

**DETERMINATION OF THE PROBABILITY OF THE
OSCILLATION OF ELECTRON NEUTRINO TO MUON
NEUTRINO.**

BY

IMARHIA BODON KISMET

PSC1707905

**A PROJECT SUBMITTED TO THE DEPARTMENT OF
PHYSICS, FACULTY OF PHYSICAL SCIENCES, IN PARTIAL
FULFILMENT OF THE REQUIREMENTS FOR THE AWARD OF
THE DEGREE OF BACHELOR OF SCIENCE (B.SC) OF THE
UNIVERSITY OF BENIN, BENIN CITY, EDO STATE, NIGERIA.**

SUPERVISOR: PROF. AIYOHUYIN E.O.

JANUARY 2023

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CERTIFICATION

This is to certify that this project work was carried out by Imarhia Bodon Kismet in the Department of Physics, University of Benin city, Edo state Nigeria under my supervision.

PROF. AIYOHUYIN E.O.
Project Supervisor

DATE

PROF. OSAHON O.D.
Head of Department

DATE

External Examiner

DATE

DEDICATION

This research work is dedicated to God Almighty for his infinite mercy and grace bestowed upon me throughout this study period.

CERTIFICATION OF DISSERTATION ON PLAGIARISM

We the undersigned attest and declare that the dissertation of **IMARHIA BODON KISMET** on the DETERMINATION OF THE PROBABILITY OF THE OSCILLATION OF ELECTRON NEUTRINO TO MUON NEUTRINO has successfully passed the anti-plagiarism test and does not violate any copyright regulations.

PROF. AIYOHUYIN E.O.

Date

Project Supervisor

PROF. OSAHON O.D.

Date

Head of Department

ACKNOWLEDGEMENT

I would want to express my gratitude to the Almighty God for his assistance and protection throughout the entirety of my study. Because of my supervisor, Professor Aiyohuyin E.O.'s patience and contribution during the duration of this project, this study work would not have been able to be finished. I am grateful to him for both of these qualities.

My unconditional gratitude goes out to my kind and great parents, Mr. and Mrs. Imarhia-Igharo for the assistance they have given me financially, spiritually, and morally.

I also want to thank all of my siblings, Mrs. Ebilenjor Atole, Miss. Dumebi Blossom, Miss. Ewere Dawn and my brother in-law Mr. Ehikioya Atole for the support, care, and encouragement they have given me, and to my lovely nieces, Wealth and Jedidiah for bringing joy and happiness into my life.

In addition, I would want to express my gratitude to all of my close friends and classmates cum family; Pst. Kelvin, Comr. Martin, Jerry, Jeffery, Bisi, Temi and Comr. David Dosu, who have contributed to and assisted me during the process of carrying out and completing my research work, may the omnipotent God shower his blessings upon you Amen.

ABSTRACT

This research is concerned with neutrino physics in general, and specifically with neutrino mixing and oscillations. Neutrinos are massless in the standard model of particle physics, so they do not mix or oscillate. However, many experimental results now appear to support neutrino oscillations, necessitating the extension of the standard model to include neutrino masses and mixing among different neutrino flavors. In calculating the probability of electron neutrino oscillating into muon neutrino, the model equation below was used:

$$P(\nu_e \rightarrow \nu_\mu) = \sin^2(2\theta_{13})\sin^2(\theta_{23})\sin^2\left(1.27\Delta m_{23}^2(eV^2)\frac{L(km)}{E(GeV)}\right)$$

The probability was calculated using the model equation by taking arbitrary values for L/E ranging from 100-10000, using the Microsoft excel spreadsheet. The value of the probability varies between 0.093-0.24 and a graph of probability, P against ranging values of L/E was also plotted using the Microsoft excel spreadsheet.

The probability of a neutrino changing type is related to the distance travelled by the neutrino from its point of production to its point of detection. As a general rule, neutrinos travelling greater distances will exhibit greater depletion from oscillation. Moreover, the oscillation varies smoothly over increasing distance.

Keywords: Neutrino oscillation, neutrino mixing.

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CHAPTER ONE

INTRODUCTION

NEUTRINOS

Neutrino is a fermion that interacts only through weak interaction and gravity (Close & Frank, 2010). The neutrino is so named because it is electrically neutral and because its rest mass is so small that it was long thought to be zero. The rest mass of the neutrino is much smaller than that of the other known elementary particles excluding massless particles (Mertens & Susanne, 2016).

The weak force has a very short range, the gravitational interaction is extremely weak, and neutrinos do not participate in the strong interaction. Thus, neutrinos typically pass through normal matter unimpeded and undetected. Neutrino oscillation is a quantum mechanical phenomenon in which a neutrino created at the source with a specific flavor, after travelling a distance to a detector can be measured to be of a different flavor (Barger et al 2012).

Neutrinos are produced by numerous types of radioactive decay, including the beta decay of atomic nuclei and spontaneous nuclear processes in stars.

- An intentionally generated nuclear chain reaction in atomic bombs.
- During supernova explosions.
- During the spin-down of the neutron star

The sun's nuclear processes are the primary source of the Earth's neutrinos. About 65 billion (6.5×10^{10}) solar neutrinos per square centimeter per second are emitted from the Sun and strike Earth's surface. The Earth's interior can be imaged via tomography created with neutrinos.

Neutrinos are unique in that they possess extraordinary qualities within themselves. Neutrinos have masses that are many orders of magnitude smaller than those of their relatives (electron, muon, tau).

Neutrino oscillations, which involve periodic transitions between different types of neutrinos in vacuum or matter, are possible due to the negligible masses of neutrinos.

Neutrinos have a special purpose in both particle physics and astronomy. Their extraordinary penetrating power enables us to investigate the sun's nuclear structure, the hidden core where solar energy is generated.

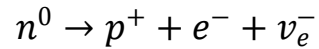
There are four possible origins for neutrinos: the sun, the atmosphere, a nuclear reactor, or a particle accelerator. The existence of flavor change in each of these neutrinos has been confirmed by a number of experiments over the past few decades.

HISTORY

In 1930, Wolfgang Pauli proposed the neutrino to explain how beta decay could save energy, momentum, and angular momentum (spin). Unlike Niels Bohr, who proposed a statistical interpretation of conservation laws to explain the observed continuous energy spectrum in beta decay, Pauli proposed an unobserved particle called a "neutron," which has the same - ending as the proton and the electron. He believed that during beta decay, the new particle, which had a mass similar to the electron, was emitted from the nucleus along with the electron or beta particle.

James Chadwick found a somewhat heavier neutral nuclear particle in 1932 and termed it a neutron, thus now there are two sorts of particles with the same name. The term "neutrino" was created by Enrico Fermi and first used by him at a conference in Paris in July 1932 and then again at the Solvay Conference in October 1933, where it was also used by Pauli. The name "little neutral one" (the Italian translation of "small neutron") was jocularly coined by Edoardo Amaldi during a conversation with Fermi at the Institute of Physics via Panisperna in Rome.

In Fermi's theory of beta decay, Chadwick's large neutral particle could decay into a proton, electron, and the smaller neutral particle (antineutrino)

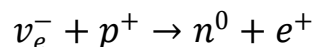


The theoretical groundwork for subsequent experimental research was laid by Fermi's 1934 paper, which integrated Pauli's neutrino, Paul Dirac's positron, and Werner Heisenberg's neutron-proton model. Fermi's work was deemed "too distant from reality" and hence rejected by Nature. Although the material he submitted was accepted by an Italian publication, Einstein ultimately decided to focus on experimental physics rather than his then-unpopular theory.

Measurements of beta particle (electron) energies published at the Solvay conference in 1934 disproved Bohr's hypothesis that energy conservation does not apply to beta decay. A statistically insignificant quantity of energy may become available in at most a handful of decays if the principle of conservation of energy is faulty, hence such a restriction appears implausible. The beta decay spectrum was first seen in 1934; its natural interpretation was that a new particle was occasionally consuming a variable fraction of the available energy, leaving the beta particle with the remainder. Pauli seized the moment to declare publicly that the hypothetical "neutrino" must exist even if it has not yet been discovered. The existence of neutrinos was originally demonstrated in 1938 with the simultaneous detection of electron and nucleus recoil in a cloud chamber.

In 1942, Wang Ganchang proposed employing beta capture to detect neutrinos in an experiment. Almost forty years after their discovery, in 1995, Clyde Cowan, Frederick Reines, Francis B. "Kiko" Harrison, Herald W. Kruse, and Austin D. McGuire were awarded the Nobel Prize for their detection of the neutrino in the July 20, 1956 issue of Science.

In this experiment, now known as the Cowan–Reines neutrino experiment, antineutrinos created in a nuclear reactor by beta decay reacted with protons to produce neutrons and positrons:



The positron quickly locates an electron and annihilates the two. The two gamma rays produced can be seen. The neutron can be detected by capturing it on a suitable gamma-ray emitting nucleus. Because both positron annihilation and neutron capture occur at the same time, the antineutrino interaction has a distinct signature.

In February 1965, Jacques Pierre Friederich (Friedel) Sellschop was part of a team that discovered the first neutrino in nature. The experiment took place in a specially built chamber at a depth of 3 kilometers in the East Rand ("ERPM") gold mine in Boksburg, South Africa. A plaque in the main structure commemorates the discovery. The experiments also investigated neutrino physics and weak interactions, as well as performing rudimentary neutrino astronomy.

PROPERTIES

Electric Charge

As a result of being exposed to an electromagnetic field, matter will take on an electric charge and feel a force. There can be either a positive or negative electric charge (commonly carried by protons and electrons respectively). Polar charges are attracted to each other, while like charges are repelled. Neutral objects are ones that do not possess any net charge. This early knowledge of the interactions between charged substances, known as classical electrodynamics, holds valid for problems that do not include quantum processes.

As a conserved property, a system's net charge, defined as the sum of its positive and negative charges, remains constant over time. There is a subatomic particle that carries an electric charge. Ordinary matter carries a negative charge in the form of electrons, while atom nuclei carry a positive charge in the form of protons. If an object has more electrons than protons, it will be negatively charged; if the opposite is true, it will be positively charged; and if the two numbers are about the same, the object will be neutrally charged. The elementary charge e , which is equal to about 1.602×10^{-19} coulombs, is the smallest charge that can exist freely (smaller charges, such as multiples of $\frac{1}{3}e$, can be found in particles called quarks, but they are only found in combination, and they always combine to form particles with integer charge). The electron has a negative charge of $-e$ while the proton has a positive charge of $+e$.

In the fundamental particle physics, neutrinos have no charge.

Spin

A significant degree of freedom exists in the direction of spin, which is an essential feature of elementary particles. At subatomic scales, it's commonly depicted as an item rotating around its own axis (hence the name "spin"), but this is a little misleading because basic particles are thought to be point-like.

When indicating spin, a vector whose length is expressed in terms of the reduced Planck constant (\hbar) is commonly utilized.

Neutrinos are categorized as fermions due to their half-integer ($\frac{1}{2}$) spin.

Weak Interaction

Along with electromagnetism, the strong interaction, and gravity, the weak interaction (also known as the weak force or weak nuclear force) is considered to be one of the four fundamental interactions. Atoms decay radioactively due to the weak contact between subatomic particles, which is used in both nuclear fission and nuclear fusion.

A neutrino of one flavor can only transform into a neutrino of a different flavor through the weak interaction.

Strong Interaction

The other three fundamental interactions are electromagnetism, the weak interaction (weak nuclear force), and gravitation. At a distance of 10^{-15} m, the strong force is approximately 137 times stronger than electromagnetism, 106 times stronger than the weak interaction, and 10^{38} times stronger than gravitation (little more than the radius of a nucleon).

Neutrinos do not interact strongly. As a result, neutrinos typically pass through normal matter unimpeded and undetected.

Mass

Compared to their nuclear family members, neutrino masses are extremely small (electron, muon, tau). Neutrino oscillations, which involve periodic transitions between different types of neutrinos in vacuum or matter, are possible due to the negligibly small masses of neutrinos.

Yet another way to think about neutrinos is in terms of their mass, or "mass state." Although ν_1 , ν_2 , and ν_3 are also acceptable nomenclature for these neutrinos, the names mass m_1 , mass m_2 , and mass m_3 have gained wider acceptance.

At first glance, it would seem that each of the three neutrino masses corresponds to a different flavor, but in reality, the mass state of a neutrino does not correspond exactly to the flavor state of a neutrino.

Neutrinos come in three different masses: 1, 2, and 3, and each mass contributes to the probability that a neutrino will interact with a detector and produce a specific charged particle of a given flavor, such as the electron neutrino (electron, muon, or tau).

When it comes to neutrino masses, scientists know a little bit but still have a lot of questions. They've calculated that the combined masses of the three neutrinos are quite tiny. They also have a basic understanding of how the flavor mixture of each mass neutrino is separated out. Mass m_1 is predominantly composed of electron, mass m_2 of a roughly equal mix of electron, muon, and tau, and mass m_3 of primarily muon and tau. Also, ν_1 and ν_2 have comparable masses, while ν_3 is either extremely heavy or extremely light.

Before recently, it was believed that all three types of neutrinos were massless. However, the discovery of neutrino oscillations, a phenomenon in which neutrinos change type in flight, has revealed that not only do neutrinos have mass, but that the masses of the three types (m_1 , m_2 , and m_3) are distinct.

The squared difference in mass between neutrino types m_1 and m_2 and $m_{12}-m_{22}$ has been measured by observing solar neutrino oscillations, while the squared difference in mass between m_1 and m_3 has been measured by observing Earth-produced neutrino oscillations.

Since oscillation experiments can only measure the squared difference of the masses, we still don't know what m_1 , m_2 , and m_3 are in absolute terms or whether m_2 is heavier than m_3 .

This latter issue is commonly referred to as the "neutrino mass hierarchy problem." It is said that the hierarchy is "normal" if m_2 is less massive than m_3 , and "inverted" if the reverse is true.

NEUTRINO FLAVORS

Weak interactions create neutrinos in one of three leptonic flavors: electron neutrinos (ν_e), muon neutrinos (ν_μ), or tau neutrinos (ν_τ), associated with the corresponding charged leptons, the electron (e^-), muon (μ^-), and tau (τ^-), respectively.

Electron Neutrino

The electron neutrino (ν_e) is an elementary particle which has zero electric charge and a spin of $\frac{1}{2}$. Together with the electron, it forms the first generation of leptons, hence the name electron neutrino. It was first hypothesized by Wolfgang Pauli in 1930, to account for missing momentum and missing energy in beta decay, and was discovered in 1956 by a team led by Clyde Cowan and Frederick Reines.

Muon Neutrino

The muon neutrino is a lepton, an elementary subatomic particle which has the symbol (ν_μ) and no electric charge. Together with the muon, it forms the second generation of leptons, hence the name muon neutrino. It was discovered in 1962 by Leon Lederman, Melvin Schwartz and Jack Steinberger. The discovery was rewarded with the 1988 Nobel Prize in Physics.

The muon neutrino or "neutretto" was hypothesized to exist by a number of physicists in the 1940s. The first paper on it may be Shoichi Sakata and Takesi Inoue's two-meson theory of 1942, which also involved two neutrinos.

Tau Neutrino

The tau neutrino is a subatomic elementary particle which has the symbol ν_τ , and no net electric charge. Together with the tau (τ), it forms the third generation of leptons, hence the name tau neutrino. Its existence was immediately implied after the tau particle was detected in a series of experiments between 1974 and 1977 by Martin Lewis Perl with his colleagues at the SLAC–LBL group. The discovery of the tau neutrino was announced in July 2000 by the DONUT collaboration (Direct Observation of the Nu Tau). The tau neutrino is the last of the leptons and is the second most recent particle of the Standard Model to be discovered. Its mass is very small but not zero.

NEUTRINO OSCILLATION

Neutrino oscillations arise from a quantum mechanical phenomenon associated with the fact that the neutrinos have mass. For the three neutrinos species that we know to exist, the principle of superposition allows "flavor" states, namely neutrinos that interact to produce electrons, muons, or taus, to be (orthogonal) combinations of three neutrino states with definite mass. In other words, when a weak interaction produces a flavor state, such as a muon neutrino, that state is a mixture of states with different mass. These states evolve at different rates so that a later time, the state may acquire some component of a new flavor state, such that if it interacts, it may do so as a flavor state different from its original flavor.

This possibility of flavor change, namely that a neutrino is created in one flavor and interacts sometime later as another, is the primary manifestation of neutrino oscillations.

STATEMENT OF THE PROBLEM

The observations in the 1990s from the Super-Kamiokande; an enormous underground detector in Japan, finds neutrinos apparently “disappearing”. Since it is unlikely that momentum and energy are actually vanishing from the universe, a more plausible explanation is that the types of neutrinos we can detect are changing into types we cannot detect. This phenomenon is known as neutrino oscillation.

The probability of a neutrino changing type is related to the distance travelled by the neutrino from its point of production to its point of detection. The reason neutrino oscillation is relevant to the question of neutrino mass, is that massless neutrinos cannot oscillate. The implication of the existence of neutrino oscillation is that neutrinos have finite masses and mixings, which are not accounted for in the framework of the standard model of elementary particles.

AIM AND OBJECTIVES

- **AIM**

TO DETERMINE THE PROBABILITY OF OSCILLATION FROM ELECTRON NEUTRINO INTO MUON NEUTRINO

- **OBJECTIVES**

- I. Determine the values of the mixing angles θ_{13} and θ_{23}
- II. Determine the value of the distance per energy $\frac{L}{E}$
- III. Determine the value of the mass squared difference Δm^2_{23}
- IV. Use the model equation to calculate the probability of the oscillation.
- V. Plot the graph of probability(P) against distance per energy(L/E).
- VI. Determine the energy of the oscillation.

CHAPTER TWO

METHODOLOGY

To calculate the probability of electron neutrino oscillating into muon neutrino, the model equation below was used:

$$P(\nu_e \rightarrow \nu_\mu) = \sin^2(2\theta_{13})\sin^2(\theta_{23})\sin^2\left(1.27\Delta m_{23}^2(eV^2)\frac{L(km)}{E(GeV)}\right)$$

- **The angle θ :** This is the so-called mixing angle. It defines how different the flavor states are from the mass states. If $\theta = 0$, the flavor states are identical to the mass states (that is, the ν_e will propagate from source to detector as a ν_e with definite momentum.

Clearly in this case, oscillations cannot happen. If $\theta = \frac{\pi}{4}$ then the oscillations are said to be maximal and at some 1 point along the path between source and detector all of the ν_e we started with will oscillate to ν_μ .

- **The mass squared difference, Δm^2 :** If there are two flavours there will be two mass states. This parameter is the difference in squared masses of each of these states: $\Delta m^2 = m_1^2 - m_2^2$. For neutrino oscillations to occur, at least one of the mass states must be non-zero. This simple statement has huge implications for oscillations to happen, the neutrino must have mass. Also, the masses of the mass states must be different, else $\Delta m^2 = 0$ and $P(\nu_e \rightarrow \nu_\mu) = 0$.

You can see why this is: the masses control the relative phase of the two mass wavefunctions.

If they are the same, then the mass states will never get out of phase and you will measure the same linear combination of mass states at the detector as you generated at the source.

L/E: This is the parameter we, as experimentalists control. L is the distance between the source and the detector, and E is the energy of the neutrino. For a given Δm^2 , the probability of oscillation will change as one moves away from the detector, or scans over different neutrino energy. Experimentally, if we suspect that Δm^2 has a particular value, then we should build our experiment to be maximally sensitive to the oscillation probability. That is, we want to build it such that $1.27\Delta m^2 \frac{L}{E} = \frac{\pi}{2}$ or $\frac{L}{E} = \frac{\pi}{2.54\Delta m^2}$. We are free then to either change the beam energy, or the baseline (L), or both. Ideally, we want to maximize L and minimize E. This all sounds very nice, but practicalities tend to intrude. Neutrino beams diverge like an electric field from a point source, so the surface area of a detector placed at a distance L has to grow by L^2 , and so does the cost. At the same time, the neutrino cross-section decreases as the neutrino energy decreases and so the running time to collect a useful number of events increases linearly (and so does the cost). On the flip-side, if $\frac{L}{E}$ is fixed for us by nature (as it is, for example, in solar neutrinos), then we can only probe a certain range of $(\Delta m^2, \theta)$ combinations, since other choices for the values of these parameters will yield too small a probability of oscillation for observation to be feasible (we may have to wait decades to get enough events).

There are two types of neutrino oscillation experiments one could think of doing. The first is to start with a pure beam of known flavor, say; ν_x , and look to see how many have disappeared. This is a “disappearance” experiment and measures the survival. The second type of experiment is an “appearance” experiment, in which one starts with a pure beam of known flavor, say; ν_x and looks to see how many neutrinos of a different flavor, say; ν_y are detected.

3-NEUTRINOS OSCILLATION PROBABILITIES

For three neutrino oscillation, the mixing matrix can be (i) 3×3 , (ii) complex and unitary. In this case we have:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \quad (1)$$

This is known as the Pontecorvo-Maka-Nakagawa-Sakata (PMNS) matrix and does the same job in the neutrino sector as the CKM matrix does in the quark sector.

The fact that the matrix is unitary means that

$$U^\dagger U = I \rightarrow U^\dagger = U^{-1} = (U^*)^T \quad (2)$$

And from this

$$\begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} = \begin{pmatrix} U^*_{e1} & U^*_{\mu1} & U^*_{\tau1} \\ U^*_{e2} & U^*_{\mu2} & U^*_{\tau2} \\ U^*_{e3} & U^*_{\mu3} & U^*_{\tau3} \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} \quad (3)$$

The unitarity of the PMNS matrix gives several useful relations:

$$\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix} \begin{pmatrix} U_{e1}^* & U_{\mu1}^* & U_{\tau1}^* \\ U_{e2}^* & U_{\mu2}^* & U_{\tau2}^* \\ U_{e3}^* & U_{\mu3}^* & U_{\tau3}^* \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (4)$$

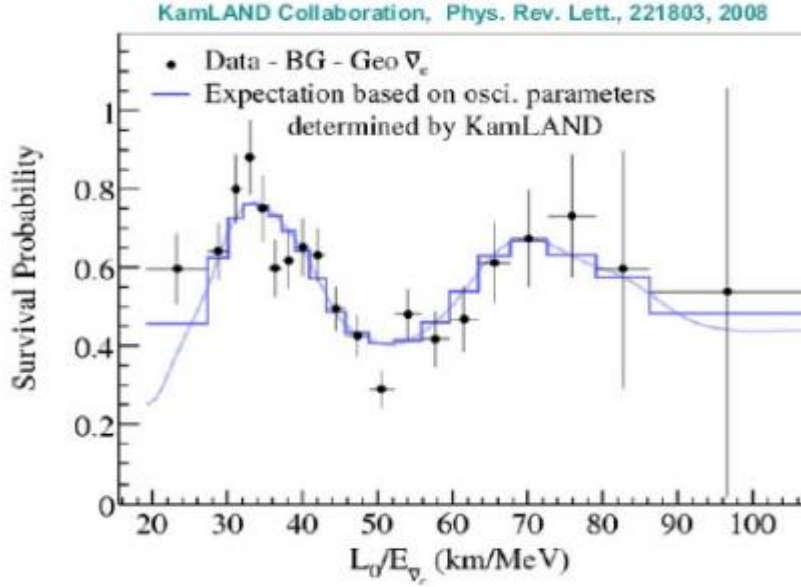


Fig. 1 Ratio of observed to expected antineutrino rates as a function of the average L/E at KamLAND

So

$$U_{e1}U_{e1}^* + U_{\mu1}U_{\mu1}^* + U_{\tau1}U_{\tau1}^* = 1 \quad (4a)$$

$$U_{e2}U_{e2}^* + U_{\mu2}U_{\mu2}^* + U_{\tau2}U_{\tau2}^* = 1 \quad (4b)$$

$$U_{e3}U_{e3}^* + U_{\mu3}U_{\mu3}^* + U_{\tau3}U_{\tau3}^* = 1 \quad (4c)$$

$$U_{e1}U_{e1}^* + U_{e2}U_{\mu2}^* + U_eU_{\mu3}^* = 0 \quad (4d)$$

$$U_{e1}U_{\tau1}^* + U_{e2}U_{\tau2}^* + U_{e3}U_{\tau3}^* = 0 \quad (4e)$$

$$U_{\mu1}U_{\tau1}^* + U_{\mu2}U_{\tau2}^* + U_{\mu3}U_{\tau3}^* = 0 \quad (4f)$$

The oscillation probability is calculated just before. Assume that a time $t=0$, we create a neutrino in a pure $|v_a\rangle$ state.

$$|\psi(x=0)\rangle = U_{a1}|v_1\rangle + U_{a2}|v_2\rangle + U_{a3}|v_3\rangle \quad (5)$$

The wavefunction evolves as

$$|\psi(t)\rangle = U_{a1}|v_1\rangle e^{-ip_1x} + U_{a2}|v_2\rangle e^{-ip_2x} + U_{a3}|v_3\rangle e^{-ip_3x} \quad (6a)$$

Where $pi.x = E_i - p_i.x$. After travelling a distance L the wavefunction is (assuming that the neutrino is relativistic)

$$|\psi(L)\rangle = U_{a1}|v_1\rangle e^{-i\theta_1} + U_{a2}|v_2\rangle e^{-i\theta_2} + U_{a3}|v_3\rangle e^{-i\theta_3} \quad (6b)$$

With $\phi_i = pi.x = E_it - |p_i|L \approx (E_i - |p_i|)L$. As before, we can approximate E_i by

$$E_i \approx p_i + \frac{m_i^2}{2E_i} \quad (7a)$$

So

$$\phi_i = (E_i - |p_i|)L \approx \frac{m_i^2}{2E_i}L \quad (7b)$$

Expressing the mass eigenstates in terms of the flavor eigenstates:

$$|\psi(L)\rangle \geq U_{a1}e^{-i\phi_1}(U_{ei}^*|v_e\rangle + U_{\mu1}^*|v_\mu\rangle + U_{\tau1}^*|v_\tau\rangle) + U_{a2}e^{-i\phi_2}(U_{e2}^*|v_e\rangle + U_{\mu2}^*|v_\mu\rangle + U_{\tau2}^*|v_\tau\rangle) + U_{a3}e^{-i\phi_3}(U_{e3}^*|v_e\rangle + U_{\mu3}^*|v_\mu\rangle + U_{\tau3}^*|v_\tau\rangle) \quad (8)$$

Which can be arranged to give

$$|\psi(L)\rangle = (U_{a1}e^{-i\phi_1} + U_{a2}e^{-i\phi_2} + U_{a3}e^{-i\phi_3})|v_e\rangle + (U_{a1}U_{\mu1}^*e^{-i\phi_1} + U_{a2}U_{\mu2}^*e^{-i\phi_2} + U_{a3}U_{\mu3}^*e^{-i\phi_3})|v_\mu\rangle + (U_{a1}U_{\tau1}^*e^{-i\phi_1} + U_{a2}U_{\tau2}^*e^{-i\phi_2} + U_{a3}U_{\tau3}^*e^{-i\phi_3})|v_\tau\rangle \quad (9)$$

From which we can get the oscillation probability $P(v_\alpha \rightarrow v_\beta)$:

$$P(v_\alpha \rightarrow v_\beta) = |\langle v_\beta | \psi(L) \rangle|^2 = (U_{\alpha1}U_{\beta1}^*e^{-i\phi_1} + U_{\alpha2}U_{\beta2}^*e^{-i\phi_2} + U_{\alpha3}U_{\beta3}^*e^{-i\phi_3})^2 \quad (10a)$$

The terms in this expression come from the diagrams in Figure. The mathematics do not really concern us. Using the complex relationship

$$|z_1 + z_2 + z_3|^2 = |z_1|^2 + |z_2|^2 + |z_3|^2 + 2\Re(z_1z_2^* + z_1z_3^* + z_2z_3^*) \quad (10b)$$

We can write Equation as (eventually)

$$P(v_\alpha \rightarrow v_\beta) = \delta_{\alpha\beta} - 4 \sum_{i>j} \Re(U_{\alpha i}^*U_{\beta i}U_{\alpha j}U_{\beta j}^*) \sin^2\left(\Delta m_{ij}^2 \frac{L}{4E}\right) + 2 \sum_{i>j} \Im(U_{\alpha i}^*U_{\beta i}U_{\alpha j}U_{\beta j}^*) \sin^2\left(\Delta m_{ij}^2 \frac{L}{4E}\right) \quad (11)$$

The PMNS matrix is usually expressed by 3 rotation matrices and complex phase:

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_{23} & \sin \theta_{23} \\ 0 & -\sin \theta_{23} & \cos \theta_{23} \end{pmatrix} \begin{pmatrix} \cos \theta_{13} & 0 & \sin \theta_{13} e^{-i\delta_{CP}} \\ 0 & 1 & 0 \\ -\sin \theta_{13} e^{i\delta_{CP}} & 0 & \cos \theta_{13} \end{pmatrix} \begin{pmatrix} \cos \theta_{12} & \sin \theta_{12} & 0 \\ -\sin \theta_{12} & \cos \theta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Which when multiplied out (and purely for reference for later equations) is

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta_{CP}} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}s_{13}s_{23}e^{i\delta_{CP}} & c_{12}c_{23} \end{pmatrix} \quad (12)$$

Where $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$. The first matrix is called the “12-sector”. The second matrix is the “13-sector”, and the third is the “23-sector”.

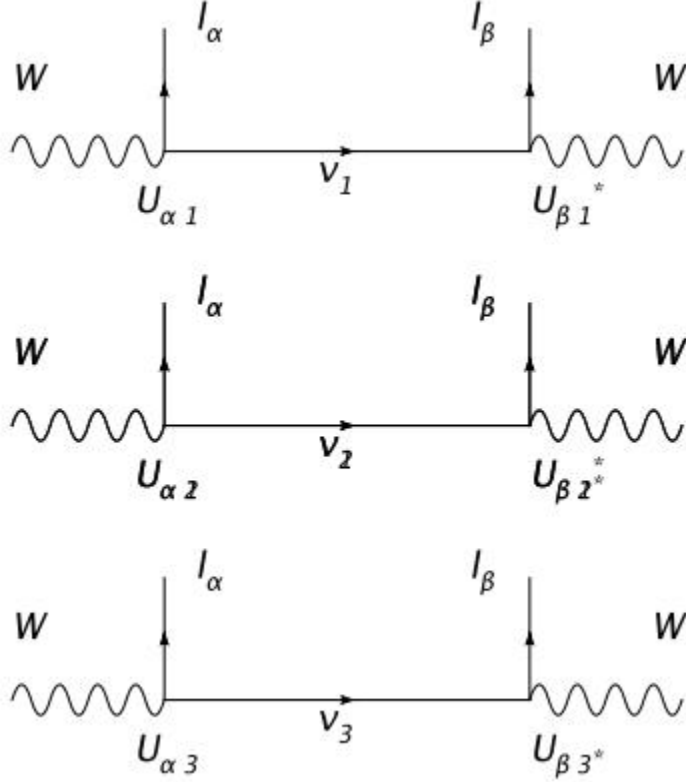


Fig 2. The diagrams for each term in three neutrino oscillation probability

The second matrix is responsible, possibly for CP-violation. We’ll talk about that below. For now, let’s set δ_{CP} to zero. If this is the case, then the imaginary term in Equation vanishes and we are left with

$$P(v_\alpha \rightarrow v_\beta) = \delta_{\alpha\beta} - 4 \sum_{i>j} (U_{\alpha i}U_{\beta i}U_{\alpha j}U_{\beta j} \sin^2) \left(\Delta m_{ij}^2 \frac{L}{4E} \right) \quad (13)$$

We also must remember the squared mass difference. There are 3 neutrino mass eigenstates and hence 2 independent mass splitting, called Δm_{23}^2 and Δm_{12}^2 . The other splitting is defined by the relationship

$$\Delta m_{12}^2 + \Delta m_{23}^2 + \Delta m_{31}^2 = 0 \quad (14)$$

We know from solar and atmospheric neutrino problems that splitting have values around $+8 \times 10^{-5} eV^2$ and $3 \times 10^{-3} eV^2$. To go ahead a bit, the mass splitting relating to 23 sector is the atmospheric neutrino mass difference: $\Delta m_{23}^2 = 3 \times 10^{-3} eV^2$ and the splitting relating to 12 sector is the solar mass difference: $\Delta m_{12}^2 = 8 \times 10^{-5} eV^2$

Let's consider an appearance e experiment. In this case $\alpha \neq \beta$. Then

$$\begin{aligned} P(v_{\alpha \rightarrow v_{\beta}}) &= -4 \sum_{i>j} (U_{\alpha i} U_{\beta i} U_{\alpha j} U_{\beta j}) \sin^2 \left(1.27 \Delta m_{ij}^2 \frac{L}{E} \right) \\ &= 4[(U_{\alpha 1} U_{\beta 1} U_{\alpha 2} U_{\alpha 2}) \sin^2 \left(1.27 \Delta m_{12}^2 \frac{L}{E} \right) \\ &\quad + (U_{\alpha 1} U_{\beta 1} U_{\alpha 3} U_{\alpha 3}) \sin^2 \left(1.27 \Delta m_{13}^2 \frac{L}{E} \right) \\ &\quad (U_{\alpha 2} U_{\beta 2} U_{\alpha 3} U_{\alpha 3}) \sin^2 \left(1.27 \Delta m_{23}^2 \frac{L}{E} \right)] \end{aligned} \quad (15a)$$

Equation can be split into two cases. The first occurs for experiment where L/E is small. In this case

$$\sin^2 \left(1.27 \Delta m_{23}^2 \frac{L}{E} \right) \rightarrow 0$$

And, with the reasonable approximation that $\Delta m_{13}^2 \approx \Delta m_{23}^2$.

$$P(v_{\alpha \rightarrow v_{\beta}}) = -4[U_{\alpha 1} U_{\beta 1} U_{\alpha 3} U_{\alpha 3} + U_{\alpha 2} U_{\beta 2} U_{\alpha 3} U_{\alpha 3}] \sin^2 \left(1.27 \Delta m_{23}^2 \frac{L}{E} \right) \quad (15b)$$

The other case occurs when L/E is large. Then the terms involving Δm_{23}^2 and Δm_{23}^2 are rapidly oscillating and average to 0.5

$$\sin^2\left(1.27\Delta m_{23}^2\frac{L}{E}\right) \rightarrow \langle \sin^2\left(1.27\Delta m_{23}^2\frac{L}{E}\right) \rangle = \frac{1}{2} \quad (16a)$$

$$\sin^2\left(1.27\Delta m_{23}^2\frac{L}{E}\right) \rightarrow \langle \sin^2\left(1.27\Delta m_{23}^2\frac{L}{E}\right) \rangle = \frac{1}{2} \quad (16b)$$

And

$$\begin{aligned} P(\nu_{\alpha} \rightarrow \nu_{\beta}) = & \\ & -4 \left[U_{\alpha 1} U_{\beta 1} U_{\alpha 2} U_{\beta 2} + \sin^2\left(1.27\Delta m_{23}^2\frac{L}{E}\right) \right. \\ & \left. + \frac{1}{2}(U_{\alpha 1} U_{\beta 1} U_{\alpha 3} U_{\beta 3} + U_{\alpha 2} U_{\beta 2} U_{\alpha 3} U_{\beta 3}) \right] \end{aligned} \quad (17)$$

Replacing the elements U_{ij} with the mixing angle terms from the PMNS matrix, and remembering that $\delta_{CP} = 0$, we can use the unitarity relations in Equation set to get (after a while) for the small L/E case

$$P(\nu_{\mu} \rightarrow \nu_{\tau}) = \cos^2(\theta_{13}) \sin^2(2\theta_{23}) \sin^2\left(1.27\Delta m_{23}^2\frac{L}{E}\right) \quad (18a)$$

$$P(\nu_{e} \rightarrow \nu_{\mu}) = \sin^2(\theta_{13}) \sin^2(2\theta_{23}) \sin^2\left(1.27\Delta m_{23}^2\frac{L}{E}\right) \quad (18b)$$

$$P(\nu_{e} \rightarrow \nu_{\tau}) = \sin^2(\theta_{13}) \cos^2(2\theta_{23}) \sin^2\left(1.27\Delta m_{23}^2\frac{L}{E}\right) \quad (18c)$$

Thus, from the model equation derived, the probability of oscillation from electron neutrino to muon neutrino is given by:

$$P(\nu_e \rightarrow \nu_{\mu}) = \sin^2(2\theta_{13}) \sin^2(\theta_{23}) \sin^2\left(1.27\Delta m_{23}^2(eV^2) \frac{L(km)}{E(GeV)}\right) \quad (18b)$$

From the model equation, the probability of the oscillation can be obtained if the variables; θ_{13} , θ_{23} , Δm_{23}^2 , $\frac{L}{E}$ can be determined.

Experimental works have been carried out extensively in this regard, the experimental data obtained were thereby used.

Parameter	Unit	(1)	(2)
$\sin^2 \theta_{12}$	-	$0.302^{+0.013}_{-0.012}$	$0.311^{+0.013}_{-0.013}$
θ_{12}	[°]	$33.36^{+0.81}_{-0.78}$	$33.87^{+0.82}_{-0.80}$
$\sin^2 \theta_{23}$	-	$0.413^{+0.037}_{-0.025} \oplus 0.594^{+0.021}_{-0.022}$	$0.416^{+0.036}_{-0.029} \oplus 0.600^{+0.019}_{-0.026}$
θ_{23}	[°]	$40.0^{+2.1}_{-0.15} \oplus 50.4^{+1.3}_{-1.3}$	$40.1^{+2.1}_{-1.6} \oplus 50.7^{+1.2}_{-1.5}$
$\sin^2 \theta_{13}$	-	$0.0227^{+0.0023}_{-0.0024}$	$0.0255^{+0.0024}_{-0.0024}$
θ_{13}	[°]	$8.66^{+0.44}_{-0.46}$	$9.2^{+0.41}_{-0.45}$
δ	[°]	300^{+66}_{-138}	298^{+59}_{-145}
Δm_{21}^2	[10^{-5}eV^2]	$7.50^{+0.18}_{-0.19}$	$7.51^{+0.21}_{-0.15}$
Δm_{31}^2 (NH)	[10^{-3}eV^2]	$2.473^{+0.070}_{-0.067}$	$2.489^{+0.055}_{-0.051}$
Δm_{32}^2 (IH)	[10^{-3}eV^2]	$-2.427^{+0.042}_{-0.065}$	$-2.468^{+0.073}_{-0.065}$

Using the experimental data table above and substituting for the values into our model equation, the probability of oscillation can be calculated with ranging values of L/E using the Microsoft excel spreadsheet and a graph of probability, P against ranging values of L/E plotted also using the Microsoft excel spreadsheet.

CHAPTER THREE

RESULTS AND DISCUSSION

The following values obtained from literature were used:

- $\theta_{13} = 8.52^\circ$,
- $\theta_{23} = 48.7^\circ$
- $\Delta m^2_{13} = 2.515 \times 10^{-3} \text{eV}^2$

where; θ_{13} and θ_{23} are the respective mixing angles

Δm^2_{13} is the mass difference

The following results were obtained using the Microsoft excel spreadsheet.

L/E	P($\nu_e \rightarrow \nu_\mu$)
100	0.09308
150	0.200618
200	0.33561
250	0.4844
300	0.631939
350	0.7633
400	0.865198
450	0.927324
500	0.943393
550	0.911781
600	0.835684
650	0.722802
700	0.584551
750	0.434918
800	0.289038
850	0.161667
900	0.065691
950	0.010816
1000	0.002595
1050	0.041859
1100	0.124636

1150	0.242553
1200	0.383682
1250	0.533746
1300	0.677567
1350	0.800596
1400	0.890388
1450	0.937859
1500	0.938209
1550	0.891402
1600	0.802172
1650	0.679545
1700	0.535927
1750	0.385844
1800	0.244478
1850	0.12613
1900	0.04277
1950	0.002831
2000	0.010353
2050	0.064575
2100	0.160013
2150	0.287012
2200	0.432725
2250	0.582414
2300	0.720935
2350	0.834278
2400	0.910976
2450	0.943272
2500	0.927899
2550	0.866411
2600	0.765028
2650	0.634007
2700	0.486599
2750	0.337717
2800	0.202421
2850	0.094396
2900	0.024569
2950	5.13E-06
3000	0.023188
3050	0.091772

3100	0.198821
3150	0.333505
3200	0.482201
3250	0.629867
3300	0.761566
3350	0.863976
3400	0.926738
3450	0.943503
3500	0.912575
3550	0.837083
3600	0.724663
3650	0.586687
3700	0.437112
3750	0.291068
3800	0.163328
3850	0.066815
3900	0.01129
3950	0.00237
4000	0.040958
4050	0.123151
4100	0.240633
4150	0.381521
4200	0.531564
4250	0.675584
4300	0.799013
4350	0.889364
4400	0.937499
4450	0.938549
4500	0.892407
4550	0.803741
4600	0.681519
4650	0.538106
4700	0.388008
4750	0.246408
4800	0.12763
4850	0.043689
4900	0.003077
4950	0.0099
5000	0.063469

5050	0.158365
5100	0.28499
5150	0.430533
5200	0.580273
5250	0.719063
5300	0.832864
5350	0.910163
5400	0.943142
5450	0.928464
5500	0.867615
5550	0.76675
5600	0.636071
5650	0.488798
5700	0.339828
5750	0.20423
5800	0.09572
5850	0.025275
5900	2.05E-05
5950	0.022512
6000	0.090473
6050	0.19703
6100	0.331403
6150	0.480001
6200	0.627792
6250	0.759825
6300	0.862746
6350	0.926143
6400	0.943603
6450	0.91336
6500	0.838474
6550	0.726519
6600	0.58882
6650	0.439306
6700	0.293102
6750	0.164996
6800	0.067947
6850	0.011773
6900	0.002155
6950	0.040066

7000	0.121672
7050	0.238718
7100	0.379363
7150	0.529381
7200	0.673597
7250	0.797423
7300	0.888332
7350	0.937129
7400	0.938879
7450	0.893403
7500	0.805302
7550	0.683488
7600	0.540284
7650	0.390174
7700	0.248343
7750	0.129139
7800	0.044619
7850	0.003332
7900	0.009457
7950	0.062372
8000	0.156725
8050	0.282972
8100	0.428342
8150	0.578131
8200	0.717186
8250	0.831441
8300	0.90934
8350	0.943001
8400	0.92902
8450	0.868811
8500	0.768465
8550	0.638133
8600	0.490997
8650	0.341941
8700	0.206044
8750	0.097052
8800	0.02599
8850	4.61E-05
8900	0.021845

8950	0.089182
9000	0.195244
9050	0.329305
9100	0.477801
9150	0.625713
9200	0.758078
9250	0.861508
9300	0.925538
9350	0.943693
9400	0.914136
9450	0.839856
9500	0.728369
9550	0.59095
9600	0.441501
9650	0.295139
9700	0.16667
9750	0.069089
9800	0.012266
9850	0.00195
9900	0.039184
9950	0.120202
10000	0.236808

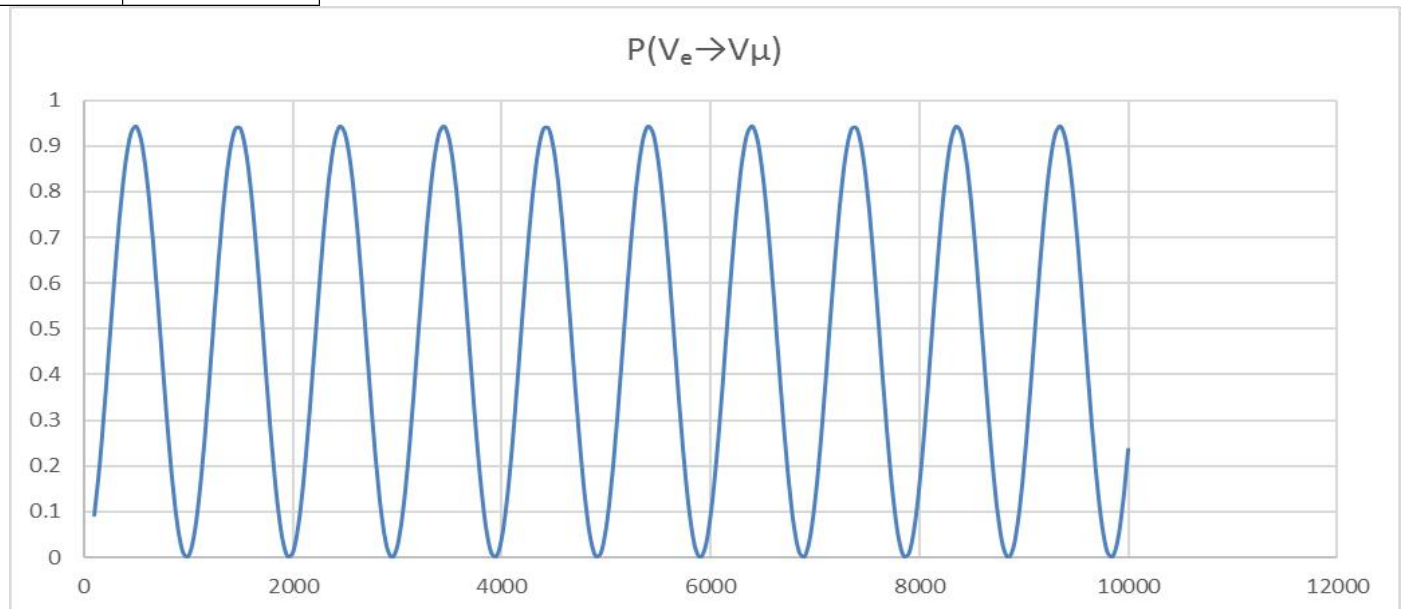


Fig. 3 Graph of probability, P on the Y-axis against distance per energy, L/E on the X-axis obtained using Microsoft excel spreadsheet.

The probability was calculated by taking arbitrary values for L/E ranging from 100-10000, using the Microsoft excel spreadsheet.

The probability of a neutrino changing type is related to the distance travelled by the neutrino from its point of production to its point of detection.

As a general rule, neutrinos travelling greater distances will exhibit greater depletion from oscillation. Moreover, the oscillation varies smoothly over increasing distance.

ENERGY OF THE OSCILLATION

To calculate the energy of the neutrino, the equation of the wavelength of the oscillation is used, which is given by:

$$\lambda_{osc} = 2.47 \frac{E(GeV)}{\Delta m_{23}^2(eV^2)}$$

$$E = \lambda_{osc} \times \frac{\Delta m_{23}^2}{2.47}$$

The value of the wavelength of the oscillation, λ_{osc} , was obtained from the graph in fig.3 as 950.

The energy of the neutrino is therefore:

$$E = \frac{950 \times 2.5 \times 10^{-3}}{2.47}$$

$$E = 0.9620 GeV$$

This implies that the neutrino has a very low energy and it could be from a natural source of neutrino such as solar neutrino, supernova neutrino, atmospheric neutrino etc.

CHAPTER FOUR

FINDINGS AND CONCLUSION

FINDINGS

1. The probability was calculated using the model equation, value of the probability varies between 0.093-0.24 as shown in the table
2. The graph of the probability, (P) against distance per energy, (L/E) is sinusoidal wave form as shown in fig.3
3. The value of the energy of the neutrino for the probability, $P(\nu_e \rightarrow \nu_\mu)$, is given as;

$$E = \frac{\lambda_{osc} \times \Delta m_{23}^2 (eV^2)}{2.47 (GeV)}$$

$$E = 0.9620 GeV$$

CONCLUSION

The probability of a neutrino changing type is related to the distance travelled by the neutrino from its point of production to its point of detection.

The oscillation varies smoothly over increasing distance. Neutrino oscillation is relevant to the question of neutrino mass, being that massless neutrinos cannot oscillate. It implies that the masses of neutrinos involved cannot be equal to one another. Since they cannot be equal to one another, they cannot both be zero.

There is need to extend the standard model of elementary particles to include this new information of finite masses and mixings of neutrino, through the knowledge gotten from neutrino oscillations.

SUGGESTION(S) FOR FURTHER STUDIES

Contrary to the observations from the Super-Kamiokande, neutrinos weren't apparently "disappearing" but in truth it was just a case of neutrinos changing type from flavors that could be detected to the ones that couldn't be detected as at that time.

The phenomenon of neutrino oscillation is relevant to the topic of neutrino masses and mixing because the existence of neutrino oscillation points to the fact that neutrinos have finite masses and mixings; which were not accounted for in the framework of the standard model of elementary particles.

Therefore, there is need to extend the standard model of elementary particles to include this new information of finite masses and mixings of neutrinos, through the knowledge gotten from neutrino oscillations.

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