A STUDY METHODS OF ESTIMATING THE PARAMENTERS OF AUTOREGEGRSSIVE PROCESS IN TIME SERIES MODELLING

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SEPTEMBER, 2023.

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A PROJECT WORK SUBMITTED TO THE DEPARTMENT OF STATISTICS, FACULTY OF PHYSICAL SCIENCES, UNIVERSITY OF BENIN, BENIN CITY, IN PARTIAL FULFILLMENT OF THE REQUIREMENT FOR THE AWARD OF BACHELOR OF SCIENCE (B.Sc.) DEGREE IN STATISTICS.

SEMPTEMBER, 2023.

UNDERTAKING

This project was project work was carried out by me, VICTORIA OLAYINKA ADEKOYA (PSC1809294) .I have not plagiarized any existing work. All published work used in this project work have been cited and referenced appropriately.

Victoria Olayinka Adekoya **Date**

CERTIFICATION

This is to certify that this project work was carried out by me, Victoria Olayinka Adekoya of the the department of Statistics, Faculty of Physical Sciences, University of Benin, Benin City.

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Head of Department

DEDICATION

This project work is dedicated to God for His unwavering divine support throughout my course of study.

ACKNOWLEDGEMENT

I am immensely indebted to God Almighty, who made it possible for me to

initiate and accomplish this research work. I articulate my sincere gratitude with special thanks to my project supervisor Prof. NEKHOSUEHI whose professional advice corrections, support , contributions , patience , encouragement and most of all availability led to the success and completion of this project . His input at the planning and execution stages, gave direction to the work, her academic , moral, and leadership qualities were propelling forces and so were his constructive criticisms.

I will also like to acknowledge the effort of the HOD ,all the lecturers in the department of Statistics for all the Knowledge you all impacted in me and

the other members of the staff, both academic and non-academic.

I am indebted my loving parent , Mrs Victoria Adekoya, for her support.

I really love you all, God bless you all.

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ABSTRACT

This research undertakes a comprehensive statistical analysis of Nigeria's Gross Domestic Product (GDP) spanning a decade, with a focus on estimating Autoregressive (AR) models using two prominent statistical methods: the Yule-Walker method and the Least Squares method. The study aims to provide statistical insights into the underlying dynamics of Nigeria's economic performance during this period.

The research commences by delineating the statistical framework of AR models, which offer a statistical representation of a time series based on its past values. Subsequently, the Yule-Walker method is introduced, a statistical technique leveraging autocorrelation functions to estimate AR model parameters. The statistical properties of Yule-Walker estimators are elucidated in the context of Nigeria's GDP data.

In contrast, the Least Squares method is presented as an alternative statistical approach, characterized by its objective to minimize the sum of squared prediction errors. A statistical framework for the least squares estimators is outlined, providing insights into the statistical properties of parameter estimates and their significance in explaining variations in Nigeria's GDP.

The core of the research involves the statistical analysis of Nigeria's GDP time series data over the 10-year period. Both the Yule-Walker and Least Squares methods are applied to estimate AR models tailored to the GDP data. The statistical comparison is based on goodness-of-fit statistics, such as the Akaike Information Criterion (AIC), to evaluate the models' adequacy in capturing the statistical patterns within the GDP dataset.

CHAPTER ONE

INTRODUCTION

Time series is a set of observations $\{xt\}$ which is recorded at a specific time t sequentially, over equal time increments or continuous time. If the set is of single observations, the series is called a univariate time series (Tingyan, 2010).

Autoregressive models, often abbreviated as AR models, are a type of statistical time series model used to predict future values based on past observations of a variable. In an autoregressive model, the current value of a variable is assumed to be linearly dependent on its past values with some added random noise. The "auto" in autoregressive refers to the fact that the model relies on its own past values for prediction. In recent times, there has been a lot of attention towards working with time series data. There are two main types of methods for analyzing time series: frequency-domain methods and time-domain methods. The frequency-domain methods include spectral analysis and wavelet analysis, while the time-domain methods consist of autocorrelation and cross-correlation analysis. These methods find broad applications in studying astronomical events, weather patterns, financial asset prices, economic activities, and more.

Various time series models have been introduced to understand and represent the data. These models include simple autoregressive (AR) models, simple moving-average (MA) models, mixed autoregressive moving-average (ARMA) models, seasonal models, unit-root non-stationarity, and fractionally differenced models for long-range dependence. Among these, the autoregressive moving average model (ARMA) is one of the most fundamental and widely used approaches in time series analysis (Tingyan, 2010).

In this project work, we will study some of the methods of estimating the model parameters of the AR models and apply this estimation techniques on real life time series data.

1.1.BACKGROUND OF THE STUDY

The autoregressive (AR) model, created by Box and Jenkins in 1970, is a way to express how a current value of a series depends on its past values using a linear regression approach. In the mid seventies, autoregressive modeling was first introduced in nuclear engineering and quickly gained popularity in other industries (Beran, J., 1994). Nowadays, autoregressive modeling is widely used for identifying, monitoring, detecting malfunctions, and diagnosing system performance.

To build an autoregressive model, we estimate a limited number of parameters from time series data. There are various techniques available for calculating the AR coefficients, with the main two categories being Least Squares and Burg's method

(Brockwell, P. J., & Davis, R. A., 2016), which are used to determine the best fit for the model based on statistical analysis. The Least Squares method seeks to minimize the sum of squared errors between the predicted values and the actual values in the time series. On the other hand, Burg's method relies on a recursive algorithm to estimate the coefficients based on the Yule-Walker equations. Among all of the methods, the most common method is the so called Yule-Walker method which applies the least squares regression method on the Yule-Walker equations system. This project work seeks to contribute to the existing body of knowledge by comprehensively examining the performance of the Least Squares and the Yule-Walker method in estimating autoregressive models for time series analysis.

1.2. AIM AND OBJECTIVES OF THE STUDY

The aim of this project work is to make a comparison between the Least squares method and the Yule-Walker method of estimating the parameter of an Autoregressive model. The objectives of this project work are as follows:

1. To review the foundational concepts and theories of Autoregressive modeling and its significance in time series analysis.

2. To study the estimation methods for autoregressive models, with a particular focus on the Least Squares and Yule-Walker method.

3. To empirically compare the performance of the estimation methods on selected datasets, analyzing and interpreting the results to identify their respective strengths and weaknesses.

1.3 SIGNIFICANCE OF THE STUDY

The study of Autoregressive (AR) models holds immense significance in the field of time series analysis and various application domains. Understanding and accurately estimating AR models offer valuable insights and practical benefits that contribute to advancements in research, decision-making, and problem-solving. Thus, this project work offers valuable insights and benefits in several key areas, making it a critical subject of investigation for statisticians and researchers. Some of the significance of estimating AR models are outlined below:

1. Forecasting and Prediction: AR models provide a powerful framework for forecasting future values based on past observations. Accurate predictions are crucial in decision-making processes for industries such as finance, economics, and meteorology. AR models enable stakeholders to anticipate trends, identify potential risks, and make informed strategic choices, leading to improved planning and resource allocation. (Box et al., 2015) (Taylor, 2017)

2. Process Control and Quality Assurance: In industries where continuous monitoring is important, such as manufacturing and healthcare, AR models

facilitate real-time process control and quality assurance. By detecting deviations from expected patterns, these models help identify anomalies and malfunctions, allowing for timely interventions and optimization of processes. (Montgomery, 2019) (Box et al., 2020)

3. Financial Market Analysis: AR models play a fundamental role in financial time series analysis. They assist in modeling stock prices, exchange rates, and other financial indicators, aiding investors, traders, and financial analysts in making well-informed decisions. Moreover, AR models contribute to risk assessment and volatility prediction, supporting portfolio diversification and risk management strategies. (Tsay, 2018) (Brooks, 2019)

4. Economic Policy and Planning: In the realm of economics, AR models are utilized to analyze economic indicators, such as GDP, inflation rates, and unemployment. These models help policymakers understand the dynamics of the economy, identify trends, and evaluate the effectiveness of various policy interventions. The insights derived from AR models guide economic planning and contribute to achieving stable and sustainable economic growth. (Enders, 2015) (Hamilton, 2020).

1.4. SCOPE OF THE STUDY

In this project work, the main focus will be on the method for parameter estimation for Autoregressive (AR) model parameters. Particularly, we shall focus on the Least squares estimation method and the Yule-Walker estimation method.

Furthermore, this study will be limited to application of AR models on secondary time series data on Nigeria GDP, sourced from World-bank official site.

1.5. ORGANIZATION OF THE STUDY

The organization of the study is as follows:

The first chapter gives concise information on the background of the study. The second chapter will present a brief history and literature review on Autoregressive (AR) models by many scholars. The third chapter will examine the methodologies and statistical assumptions of AR models followed by the theories and concepts behind AR models parameter estimation techniques considered in this project work. The fourth chapter shows the application of AR models on real life time series data. The fifth chapter gives a brief discussion and conclusion drawn based on the results obtained in fourth of this work.

1.6. DEFINITION OF BASIC TERMS

The study of the Methods of Estimating AR models will not be comprehensive enough without some Statistical terms. For the purpose of research, some terms have been chosen and the meaning given to enable us carry out analysis. The following are the definitions:

1. Autoregressive (AR) Model: A time series model in which the current value of a variable is regressed on one or more of its past values. The AR model captures the temporal dependencies and patterns in the data, making it a valuable tool for forecasting and time series analysis.

2. Time Series: A sequence of data points observed at successive time intervals. Time series data is collected over time and is characterized by its temporal ordering, where each observation is dependent on previous observations.

3. Lag: Refers to the time interval between consecutive data points in a time series. It represents the time delay or gap between an observation and its preceding observation. The concept of lag is essential in understanding the temporal dependencies and patterns within the time series data

4. Linear Regression: A statistical technique that models the relationship between a dependent variable and one or more independent variables by fitting a linear equation to the observed data. In the context of AR models, linear regression is

used to represent the relationship between the current value of the time series and its lagged values.

5. Parameter Estimation: The process of determining the values of model parameters that best fit the data. In AR models, parameter estimation involves finding the coefficients that minimize the difference between the actual values and the predicted values.

6. Model Order: The number of lagged observations (past values) considered in the AR model. It determines the memory or dependence of the time series on its past values. Selecting the appropriate model order is crucial in AR modeling.

7. Least Squares Method: A common approach to estimate the parameters of an AR model by minimizing the sum of squared differences between the actual data points and the predicted values obtained from the model.

8. Yule-Walker Equations: A set of linear equations that relate the auto-covariance function of the time series at different lags to the unknown AR coefficients. By solving these equations, one can obtain the estimates of the autoregressive coefficients, which define the AR model.

9. Forecasting: The process of predicting future values of a time series based on historical data and the fitted AR model. Forecasting is essential for planning, decision-making, and risk assessment.

10. Time Series Analysis: The examination of time series data to identify patterns, trends, and underlying relationships. Time series analysis involves various techniques, including AR modeling, to gain insights into the behavior of the data over time.

SUMMARY

This chapter examines the definition of Autoregressive models and gives an introduction to the project work. In the next chapter we shall examine the historical background of AR models and attempt a review of some academic scholars work.

CHAPTER TWO

LITERATURE REVIEW

2.0. INTRODUCTION

Autoregressive (AR) and Autoregressive Moving Average (ARMA) models are essential components of time series analysis. This literature review aims to explore the definitions, concepts, applications, and differences between AR and ARMA models, by appraising, comparing, and contrasting several works by different authors. Furthermore, Estimating the parameters of AR models is essential for accurate forecasting and understanding the underlying temporal dependencies in the data. Two popular methods for parameter estimation in AR models are the Least Squares method and the Yule-Walker method. This literature review also aims to appraise, compare, and contrast several works by different authors on these estimation techniques. Finally, this literature review would attempt to appraise several authors work on the application of Autoregressive Integrated Moving Average Models (ARIMA), another class of AR models used in time series analysis.

2.1. LITERATURE REVIEW OF ARMA MODELS

According to Tingyan (2010), A Time series model for the observed time series $\{xt\}$ is a specification of the joint distributions of the sequence of random variables $\{xt\}$. Time series models takes up different forms and explains different stochastic processes, but are broad categorized into three categories namely: Autoregressive (AR) models, Moving Average (MA) models and Integrated (I) models. In the coming sections, we shall consider the definitions and concepts behind the above mentioned Time series models as opined by several authors and scholars in the past.

2.1.1 DEFINITIONS AND CONCEPTS OF AUTOREGRESSIVE (AR) AND AUTOREGRESSIVE MOVING AVERAGE MODELS (ARMA)

AR models represent a time series as a linear combination of its past values, where the current value depends on a fixed number of lagged observations. Brockwell and Davis (2019) provide a comprehensive definition of AR models and explain their mathematical formulation. They highlight that the order of an AR model, denoted by "p," determines the number of past observations used in the model. Thus an AR model of order p can be denoted as AR(p). Mathematically, an AR(p) model can be formulated as shown below: $xt=\propto 0+\propto 1xt-1+\propto 2xt-2+\cdots+\propto pxt-p+\varepsilon t$

If we assume $\propto 0=0$, $we\ have\ xt= \propto 1xt-1+\propto 2xt-2+\cdots+\propto pxt-p+\varepsilon t$

Which can be written as in linear form as $\propto(\boldsymbol{\beta})\boldsymbol{Xt}= \boldsymbol{\varepsilon}t$

Where $\propto(\boldsymbol{\beta})=1-\propto1\beta-\propto1\beta2-\cdots-\propto1\beta p$ is a polynomial of degree p. The roots of $\propto(\boldsymbol{\beta})=\mathbf{0}$ must lie outside the unit circle to ensure stationarity.

Where, xt represents the current value of the time series at time t.$\propto0$ is the intercept or constant term in the model, representing the baseline level of the time series when all lagged terms are zero. $\propto1,\propto2,\ldots,\propto p$, are the autoregressive coefficients representing the influence of past values on the current value.

$xt-1,t-2,\ldots,xt-p$, are the lagged values of the time series up to order p, which are used to predict the current value.

εt is the error term at time t, representing the unpredictable or residual part of the time series not explained by the lagged terms.

AR(p) models are widely used in various fields, including finance, economics, and signal processing, for their ability to capture temporal dependencies and forecast future values. Box and Jenkins (1970), in their seminar work introduced the autoregressive (AR) model as a linear regression of a time series against one or more of its past values. They defined an AR(p) model of order p as follows:

"The AR(p) model expresses the current value of a time series as a weighted sum of its p previous values, plus an error term. The model captures the temporal dependencies within the data, where the weights (coefficients) represent the influence of past observations on the current one." Box and Jenkins (1970)

The definition provided by Box and Jenkins is clear and concise, introducing the concept of the autoregressive model as a linear regression of a time series against its past values. The mention of order "p" and the influence of past observations on the current one highlights the fundamental elements of AR models. This definition is foundational and serves as a basis of other definitions of AR models of time series analysis.

Hamilton (1994) opined that in an AR(p) model, the current value of a time series is determined by a linear combination of its p past values, plus an error term. The model captures the persistence of the series over time, as past values influence future outcomes. Hamilton statement makes it rather obvious to see that the value of the order of the AR model p, affects the accuracy of the model predictions. Thus, there is a need to select an appropriate value for p, so that the AR(p) model estimates are not biased.

According to Granger (1980), an Autoregressive model of order p, denoted as AR(p), represents a time series as a linear combination of its p previous observations, each weighted by a coefficient. The model captures the persistence and memory of the time series, as it depends on its own past values. This definition is consistent with Hamilton (1994) statement. Granger's definition focuses on the concept of autoregressive models in the context of causality and time series analysis. It introduces the idea of autoregressive models as a means to study relationships between variables. The definition is more specialized and may be particularly relevant for researchers understanding causality and interdependencies in time series data. Moving average (MA) model is a linear regression relationship of the current value of the series against the error terms of one or more past values of the series (Tingyan, 2010). The MA model just like its AR counterpart has an order (q). The order q determines how many past error terms are used in the model. The selection of the order q is essential for capturing the short-term dependencies and fluctuations in the time series. Thus, the mathematical formulation of the MA(q) model is shown below: $xt=\mu+\varepsilon t+\theta 1\varepsilon t-1+\theta 2\varepsilon t-2+\ldots+\theta q\varepsilon t-q$ $if\ we\ assume$ $\mu=0$, $therefore$ $\boldsymbol{Xt}= \boldsymbol{\theta}(\boldsymbol{\beta})\boldsymbol{\varepsilon}t$

$where$ $(\boldsymbol{\beta})=1+\theta 1\ \beta 1+\ \theta 2\beta 2+\cdots+\boldsymbol{\theta} q\beta q$

Where, μ the mean or constant term in the model, representing the expected or average value of the time series, $\varepsilon t-1$, $\varepsilon t-2$, …, $\varepsilon t-q$, are lagged values of the error terms and $\theta 1,\theta 2,\ldots,\theta q$ are the parameters of the model, known as the moving average coefficients. They represent the weights assigned to each lagged error term (ε) in the model.

It is worthy to note that AR(p) models are easy to estimate compared to the MA(q) models.

The combination of the AR model and the MA model produces the Autoregressive Moving Average Model (ARMA). ARMA models extend AR models by incorporating the concept of Moving Average (MA) components. These models represent a time series as a combination of its past values and a linear combination of past white noise error terms. Shumway and Stoffer (2017) provide an in-depth explanation of ARMA models, highlighting the importance of both the autoregressive and moving average components. They discuss the selection of the model orders "p" and "q," which represent the number of autoregressive and moving average terms, respectively, hence we have an ARMA(p,q) Model. ARMA models are particularly useful for capturing both the autocorrelation (a phenomenon that describes the correlation of past values of an error term with its current value) and the moving average effects in the data. A mathematical representation of an ARMA(p,q) model is shown below: $x_t=\mu+\propto_1 x_{t-1}+\propto_2 x_{t-2}+\cdots+\propto_p x_{t-p}+\theta_1\varepsilon_{t-1}+\theta_2\varepsilon_{t-2}+\ldots+\theta_q\varepsilon_{t-q}+\varepsilon_t$ *if we assume* $\mu=0$, *we have* $\propto(\boldsymbol{\beta})\boldsymbol{X_t}=\boldsymbol{\theta}(\boldsymbol{\beta})\boldsymbol{\varepsilon_t}$

Where all of the above have their usual meaning from the AR and MA models.

AR and ARMA models have their application in various fields, for example, in climate science AR and ARMA models are applied to forecast various meteorological variables. Bermejo et al. (2019) used ARMA models to predict monthly rainfall patterns in a specific region. The authors emphasized the importance of considering the lagged autocorrelations and the white noise errors to capture the seasonal patterns and short-term fluctuations accurately. However, the study also highlighted the challenges of model selection and the need for robust diagnostics to ensure model adequacy. Furthermore, in finance, AR and ARMA models are commonly used for forecasting financial time series. Bollerslev (2018) studied the application of ARMA models in modeling and forecasting volatility in financial asset returns. The study demonstrated that ARMA models can capture the time-varying volatility patterns in financial data, making them valuable tools for risk management and option pricing. However, the author also acknowledged the limitations of ARMA models in capturing long-term persistence in volatility, which might require more complex models like GARCH (Generalized Autoregressive Conditional Heteroskedasticity).

AR and ARMA models are fundamental tools in time series analysis, providing valuable insights into temporal dependencies and facilitating accurate forecasting. AR models consider past values to model the current value, while ARMA models extend this concept by incorporating moving average components. Both models have widespread applications in various fields, including finance, climate science, and economics.

2.1.2. APPLICATION OF AR MODELS AND AUTOREGRESSIVE INTEGRATED MOVING AVERAGE MODELS (ARIMA)

ARIMA models (AutoRegressive Integrated Moving Average) are a generalization of both AR and MA models, encompassing their characteristics. The "I" in ARIMA represents the integrated component, indicating the number of times differencing is applied to the time series to achieve stationarity. Time differencing, is a common technique used in time series analysis to remove trends and seasonality from a time series. It involves calculating the differences between consecutive observations to transform a non-stationary time series into a stationary one. Aghabozorgi and Talei (2021) provide a comprehensive overview of ARIMA models, emphasizing their flexibility in handling non-stationary data through time differencing. The authors highlight the importance of identifying the appropriate differencing order "d" to ensure the stability of the model and accurate forecasts. The mathematical form of the ARMA(p,q) model and the ARIMA(p,d,q) model are the same, the only difference in both models is that ARIMA is used on differenced data while ARMA is used on non-differenced data. Some applications of AR(p) models and ARIMA(p,d,q) models as seen in the works of several authors include: Chen and Zheng (2018) work, which demonstrated the use of AR models for predicting stock price movements. Their study showed that AR models can capture short-term trends and provide valuable insights to investors. However, they also noted that AR models might not perform well during periods of high volatility or non-stationary behavior. Ahmed and Rahman (2020) analyzed the impact of foreign direct investment on economic growth using ARIMA modeling. Their findings suggested a significant positive relationship between the two variables. In contrast, Hatemi-J (2018) investigated the relationship between exchange rates and interest rates using AR models. The author demonstrated that AR models can be powerful tools in uncovering dynamic interactions in economic time series. Finally we consider the work of Sung et al. (2017), they applied ARIMA modeling to predict tourist arrivals in a popular tourist destination. The study showed that ARIMA models can provide accurate short-term forecasts, supporting tourism management and resource planning. However, the authors noted that long-term forecasting with ARIMA models may be less reliable due to the potential impact of external events and changing patterns.

AR(p) models and ARIMA(p,d,q) models are powerful tools in time series analysis and forecasting. AR models are effective in capturing short-term trends and patterns, while ARIMA models are suitable for handling non-stationary data. Their applications span across various domains, including finance, economics, tourism, and energy forecasting. However, researchers must carefully consider model assumptions, and the appropriate selection of model orders (p, d, q) to ensure accurate and reliable forecasts.

SUMMARY

In this chapter the literatures of various scholars and authors about AR, MA, ARMA and ARIMA models were presented and reviewed and some examples on the applications of these models in various fields was considered. In the next chapter we shall study some methodologies of AR models and the methods of parameter estimation of AR models.

CHAPTER THREE

METHODOLOGY

3.0. INTRODUCTION

This study focuses on one specific type of time series model: The Autoregressive (AR) model. The AR(p) model was introduced by Box and Jenkins in 1970 (Box, 1994). As mentioned in the chapter one of this study, AR (p) model is a linear regression that establishes a relationship between the current value of a series against past values of the series. The value of p is known as the order of the AR(p) model, which means that the current value is dependent on p past values in the series. In this chapter we shall focus on the methods of estimating the parameters of the AR(p) model. We shall consider two of these methods namely: Yule-walker estimation method and Least squares estimation method; considering their assumptions, theories and concepts.

3.1. STATISTICAL ASSUMPTIONS OF AUTOREGRESSIVE MODELS

In order to model a time series $\{Xt\}$ using the AR(p) model, there are certain assumptions that must hold regarding the nature of $\{Xt\}$. These assumptions are given below:

3.1.1. STATIONARITY

The bedrock of time series analysis is stationarity. A time series is considered stationary if its statistical properties, such as mean, variance, and autocorrelation, remain constant over time. In simpler terms, the characteristics of the data do not change with respect to time. There are basically two types of stationarity namely:

1. Strict Stationarity: A time series is strictly stationary if the joint distribution of any subset of its time points is the same for all time points. This implies that the statistical properties of the data do not change even when considering different time intervals. Mathematically; $\{Xt\}$ to be strictly stationary if the joint distribution of ($Xt1$, ..., Xtk) is identical to that of

($Xt1$, ..., Xtk+1) for all t, where k is an arbitrary positive integer and

($t1$, , ..., tk) is a collection of k positive integers represent the recorded time (You, 2010).

2. Weak Stationarity: A time series is weakly stationary if its mean and variance are constant over time and the autocovariance function (or autocorrelation function) between any two time points depends only on the time lag between them. In other words, the overall pattern of the data remains consistent, and it exhibits constant mean, variance, and covariance

structure. That is to say, for a time series $\{Xt\}$ to meet the requirement of weakly stationary, it should satisfy two conditions:

(a) Constant mean: E(Xt) = μ; and (b) Cov(Xt , $Xt-i$) = γi only depends on i. In order to model a time series using AR(p) model, the time series must satisfy the condition of being Stationary; Either strictly or weak stationary. There are various methods of detecting stationarity in time series. These methods are broadly categorized into: Visualizations and Statistical testing methods.

3.1.1.1. Visualization Method of Detecting Stationarity

One widely used visualization method is by taking a time plot. A time plot is a plot of the observations in a time series against time. A time plot helps illustrate the existence of stationarity in a time series. The existence of a trend in a time series plot implies that the time series is not stationary, while the existence of no trend in time series plot indicate that the series in stationary. An illustration is shown below.

3.1.1.2. Statistical testing methods

Testing for the presence of stationarity in time series using plots is not efficient enough. Thus more robust statistical techniques are required to capture the presence of stationarity. One major statistical testing tool employed in testing for stationarity otherwise known as unit root in time series is the Augmented Dickey Fuller test. Where the null hypothesis of the test is there exist non-stationarity and the Alternative hypothesis is the existence of stationarity. i.e. Augmented Dickey Fuller Test: $H0{:}Time\ series\ is\ stationary\ H1{:}Time\ series\ is\ non{-}stationary$

Thus, in the advent of rejection of the null hypothesis, we may conclude that the time series is stationary and thus good for the AR(p) model.

3.1.2. AUTOCORRELATION

Autocorrelation in an AutoRegressive (AR) model refers to the correlation between a time series observation and its past values at different lags. In other words, it measures how a value in the time series is related to its own previous values.

In an AR(p) model, where "p" represents the order of autoregression, the autocorrelation is a key factor in understanding the dependence structure of the time series. The autocorrelation function (ACF) and Partial autocorrelation function (PACF) are commonly used to visualize the autocorrelation at various lags.

1. AutoCorrelation Function (ACF): ACF measures the correlation between a time series and its lagged values. It helps identify the nature of the correlation between the current value and past values at different lags. In an AR(p) model, ACF can help determine the order of the autoregressive component (p) by observing which lag values have significant correlations.

1. If the ACF shows a significant correlation at lag 1 and a gradual decline for subsequent lags, it suggests an AR(1) model.

2. If the ACF shows significant correlations at lags 1, 2, and a gradual decline afterward, it suggests an AR(2) model.

3. Similarly, significant correlations at specific lags can suggest the appropriate order for the AR model.

2. Partial AutoCorrelation Function (PACF): PACF measures the direct correlation between a time series value and its lagged values, while controlling for the influence of other intermediate lag values. It helps identify the direct effect of each past value on the current value, which is particularly important for determining the AR order in AR(p) models.

1. If the PACF shows a significant correlation only at lag 1 and no significant correlations at other lags, it suggests an AR(1) model.

2. If the PACF shows significant correlations at lags 1 and 2 and no significant correlations afterward, it suggests an AR(2) model.

3. The PACF helps differentiate the direct influence of each lag on the current value, allowing for a more accurate determination of the AR order.

3.2. PARAMETER ESTIMATION IN AUTOREGRESSIVE MODELS

The AR model is widely used in science, engineering, econometrics, biometrics, geophysics, etc. When a series is to be modeled by the AR model, the appropriate order p should be determined using the ACF and PACF plot and the parameters of the model must be estimated after testing for stationarity.

There are a number of methods available for estimating its parameters for this model and of these the following two maybe the most commonly used.

3.2.1. Least Squares Estimation Method Regression analysis is perhaps the most commonly employed statistical technique in data analysis. Among the assortment of regression methods, Least Squares is extensively applied in linear regression models and is frequently utilized for estimation purposes. The core principle of the Least Squares method involves minimizing the sum of squared error terms (εt). An autoregressive (AR) model, a basic linear regression model, operates by making use of the principle of the least squares method to fit a model by minimizing the sum of square errors for estimating parameters. Consider the following AR(p) model: $xt = \propto 1xt-1+\propto 2xt-2+\cdots+\propto pxt-p+\varepsilon t$ *here we assume that xthas mean zero or it has been demean i.e* $xt = xt* - \bar{x}$

Here, xt represents the current value of the time series at time t.

$\propto 1, \propto 2, \ldots, \propto p$, are the autoregressive coefficients representing the influence of past values on the current value.

$xt-1, t-2, \ldots, xt-p$, are the lagged values of the time series up to order p, which are used to predict the current value.

ε_t is the error term at time t, representing the unpredictable or residual part of the time series not explained by the lagged terms.

The white noise ε_t is under the following assumptions:

1. $E(\varepsilon_t) = 0$

2. $E(\varepsilon_t^2) = \sigma^2$

3. $E(\varepsilon_t \varepsilon_k) = 0$ for $t \neq k$

4. $E(x_{t-k}\varepsilon_t) = 0$, k=1, 2, …..

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5. ε_t *are independently and identically Normally distribute(iid) .*

Our objective is to find the parameter estimates $\propto_0, \propto_1, \propto_2, \ldots, \propto_p$, that minimize the sum of squared errors ,$SSE = \sum_{t=1}^{n} \varepsilon_t^2$ where n is the total number of observations.

To minimize the sum of squared errors, we differentiate the sum of squared errors with respect to each parameter $\propto_i$, set the derivatives to zero, and solve for the parameter estimates. Here's the general procedure:

1. Formulate the sum of squared errors SSE as:

$$SSE = \sum_{t=1}^{n} \varepsilon_t^2 = \sum_{t=1}^{n} (x_t - \propto_0 - \propto_1 x_{t-1} - \propto_2 x_{t-2} - \cdots - \propto_p x_{t-p})^2$$

2. Differentiate SSE with respect to each parameter $\propto i$:

•Differentiate with respect to $\propto 0$: Set $\partial SSE \partial \propto 0$ = 0

•Differentiate with respect to $\propto 1, \propto 2, \ldots, \propto p$: Set $\partial SSE \partial \propto i$ =0 for each i,and write down the normal equations where $\propto 0, \propto 1, \propto 2, \ldots, \propto p$ are parameters in the system of equations

2. Solve the resulting equations for $\propto 0, \propto 1, \propto 2, \ldots, \propto p$.

3.2.2. Yule-Walker Estimation Method

Yule-Walk Method method, also called the autocorrelation method, is a numerically simple approach to estimate the AR parameters of the ARMA model. In this method, an autoregressive (AR) model is also fitted by minimizing the forward prediction error in a sense of least-squares regression. The difference is that Yule-Walker method is to solves the Yule-Walker equations, which is formed from sample covariances. (You, 2010)

A derivation of Yule-walker estimates for an AR(p) model as outlined by (Young and Jakeman 1979) is given below:

AR(p) Model: Consider an AR(p) model of order "p": $xt = \propto 1 xt-1 + \propto 2 xt-2 + \cdots + \propto p xt-p + \varepsilon t$

Covariance Structure: The Yule-Walker method is based on the covariance structure of the time series. For an AR(p) model, the covariance between xt and $xt-k$ can be expressed as:

Cov(xt,$xt-k$) = γk= $\alpha 1\gamma k-1$+ $\alpha 2$ $\gamma k-2$+ … + $\alpha p\gamma k-p$ Where γk is the covariance between xt and $xt-k$, and $\gamma k-1$ to $\gamma k-p$ are covariances at previous lags.

Yule-Walker Equations: The Yule-Walker method aims to find estimates for the coefficients $\alpha 1$ to αp using the sample covariances γk,$sing$ ρk=$\gamma k\gamma 0$. This leads to a set of p equations, known as the Yule-Walker equations: $\rho 1$= $\alpha 1$ $\rho 0$+ $\alpha 2$ $\rho -1$+ … + $\alpha p\rho -p$+1 $\rho 2$= $\alpha 1$ $\rho 1$+ $\alpha 2$ $\rho 0$+ … + $\alpha p\rho -p$+2 ⋮ … ⋮ ρp= $\alpha 1$ $\rho p-1$+ $\alpha 2$ $\rho p-2$+ … + αp $\rho 0$

i.e. ρk= $\Sigma\alpha j\rho k-jpj$=1 ,=1,2,3,4,…,p

using the fact that $\rho 0$ =1,$\rho -k$=ρk, k=1,2,3,…

In Matrix form we have: ($\rho 1\rho 2$⋮$\rho 3$)=($\rho 0\rho 1$…$\rho p-1\rho 1\rho 2$…$\rho p-2$⋮$\rho p-1$⋮⋮$\rho p-2$…⋮ρp)($\alpha 1\alpha 2$⋮αp)

thus when p = 1, we have an AR(1) model. So the yule-walker equations becomes:1=$\Sigma\alpha j\rho k-j1j$=1=$\alpha 1\rho 0$

With $\rho 0$=1 we $have$ $that$ $\rho 1$=$\alpha 1$̂

When p =2, we have an AR (2) model, so that the walker equations ρk= $\Sigma\alpha j\rho k-j2j$=1,=1,2 $\rho1$= $\alpha1\rho0$+ $\alpha2\rho-1$ $\rho2$= $\alpha1\rho1$+ $\alpha2\rho0$

i.e. ($\rho1\rho2$)=(1$\rho1\rho11$)($\alpha1\alpha2$)

When p =3, we have an AR (3) model, so that the yule-walker equations becomes ρk= $\Sigma\alpha j\rho k-j3j$=1,=1,2,3 $\rho1$= $\alpha1$ + $\alpha2$ $\rho1$+ $\alpha3$ $\rho2$

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$\gamma2$= $\alpha1$ $\rho1$+ $\alpha2$ + $\alpha3$ $\rho1$ $\rho2$= $\alpha1$ $\rho2$+ $\alpha2\rho1$ + $\alpha3$

i.e. ($\rho1\rho2\rho3$)=(1$\rho1\rho2\rho11\rho1\rho2\rho11$)($\alpha1\alpha2\alpha3$)

These equations gotten by By Multiplying both side of the AR(p) model by $xt-k$, k = 0, 1, · · · , p, then taking expectation, forming a system of linear equations in terms of the unknown coefficients $\alpha1$,..,αp.

By solving this system of equations for α, the estimates for the AR(p) model coefficients are obtained.

3.3. COMPARISON BETWEEN LEAST-SQUARE ESTIMATION METHOD AND YULE-WALKER ESTIMATIOH METHOD

Time series analysis is a critical domain in statistics and econometrics, with various estimation methods being employed to model temporal dependencies accurately. Two commonly used methods for estimating the parameters of

AutoRegressive (AR) models are the Least-Squares Estimation Method and the Yule-Walker Estimation Method. This section attempt to compare and contrast these two approaches based on their underlying principles, assumptions, applicability, efficiency, and advantages. The Least-Squares Estimation Method focuses on minimizing the sum of squared residuals between observed data and model predictions, emphasizing the overall fit of the model to the entire dataset. The Yule-Walker Estimation Method, on the other hand, leverages the autocovariance structure of the time series. It estimates parameters based on the autocovariance and autocorrelation functions, offering insights into the temporal relationships among data points. (Yule, 1927). Least-Squares method is applicable to a wide range of regression problems, but it assumes that errors are independent and identically distributed (i.i.d.). Yule-Walker method is specifically designed for stationary AR models, assuming constant mean, variance, and autocorrelation over time. Least-Squares estimation aims to provide parameter estimates that minimize the overall error, ensuring a balanced fit across the entire dataset. Yule-Walker estimation provides parameter estimates that closely match the autocovariance structure, capturing the temporal dependencies within the time series. Least-Squares method is versatile and widely applicable, but its sensitivity to outliers can affect its robustness. Yule-Walker method is particularly useful for stationary time series with short-term dependencies and offers insights into the time lag structure of the data. (Kutner, M. H., et al., 2004)

In conclusion, both the Least-Squares Estimation Method and the Yule-Walker Estimation Method have their strengths and limitations. The choice between these methods depends on the specific characteristics of the dataset, the assumptions that can be reasonably made, and the research objectives. Researchers and practitioners need to carefully consider the nature of the time series, computational efficiency, and the interpretability of results when selecting the appropriate estimation method.

SUMMARY

In this chapter we have considered the methodologies behind Autoregressive (AR) models, the method and parameter estimation and a comparison between two of these methods. In the chapter four of this project work we shall apply this method in finding parameter estimates to an AR model using data on Nigeria Gross Domestic product gotten from World Bank.

CHAPTER FOUR

APPLICATION OF AUTOREGRESSIVE MODELS TO REAL LIFE TIME SERIES DATA

4.0. INTRODUCTION

In this chapter we will be applying the Autoregressive (AR) model on real life time series data. We will be using a secondary data on The Gross Domestic product (GDP) of Nigeria, collected from the World Bank data site. Furthermore, we will be adopting the Yule-walker and Least Squares Method in estimating the model parameters and attempt a comparison between these models, using R programming Language for all of the analysis in the study.

4.1. DATASET DESCRIPTION

The dataset adopted in this study comprises of the value of Nigeria Gross Domestic product (GDP) in yearly basis, over the span of 10 years (2013 -2022). This dataset was extracted from the World Bank data site and analysis conducted.

4.2. CHECK FOR STATIONARITY

In order to model the time series observations using the Box-Jenkins method, it is appropriate to check if the time series is stationary or not. The time series plot shows that the data is non-stationary as an upward and downward is observed.

4.3. FINDING THE ORDER OF THE AR MODEL

In order to find the order (p) of the AR (p) model to be used, we will be making an Sample Autocorrelation (SACF) plot as shown below:

The SACF plot suggests that it will be appropriate to adopt orders 1,2 and 3 for this dataset, Hence, we will be adopting AR(1), AR(2) and AR(3) models and select the best model based on the AIC criterion. Thus, the 3 models can be shown mathematically as: $Let\ GDP=X\ Xt=\propto 1Xt-1+\varepsilon t$ AR(1) $Xt=\propto 1Xt-1+\propto 2Xt-2+\varepsilon t\ AR(2)\ Xt=\propto 1Xt-1+\propto 2Xt-2+\propto 3Xt-3+\varepsilon t\ AR(3)$

The model with the lowest AIC will be considered the best model and will be adopted for interpretation.

4.4.

ESTIMATION USING YULE-WALKER METHOD

Yule walker method was applied in estimating the model parameters of the AR (1) ,AR(2), AR(3) model and the results presented below:

Yule-Walker estimated parameters

AR(P) Model

Estimated Coefficients

Variance

AIC

AR(1)

0.5582

1321

9.963565

AR(2)

1.0048, -0.8003

475

8.230375

AR(3)

1.058 -0.8532 -0.8190

459

6.659628

$Xt=1.508Xt-1-0.8532Xt-2-0.8190Xt-3+\boldsymbol{\varepsilon}t$

Interpretation:

The results presented above shows that the estimated parameters of the AR (1), AR(2) and AR(3) models are (0.5582), (1.0049,-0.8003) and (1.058, -0.8532,

-0.8190) respectively the models AICs are 9.963565, 8.230375, and 6.659628 respectively. Thus, the AR(3) model will be considered the best model under the yule-walker method of estimation based on the AIC criterion.

4.5.

ESTIMATION USING LEAST SQUARES METHOD

Least squares was applied in estimating the model parameters of the AR (1), AR(2) and AR(3) models and the result presented below:

Least squares estimated parameters

AR(P) Model

Estimated Coefficients

Variance

AIC

AR(1)

0.5117

1197

10.417931

AR(2)

1.0339 -0.7270

453.9

7.692850

AR(3)

1.0567 -0.7341 -0.6777

447.9

6.101239

Xt=1.0567Xt−1−0.7341Xt−2−0.6777Xt−3+$\boldsymbol{\varepsilon t}$

Interpretation:

The results presented above shows that the estimated parameters of the AR (1), AR(2) and AR(3) models are (0.5117), (1.0339 -0.7270) and (1.0567 -0.7341 -0.6777) respectively and the models AICs are 10.417931, 7.692850 , 6.101239 respectively. Thus, the AR(3) model will be considered the best model under the Least squares method of estimation based on the AIC criterion.

4.6. COMPARISON BETWEEN YULE-WALKER METHOD AND LEAST SQUARES ESTIMATION METHOD

The Yule-walker and Least-Squares estimation methods, both yields similar AR(3) coefficients. While the yule walker estimation technique yielded an MSE of 459.3, the Least Squares estimation technique yielded an MSE of 447.9. Furthermore, the AIC of the yule walker model was 6.659628 and the AIC of the least squares model was 6.101239. Thus, we may conclude that the accuracy in predicting future values of GDP using the Least squares estimation technique is higher than the accuracy of the yule-walker method and the AR(3) model estimated using the Least squares method is the model of best fit. These results reflect on the importance of the requirements of stationarity when applying the yule-walker method, as the yule-walker method of estimations solemnly relies heavily on the stationarity of the time series data for accurate predictions compared to the least squares method.

CHAPTER FIVE

SUMMARY AND CONCLUSION

SUMMARY

In the chapter four of this project work, an Autoregressive (AR) model was applied to real-life time series data representing the Gross Domestic Product (GDP) of Nigeria over a period of 10 years (2013 - 2022). Two estimation methods, the Yule-Walker method and the Least Squares method, were employed to estimate the model parameters. The following key findings were observed:

1. Stationarity Check: A time series plot was drawn to observe the data trend over the years of study and check for stationarity. The time series plot showed that the data was not stationary as an upward and downward trend was observed.

2. Order of AR Model: To determine the order (p) of the AR model, an Autocorrelation Function (ACF) plot was created. The SACF plot suggested a appropriate orders of one, two and three (AR(1), AR(2) and AR(3)) based on the lags that remained inside the cut-off lines.

3. Estimation Using Yule-Walker Method: The Yule-Walker method was applied to estimate the parameters of the AR(1), AR(2) and AR(3) models. The estimated coefficients for the model were presented, along with a Mean Squared Error

s (MSEs) and Akaike Information Criterions (AICs). The AIC of the models suggested that the AR(3) model, with the lowest AIC, was the best fit for the data.

4. Estimation Using Least Squares Method: The Least Squares method was also employed to estimate the parameters of the AR(1), AR(2) and AR(3) models. The estimated coefficients for the model were presented, along with a Mean Squared Errors (MSEs) and Akaike Information Criterions (AICs). The AIC of the models suggested that the AR(3) model, with the lowest AIC, was the best fit for the data.

5. Comparison Between Estimation Methods: A comparison between the Yule-Walker and Least Squares estimation methods was conducted. Both methods yielded similar AR(3) coefficients, indicating the consistency of the model. However, it was observed that the Least Squares method had a lower MSE (447.9) and AIC (6.101239) compared to the Yule-Walker method MSE(459.3) and AIC(6.659628). This suggests that the Least Squares method exhibited higher accuracy in predicting future GDP values.

CONCLUSION:

This project work has demonstrated the application of AR models to real-life time series data and highlighted the importance of choosing an appropriate estimation method based on stationarity considerations. The findings contribute to the understanding of GDP fluctuations in Nigeria and provide insights into accurate modeling techniques.

This study has contributed valuable insights into the application of AR models in a real-world context. It underscores the significance of data stationarity and the choice of estimation methods, ultimately highlighting the role of rigorous statistical analysis in understanding and predicting economic phenomena. As the world of time series analysis continues to evolve, the lessons from this study remain relevant for economists, policymakers, and analysts seeking to make informed decisions based on time series data.

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