computer sofrware SPSS.

TABLE 4.1: DATA SETS

N	Age	Nx	Dx
1	12	8119	14
2	17	7950	20
3	22	6525	22
4	27	5998	23
5	32	5586	26
6	37	5245	28
7	42	4659	32
8	47	4222	37
9	52	3660	44
10	57	3012	54
11	62	2500	68
12	67	2113	87
13	72	1469	100
14	77	883	95

15	82	418	70
16	87	181	49

4.2 SOLUTION USING DIFFERENCE EQUATION

$$y_t - ay_{t-1} = 0$$

Here we are to find the general solution to the above equation.

Recall $\Sigma f_n = f_{n+1}$ (shift operator)

The equation above can be written as

$$\Sigma^0 y_t - a\Sigma^{-1} y_t = 0 \qquad let\ \Sigma^0 = 1$$

$$y_t - a\Sigma^{-1} y_t = 0$$

$$multiply\ through\ by\ \Sigma$$

$$\Sigma y_t - ay_t = 0$$

$$factorizing\ we\ have$$

$$(\Sigma - a)y_t = 0$$

$$y_t \neq 0$$

$$\Sigma - a = 0 \qquad \therefore\ \Sigma = 0$$

The general solution is given by

$$y_t = \sum_{i=1}^{t} \beta_i \Sigma_i{}^t + \frac{\phi}{p}$$

$$\sum_{i=1}^{t} \beta_i \varepsilon_i{}^{t} \textit{ is the homogeneous solution}$$

$$\frac{\phi}{p} \textit{ is the particular solution} \qquad \textit{where } \frac{\phi}{p} = 0$$

$$\boldsymbol{y_t = \beta_i a^t} \textit{ is the general solution}$$

From the data set above, we will be comparing Dx and Age

To do that we have to get the regression equation from our solved general equation

$$y_t = \beta_i a^t \quad \textit{will now be given as} \quad D_x = \beta_i a^t$$

We now take the natural logarithm of the equation

$$InD_x = In\beta_i + tIna$$

$$InD_x = y$$

$$In\beta_i = \alpha$$

$$Ina = \beta$$

Therefore we have our regression equation to be $y = \alpha + \beta t$

Table 4.2: DATA SET

N	Age	Dx	InDx
1	12	14	2.64
2	17	20	3.00
3	22	22	3.09
4	27	23	3.14
5	32	26	3.26

6	37	28	3.33
7	42	32	3.47
8	47	37	3.61
9	52	44	3.78
10	57	54	3.99
11	62	68	4.22
12	67	87	4.47
13	72	100	4.61
14	77	95	4.55
15	82	70	4.25
16	87	49	3.89

From the above data, our regression model is

$$\boldsymbol{lnDx = 2.557 + 0.023Age}$$

4.3 COMPUTATION OF THE PREDICTION VALUES

We are going to use the regression model $\boldsymbol{lnDx = 2.557 + 0.023Age}$

Predicting each of the Age value using the regression model

i. $lnDx = 2.557 + 0.023(12)$

$2.557 + 0.276$

2.833

ii. $lnDx = 2.557 + 0.023(17)$

$2.557 + 0.391$

2.948

iii. $lnDx = 2.557 + 0.023(22)$

$2.557 + 0.506$

3.063

iv. $lnDx = 2.557 + 0.023(27)$

$2.557 + 0.621$

3.178

v. $lnDx = 2.557 + 0.023(32)$

$2.557 + 0.736$

3.293

vi. $lnDx = 2.557 + 0.023(37)$

$2.557 +$ 0.851

3.408

vii. $lnDx = 2.557 + 0.023(42)$

2.557 + 0.966

3.523

viii. $lnDx = 2.557 + 0.023(47)$

$2.557 + 1.081$

3.638

ix. $lnDx = 2.557 + 0.023(52)$

$2.557 + 1.196$

3.753

x. $lnDx = 2.557 + 0.023(57)$

$2.557 + 1.311$

3.868

xi. $lnDx = 2.557 + 0.023(62)$

$2.557 + 1.426$

3.983

xii. $lnDx = 2.557 + 0.023(67)$

$2.557 + 1.541$

4.098

xiii. $lnDx = 2.557 + 0.023(72)$

$2.557 + 1.656$

4.213

xiv. $lnDx = 2.557 + 0.023(77)$

$2.557 + 1.771$

4.328

xv. $lnDx = 2.557 + 0.023(82)$

$2.557 + 1.886$

4.443

xvi. $lnDx = 2.557 + 0.023(87)$

$2.557 + 2.001$

4.558

TABLE 4.3: UPDATED TABLE

N	Age	Dx	InDx	lnDx^
1	12	14	2.64	2.833
2	17	20	3.00	2.948
3	22	22	3.09	3.063
4	27	23	3.14	3.178
5	32	26	3.26	3.293
6	37	28	3.33	3.408
7	42	32	3.47	3.523
8	47	37	3.61	3.638
9	52	44	3.78	3.753
10	57	54	3.99	3.868
11	62	68	4.22	3.983
12	67	87	4.47	4.098
13	72	100	4.61	4.213
14	77	95	4.55	4.328
15	82	70	4.25	4.443
16	87	49	3.89	4.558

SUBTRACTING lnDx^ from lnDx

TABLE 4.4: UPDATED TABLE

N	Age	Dx	InDx	lnDx^	lnDx − lnDx^
1	12	14	2.64	2.83	-0.19
2	17	20	3.00	2.95	0.05
3	22	22	3.09	3.06	0.03
4	27	23	3.14	3.18	-0.04
5	32	26	3.26	3.29	-0.03
6	37	28	3.33	3.41	-0.08
7	42	32	3.47	3.52	-0.05
8	47	37	3.61	3.64	-0.03
9	52	44	3.78	3.75	0.03
10	57	54	3.99	3.87	0.12
11	62	68	4.22	3.98	0.24
12	67	87	4.47	4.10	0.37
13	72	100	4.61	4.21	0.40
14	77	95	4.55	4.33	0.22
15	82	70	4.25	4.44	-0.19
16	87	49	3.89	4.56	-0.67

(lnDx – InDx^)^2

Squaring the values for lnDx – lnDx^ we get the last column and therefore have an updated data

TABLE 4.5: FINAL TABLE

N	Age	Dx	InDx	lnDx^	lnDx – lnDx^	(lnDx – lnDx)^2
1	12	14	2.64	11.47	-0.19	0.0361
2	17	20	3.00	24.31	0.05	0.0025
3	22	22	3.09	27.52	0.03	0.0009
4	27	23	3.14	29.30	-0.04	0.0016
5	32	26	3.26	33.58	-0.03	0.0009
6	37	28	3.33	36.08	-0.08	0.0064
7	42	32	3.47	41.07	-0.05	0.0025
8	47	37	3.61	46.07	-0.03	0.0009
9	52	44	3.78	52.13	0.03	0.0009
10	57	54	3.99	59.62	0.12	0.0144
11	62	68	4.22	67.82	0.24	0.0576
12	67	87	4.47	76.74	0.37	0.1369
13	72	100	4.61	81.74	0.40	0.1600
14	77	95	4.55	79.60	0.22	0.0484
15	82	70	4.25	68.90	-0.19	0.0361
16	87	49	3.89	56.06	-0.67	0.4489

4.4 ILLUSTRATION USING SPSS

Comparing Age and The natural logarithm of Dx using the SPSS Software

Since we have gotten a regression model from the difference equation, we are to regret the age and lnDx.

Regression deals with the relationship between two variables and allows us to predict the value of one variable if we know another. The accuracy of the prediction would depend on the strength of the correlation between the two variables.

TABLE 4.6:

N	Age	lnDx
1	12	2.64
2	17	3.00
3	22	3.09
4	27	3.14
5	32	3.26
6	37	3.33
7	42	3.47
8	47	3.61
9	52	3.78
10	57	3.99
11	62	4.22
12	67	4.47
13	72	4.61
14	77	4.55
15	82	4.25
16	87	3.89

Table 4.7: Variables Entered/Removed

Model	Variables Entered	Variables Removed	Method
1	Age[b]		Enter

This table is showing results for the variables entered and the variables removed.

Age is the independent variable, while lnDx is the dependent variable

a. Dependent lnDx

b. All requested variables entered

Table 4.8: Model Summary

Model	R	R square	Adjusted R square	Std. Error of the Estimate
1	.910[a]	.828	.816	.26065

a. Predictors (constant). Age

As the name of the table implies, it means summary of the model.

Let's consider them individually

- re will increaseR: This is the correlation coefficient between the two variables. Since our R is .910, and it is close to 1, we can say that it is a positive correlation. This can also be seen from our graph below.
- R^2 : Is the square of R i.e .910 x .910 which is .828. This can also be referred to as coefficient of determination. It can be used to denote how good your regression equation fits your data'
- Adjusted R^2 : This also denote how good your regression fits your data, but adjusts for the number of terms in a model. If you add more useful variables, adjusted r-squared will increase. Adjusted R^2 will always be less than or equal to R^2.

- Standard error of the estimate: This is a measure of variability, it is like the standard deviation which tells us how mush accuracy you will get from your prediction.

Table 4.9: Coeffficients[a]

Model	B	Unstandardized Coefficients std. Error	Standardized Coefficients Beta	t	sig.
1 (constant)	2.557	.154		16.564	.000
Age	.023	.003	.910	8.212	.000

a. Dependent Variable: lnDx

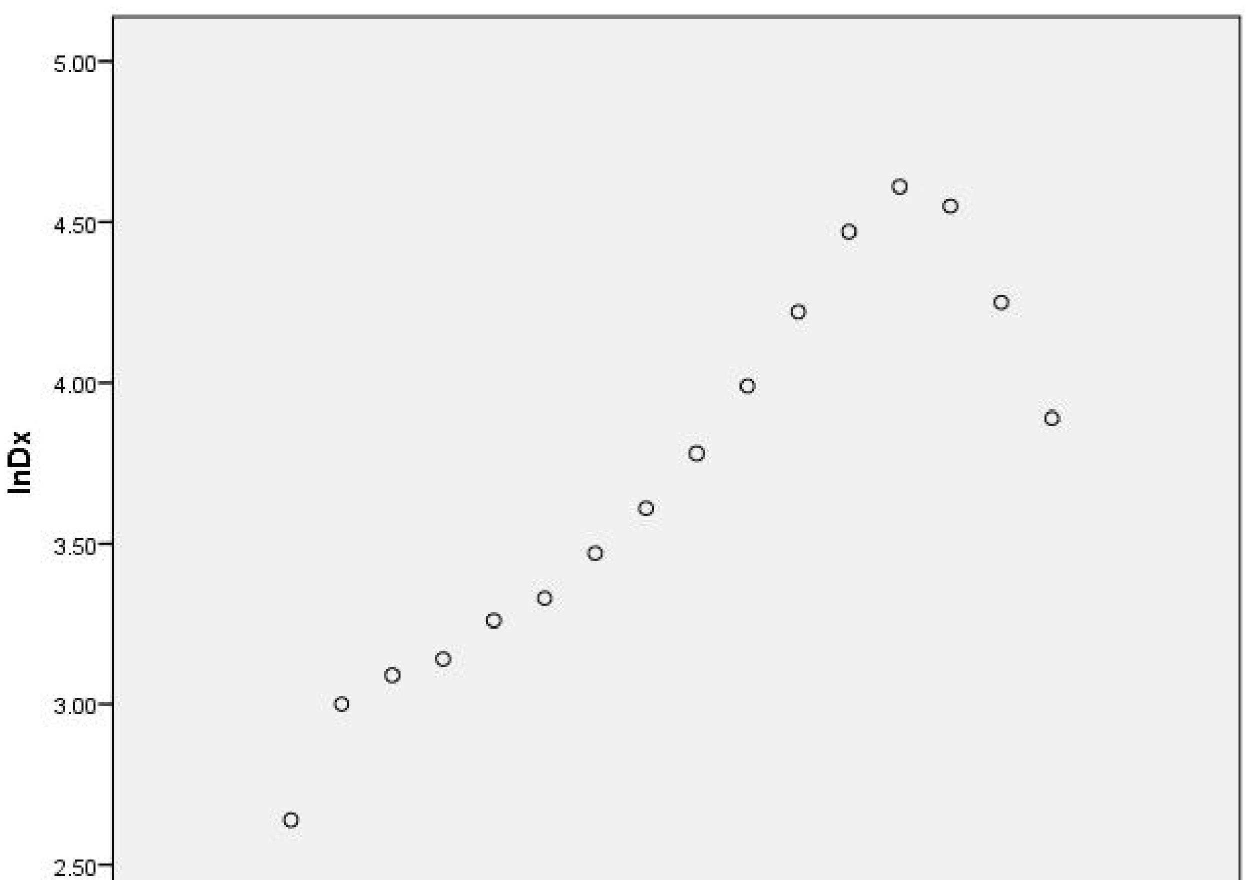
This is the table we use to construct our regression equation which is

$$\mathbf{lnDx = 2.557 + 0.023Age}$$

Age is the independent variable

lnDx is the dependent variable

Scattered plot



Conclusion

Having established that it is a strong positive correlation, we can see that from the above graph that the plots are moving from the bottom left to the top right. Which shows a positive correlation. We can therefore conclude that as the age of people increases, the probability of people dying also increases.

Test of the goodness of the graduation

	Observed N	Expected N	Residual
2.64	3	3.7	-.7
3.00	3	3.7	-.7
3.09	3	3.7	-.7
3.14	3	3.7	-.7
3.26	3	3.7	-.7
3.33	3	3.7	-.7
3.47	3	3.7	-.7
3.61	4	3.7	.3
3.78	4	3.7	.3
3.89	4	3.7	.3
3.99	4	3.7	.3
4.22	4	3.7	.3
4.25	4	3.7	.3
4.47	4	3.7	.3
4.55	5	3.7	1.3
4.61	5	3.7	1.3

Total	59		

Test Statistics

Graduation

Chi-Square	2.017[a]
df	15
Asymp. Sig.	1.000

a. 16 cells (100.0%) have expected frequencies less than 5. The minimum expected cell frequency is 3.7.

We conclude here that we accept the null hypothesis and conclude that there is a relation between graduation and the ages of the people.

Summary

In this chapter, we illustrated the application of difference equation on graduation of mortality rate, test the goodness of the graduation and also used a computer software to analyze our given data.

CHAPTER FIVE

SUMMARY AND CONCLUSIONS

5.1 SUMMARY

Graduation is the process whereby smooth mortality rates are created from crude mortality rate. Mortality is the condition of one having to die or the rate of failure or loss.

The parts of mortality rate include

- The number of deaths
- The population size in which these deaths were counted
- The time period which these death occurred

The graduation of mortality data aims to estimate death rates by carrying out a smoothing of the crude rates obtained directly from the original data. The main difference with regards to parametric models is that the assumption of an age-dependent function is unnecessary, which is advantageous when the information behind the model is unknown, as one cause of error is often the choice of an inappropriate model.

There are several different mortality rates used to monitor the level of mortality in populations. The following are most commonly used in today's world

- Crude mortality rate

- Age-specific mortality rate
- Infant mortality rate
- Maternal mortality rate
- Under-5 mortality rate

Difference equations relates to differential equations as discrete mathematics relates to continuous mathematics. Difference equation is the mathematical equality involving the differences between successive values of a function of a discrete variable.

A difference equation is a very useful tool in describing and calculating the output of the system described by the formula for a given sample n. The property of a difference equation is its ability to easily help find the transform H(z) of a system.

Difference equation in math problem is the result of subtracting one number from another. How much one number differs from another. Example: The difference between 8 and 5 is 3.

5.1 CONCLUSION

In conclusion, the study was able to show that we can find graduation of mortality rate, using difference equation. The use of illustrative examples and the analysis carried out made it possible for us to see that difference equation can be used to find graduation of mortality rate hence determining how the age of people will affect or tell when they will die.

RECOMMENDATION

1. It should be brought to the general public that the probability of people dying, is mostly dependent on how long they have lived.

2. When death occur rapidly on young people, it should be seen as abnormal and hence the cause and the cure should be provided urgently so as to make the probability of people dying correlate with how long they have lived. To prevent the occurrence of what is known as untimely death.

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monotone iterative method was given for semi-linear partial differential equation of elliptic type. The modified central finite difference scheme is introduce. Numerical solutions agree with solutions obtained by adaptive finite element method.