NUMERICAL APPRAOACH TO ANALYSE INCREMENTAL OIL “ENHANCED OIL RECOVERY”

“THESIS”

“Presented to the” Department of Petroleum Engineering

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By

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NUMERICAL APPRAOACH TO ANALYSE INCREMENTAL OIL ENHANCED Oil RECOVERY
“A THESIS APPROVED BY THE PETROLEUM ENGINEERING DEPARTMENT.”

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Lastly “to my” family; most important my sweet mother for their undying love and support financially and other areas of life.“I would not have pulled through without your support, I say ‘Medaase’”.

“DEDICATION”

“This work is dedicated to my family; for their undying love, prayers and support and also to my two lovely”.

ABSTRACT

The effectiveness and efficiency of “enhanced oil recovery EOR process” carried out “is” directly related “to the” increment of the reservoir inflow well performance. This Thesis adapted the Simpson rule, trapezoidal rule and equation of trendline in predicting the incremental oil from successful enhanced oil recovery processes.

“Incremental oil values of Enhanced Oil Recovery projects have been obtained in the past using the” Trapezoidal rule, Simpsons rule, and equation of trendline using Microsoft excel package with both former posing “approximates the area under a curve with a straight line segment” while the later posing anti derivative of the trendline equation as an approximate value for the incremental oil.

Undermining “the accuracy and significance of the incremental oil values obtained by these methods. In this work”, Simpson Rule and Trapezoidal Rule were “applied to the respective rate-time data and the concept of finite difference was introduced to account for the error term”. Experimental data was used for the analysis (Olaoye, Taiwo, & olafuyi, 2022).

“Result showed more accurate predictions of Incremental oil by the” methods used in this thesis as compared to Cubic spline and Decline curve. Since, incremental oil values rightly measures the success of any EOR project by estimating accurate volumes of oil produced via such displacement mechanism, it imply that adopting the trapezoidal and Simpson Rule will aid better estimations of the economic values of projects and effective management decision making (Olaoye, Taiwo, & olafuyi, 2022).

Reducing the step size(s) of the rate time curve showed more accurate results of incremental oil which is consequential in Trapezoidal rule and Simpson rule than equation of trendline, third methodology used in this thesis to fix conclusively the amount of incremental oil.

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CHAPTER ONE

INTRODUCTION

“General Introduction”

“A universal” economical “measure of the success of an Enhanced oil recovery EOR project is the amount of the incremental oil recovered” compare to the resources used (man, material, money, machine, methods).

Figure 1 below “defines incremental oil. A reservoir whose oil rate is declining from A to B. At B- an EOR project” was “initiated, and if successful, the rate should show a deviation from the projected decline after B” because enhanced “oil” recovery “is” not instantaneous.

The Incremental oil has been the difference between what was recovered, B to D, and what would have been recovered had the process not been initiated, B to C i.e better inflow well performance is estimated by the area under the rate-time curve; the incremental oil recovered is, therefore, the shaded region (Olaoye, Taiwo & Olafuyi, 2022).

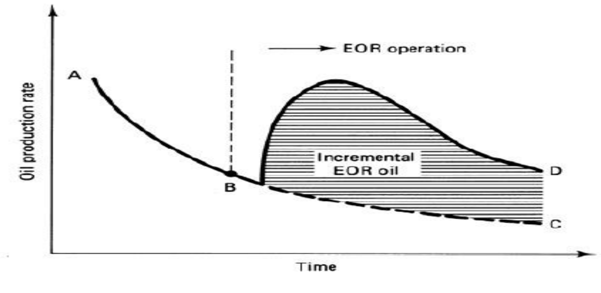


Figure 1: Profile of volumetric flow rate against time that depicts incremental oil.

Consequently, the additional amount of hydrocarbon in place produced through the successful application of an EOR technique, in the excess of the existing conventional process primary or secondary recovery processes of operation - is referred to, technically as the Incremental oil (Olaoye, Taiwo & Olafuyi, 2022).

“Estimating the incremental oil from an EOR process involves integration to evaluate the area under the rate-time curve. Ways of achieving this over the years in the industry” range “from the use of models to graphical procedures. One of the” models “is the Decline curve method and one of the graphical procedures is the Trapezoidal rule. The Decline curve is an effective tool used in predicting well performance and reservoir life based on real-life data”. However, "its adoption in estimating incremental oil” has “shown little success in the past, as its results have shown to be highly erroneous. This is attributed to the challenge of inaccuracy in shifting while; the Trapezoidal rule approximates the integral under a curve with that under a straight line segment”.These challenges have substantially impaired the accuracy of incremental oil predictions obtained from EOR projects (oloyede, olalekan & Taiwo, 2022).

Problem Definition

“The volume of the” incremental oil “as a result of the” enhancing “the reservoir” natural energy (Enhanced oil Recovery) cannot be easily and quickly determined due to the irregular curve from the plot/ graph of reservoir volumetric flow rate against time therefore numerical method such as “Trapezoidal rule and Simpson rule are” employed “to” reduce “the” difficulty in analyzing the volume of the incremental oil. These numerical methods are not 100% accurate i.e. errors are generated. “As such intensive studies and” forecasts “about their outcomes therefore the need for” other types of methods analytical and numerical methods such as cubic spline model formulation and interpolation are used.

Objectives/Aims

“Below are the outlined objectives for the work”:

- “To” provide “the design and analysis of techniques to give approximate but accurate solutions to hard problems” such as analyzing “the” volume “of” incremental oil from the volumetric flow rate against time.
- To show and display the behavior of how the volumetric flow rate changes with time on a plot/graph.
- To compare the accuracy level between different numerical methods used “to determine the volume of” the incremental “oil”.
- “To” compare “the” accuracy level between the same numerical methods with different value for step sizes used to estimate the volume of the additional oil.

“Scope and limitation of” the “work”

“This work is limited to the” analyses most important estimation of incremental oil of a hydrocarbon reservoir undergoing enhanced recovery using numerical methods which are Simpson rule, trapezoidal rule and equation of trend line from Microsoft excel.

“Organization of thesis”

“The thesis is structured in this manner:”
“Chapter two gives a brief evaluation of drive mechanisms”, and “enhanced oil recovery”. “Numerical” methods e.g. Trapezoidal and Simpson rule.
“Chapter three presents the methodology employed in this study; mathematical (numerical)”- trapezoidal “and” Simpson and “computer models formulations”- Microsoft office excel spreadsheet.
“Chapter four contains” a “discussion of the results.”
“Chapter five draws logical conclusions based on the” Numerical (mathematical) “and” computer methods result used, “and makes useful recommendations for further studies”.

“CHAPTER TWO”

“LITERATURE REVIEW”

“Natural” “Drive Mechanism”

“The natural energy of a reservoir can be used to move” hydrocarbons “toward the wellbore”. “These sources of energy are called” *primary* “*drive mechanisms*. Early determination and characterization of the drive mechanism(s) present within a reservoir may allow a greater ultimate recovery of hydrocarbons. Drive mechanisms are determined by the analysis of historical production data, primarily reservoir pressure data and fluid production ratios” such as Water “oil” ratio, gas oil ratio e.t.c.

The four “primary oil reservoir drive mechanisms are solution gas drive, gas cap drive, water drive” and gravity drainage “drive”. “A combination or mixed drive occurs when two or more of the primary drive mechanisms are present in the same reservoir. A combination drive may also occur when one or more of the primary drive mechanisms are assisted by gravity drainage.”

“Table below shows the energy sources and ultimate recovery ranges of the major drive Mechanisms.”

“Table 1 Ultimate recovery ranges by drive mechanism”

“Drive Mechanism”	**“Energy Source”**	**“Recovery (% OOIP)”**
“Solution Gas Drive”	“Evolved Solution Gas Expansion”	“5-30”
“Gas Cap Drive”	“Gas cap And Evolved Solution Gas Expansion”	“20-40”
Gravity Drainage	Gravity	5-30(incremental)

Water Drive	Aquifer Expansion	35-75

“Each reservoir is composed of a unique combination of geometric form, geological rock properties, fluid characteristics, and drive mechanism (primary) with these no two reservoirs are identical in all aspects they can be grouped according to the primary recovery mechanism by which they produce”. “There are basically five driving mechanisms that provide the natural energy necessary for oil recovery”:

“Solution Gas” drives “in a solution (or dissolved) gas drive reservoir, the oil-bearing rock is completely surrounded by impermeable” rocks. “As the reservoir pressure drops during production, expansion of the oil and its dissolved gas provides most of the reservoir's drive energy Additional energy is obtained from the expansion of the rock and its associated water”.

“Depending on its discovery pressure, a solution gas drive reservoir can be initially either” under saturated “or saturated in an” under saturated “reservoir, the reservoir pressure is greater than the” bubble point “of the oil. No free gas exists in the reservoir while the pressure remains above the” bubble point. “The reservoir drive energy is provided only by the limited expansion of the oil, rock, and water. In a saturated reservoir, the reservoir pressure is at the” bubble point. “As soon as oil is produced, the pressure drops and bubbles of solution gas form in the reservoir. This solution gas liberation causes the oil to shrink, but the oil shrinkage is more than offset by solution gas expansion, the primary source of reservoir drive energy below the” bubble point. It is also called depletion drives.

“Solution gas drive reservoirs show characteristic” rapid or quick “changes in reservoir pressure, producing gas-oil ratio, and oil and water production rates during the life of the reservoir. If the reservoir is initially” under saturated, “the reservoir pressure falls quickly during oil production because of the small” compressibility “of oil, water, and rock. Pressure drops of several hundred pounds per square inch can easily occur over a” short period “of” time. “Because the only gas produced is that which evolves from the produced oil in the wellbore, the gas-oil ratio (GOR) remains constant until the reservoir reaches the" bubble point.

"Once reservoir pressure reaches the" bubble point "pressure or if the reservoir was initially saturated, the reservoir pressure declines less quickly due to the large compressibility" and expansivity "of the gas bubbles forming in the reservoir. The producing GOR rises quickly as the bubbles link up and begin to flow and can increase to as much as ten" time "the initial GOR. If reservoir pressure continues to fall, the producing GOR will eventually drop as the gas expands less and less as it flows up the wellbore."

"Oil production rates fall quickly once the producing GOR begins to rise. Wells must be placed on artificial lift early in their life. Initially, little or no water is produced. As reservoir pressure drops, a small amount of water may be produced as the interstitial water saturation increase and exceeds the critical" water saturation "value required for flow".

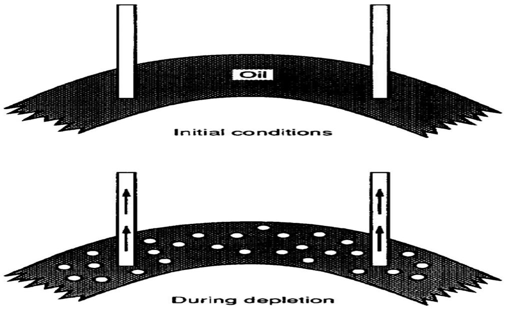


Figure 2: Depletion Drive Reservoir.

"Oil recovery from" depletion "drive reservoirs is usually low, ranging from 5 to 30% of the original oil in place OOIP". Above the bubble point "less than 5% of the original oil in place is" produced. "Better solution gas drive recoveries are obtained in reservoirs with relatively low oil viscosities and fairly homogeneous rock properties. Recovery can sometimes be improved with completion" techniques "that conserve reservoir energy by minimizing the producing gas oil ratio".

“Gas cap drive”

“In” this “reservoir, the primary source of reservoir energy is initial gas” caps, “which” increase in size by expanding “as the reservoir pressure drops. Additional energy is provided by the expansion of solution gas released from the oil”. The “drive contributions provided by the expansion of the rock and its” associates are less significant.

“Gas cap expansion causes reservoir pressure to” decline “slowly in a gas cap drive reservoir than in one producing in a solution gas drive. The rate of pressure decline is closely tied to the relative size of the gas cap to the size of the” oil reservoir, “with larger gas caps to” oil reservoir size ratios “resulting in a more gradual pressure decline as oil is produced.”

“Early in the life of a gas cap drive reservoir, the GOR rises slowly because the higher reservoir pressure keeps more gas in solution in the oil. Later, the GOR increases dramatically as the expanding gas cap reach the highest wells on structure. The GOR continues to climb as the gas-oil contact moves farther down structure and gas cap gas production increases. Oil production rates fall less quickly than in a solution gas drive reservoir due to the slower decline in reservoir pressure. Artificial lift may not be required as early in the field's life since wells tend to have longer flowing lives. As in a solution gas drive, little or no water is produced”.

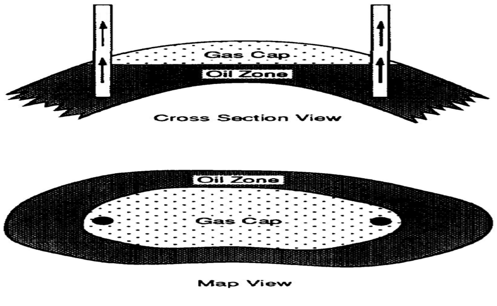


Figure 3: “Gas Cap Drive” Reservoir.

“Oil recovery from gas cap drive reservoirs typically ranges from 20 to 40% of the original oil in place. The actual recovery obtained depends on the size of the initial gas cap, the structural geometry of the reservoir, and the way the field is managed”.

“Gas cap drive recovery increases with the size of the initial gas cap if gas cap gas production can be minimized. This is done most easily in steeply dipping reservoirs or those with thick oil columns which allow the wells to be perforated as far as possible below the gas-oil contact. Recovery can also be improved by shutting-in wells when they begin to produce large amounts of gas cap gas. In addition, the produced gas may be returned to the gas cap using gas injection wells located high on structure.”

“Water drive”

“In a water drive reservoir, the oil zone is in communication with an aquifer that provides the bulk of the reservoir's drive energy. As oil is produced, the water in the aquifer expands and moves into the reservoir, displacing oil. Depending on the aquifer's strength, additional energy may be provided by solution gas expansion. Much less significant contributions are provided by the expansion of the reservoir rock and its associated water.”

“The geometry of the aquifer determines whether it is bottom water or an edge water drive in a bottom water drive, the aquifer is present below the entire reservoir and water influx moves vertically upward into the oil zone. In an edge water drive, the aquifer is located on the flanks of the reservoir and the water moves upward along the reservoir dip.”

In a “water drive”, “the reservoir pressure response to production depends on the size and permeability of the aquifer and the rate at which the reservoir is produced. If the reservoir is produced at a low rate, the aquifer is able to replace the fluid volumes produced and reservoir pressure remains fairly constant. At high production rates, the aquifer is unable to keep up with withdrawals and reservoir pressure drops. If the rate is then reduced to a low level, reservoir pressure will rise. The magnitude of “high” and “low” production rates for a particular water drive reservoir is determined by the size and permeability of its associated aquifer.

In a strong water drive reservoir, the producing GOR remains fairly constant, reflecting the stable reservoir pressure. However, if the aquifer is unable to maintain reservoir pressure, the producing GOR will rise accordingly.

Oil rates remain high under strong water drive until water breaks through in the producing wells. Water production usually occurs early in the field life of down structure wells, and the water-oil ratio (WOR) continues to increase with time as the oil-water contact moves upward. Gas lift may be required for high water cut wells to continue to flow”.

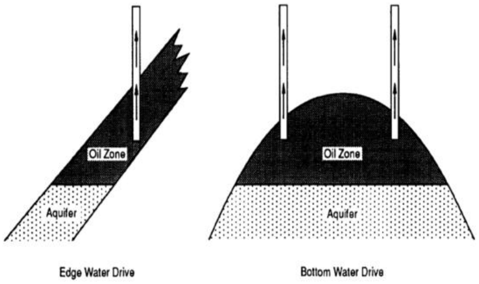


Figure 4: Water Drive Reservoir.

“Oil recovery from water drive reservoirs typically ranges from 35 to 75% of the original oil in place. The actual recovery obtained depends on the strength of the aquifer, the sweep efficiency of the encroaching water, and the way the field is managed”.

“Water drive recovery increases with the strength of the aquifer if water production can be minimized. As with gas cap drive reservoirs, this is done most easily in reservoir geometries that allow wells to be perforated a considerable distance from the fluid contact”.

“Water drive recovery also depends on the aquifer's sweep efficiency. Sweep efficiency is a measure of how effectively the encroaching water displaces oil. Higher sweep efficiencies and oil recoveries occur when the viscosity of the oil is low compared to that of the water and oil flows more easily than the encroaching water. Water drive reservoirs with high viscosity crudes have lower sweep efficiencies and oil recoveries because the water tends to move ahead of or “finger” through the oil, leaving behind” upswept “oil”.

“Water drive recovery can be improved by balancing production rates across the field so that the oil-water contact moves up as uniformly as possible. Since water drive is usually more efficient than solution gas drive, in some cases it is possible to increase recovery by producing the reservoir at a rate low enough that the aquifer is able to maintain a high reservoir pressure.”

“Combination drive”

“Most oil reservoirs produce under the influence of two or more reservoir drive mechanisms, referred to collectively as a combination drive. A common example is an oil reservoir with an initial gas cap and an active water drive”.

The “production trends of a combination drive reservoir reflect the characteristics of the dominant drive mechanism. A reservoir with a small initial gas cap and a weak water drive will behave in a way similar to a solution gas drive reservoir, with rapidly decreasing reservoir pressure and rising GORs. Likewise, a reservoir with a large gas cap and a strong water drive may show very little decline in reservoir pressure while exhibiting steadily increasing GORs and WORs. Evaluation of these production trends is the primary method a reservoir engineer has for determining the drive mechanisms active in a reservoir”.

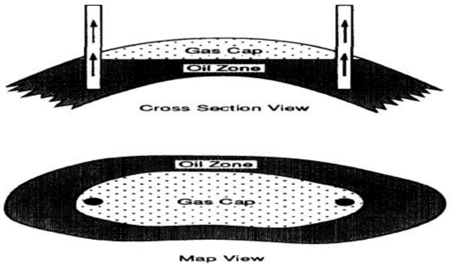


Figure 5: Combination Drive “Reservoir.”

“The ultimate recovery obtained from a combination drive reservoir is a function of the drive mechanisms active in the reservoir. The recovery may be high or low depending on whether displacement or depletion drive mechanisms dominate. Water drive and gas cap expansion are both displacement type drive mechanisms and have relatively high recoveries. Solution gas drive is a depletion type drive and is relatively inefficient.”

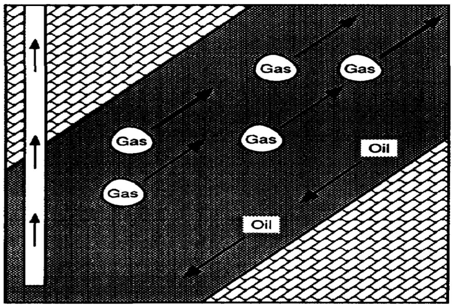
“Recovery from a combination drive reservoir can often be improved by minimizing the effect of depletion drive mechanisms by substituting or augmenting more efficient ones through production rate management or fluid injection. To do this, the drive mechanisms active in a reservoir must be identified early in its life.”

Gravity drainage

“Gravity drainage” occurs when the reservoir fluids gas, oil “and water to segregate in a reservoir during production due to their differing densities as a secondary drive mechanism” also called “gravity” segregation, gravity “drainage occurs only in combination with one or more of the primary oil reservoir drive mechanisms.”

“Conditions conducive to gravity drainage include” high “reservoirs” thickness, “high vertical” permeability “or” low “reservoirs” thickness “with steep dips. In a solution gas drive reservoir perforated down dip, gravity drainage can cause released solution gas to migrate upward and oil to flow downward” due to density difference, “conserving reservoir energy and increasing recovery to near that of a water drive.”

“The” “oil gravity drainage” rate “in the reservoir is usually low compared to field production rates. Over time, however, gravity drainage can be extremely efficient and recoveries higher than any of the primary drive mechanisms are possible.”



“Figure 6: Gravity Drainage Reservoir”

“Enhanced Oil Recovery” Methods

“The process of producing hydrocarbons from the reservoir” to another system likes well by methods other than the conventional methods primary and secondary are “called Enhanced oil recovery (EOR). It also includes re-pressurizing” processes “with gas and water. An” Enhanced oil recovery “method should generate an incremental oil recovery. Incremental oil is designated as oil Produced higher than the projected production from the reservoir” after “the” “enhanced oil recovery processes are carried out”.

Economically, “enhanced” oil “recovery” EOR techniques are advantageous when there is a high level of demand to low level of supply of the commodity which can significantly extend global oil reserves. “Given a broad consensus that we have entered a period of supply constraints, operators can at last plan on the assumption that the oil price is likely to remain relatively high. This coupled with the realization that new giant fields are becoming increasingly difficult to find, is creating the conditions for extensive deployment of” enhanced oil recovery. “This paper provides a comprehensive overview of the nature, status and prospects for” enhanced oil recovery “technologies”. It explains why the average oil recovery factor worldwide is only between 20% and 40%, because one-third of the world oil reserves is light oil and two-third are heavy oil and bitumen which require advance technology for extraction (Ojokwu and Taiwo, 2018). It also “describes the factors that contribute to these low recoveries and indicates which of those factors” enhanced oil recovery “techniques can affect the” extraction of heavy crude oil. The thesis “then summarizes the breadth of EOR processes, the history of their application and their current status. It introduces two new EOR technologies that are beginning to be deployed and which look set to enter mainstream application. Examples of existing EOR projects in the mature oil province of the North Sea are discussed. It concludes by summarizing the future opportunities for the development and deployment of EOR."

"The majority of oil companies today are focusing on optimizing the recovery factor (RF) from their oilfields as well as maintaining an economic oil production rate. This is because it is becoming increasingly difficult" and expensive "to discover new oilfields" and developed new field.

"Most of the sedimentary basins that contain oil have already been explored" (giant field) "and new" basins "discoveries tend to be small. Those basins that remain unexplored are in remote and environmentally sensitive areas of the world (e.g. the Arctic and the Antarctic). Although there are huge volumes of unconventional hydrocarbons, such as the very viscous oils, oil shale"'s, shale "gas and gas hydrates, many of the technologies for exploiting these resources are either very energy intensive e.g. steam injection into heavy oil environmentally sensitive e.g. as seen in recent adverse press coverage of 'fraccing' to recover shale gas or are not yet ready to be applied at scale."

"The average RF from mature" oil fields "around the world is somewhere between 20% and 40"% of the original oil in place. "This contrasts with a typical RF from gas fields of between 80% and 90"% of "the" original gas in place due to the fact that light hydrocarbon are less dense and less viscous than heavy hydrocarbon. At current production rates existing proven oil reserves will last 54 years (Tom Bergin, 2012). "This is probably as good as it has ever been. Improving oil recovery" of oil field "to that of" a typical "gas fields" alternatively allow for increased production rates "could more than double the time for which oil is available" by exploring new oil field. "This would provide more time for an increasingly energy-hungry world to develop alternative energy sources and technologies."

"Crude oil and natural gas are found in large" sub surface "deposits usually termed reservoirs or pools in sedimentary basins around the world". The largest oil reservoir in the world (the Arab

D limestone in Ghawar in Saudi Arabia is approximately 230 km long and 30 km wide and 90 m thick (Abdulkader M, 2005) While most commercially exploited minerals and ores exist as solid rocks and have to be physically dug out of the ground, oil and gas exist as fluids underground. "They occupy the connected pore space within strata of sedimentary rocks, typically sandstones or carbonates."

"Oil and gas are extracted by creating pressure" difference with respect to time and space called pressure gradient "within the reservoir that cause the oil and/or gas to flow through the interconnected pores to one or more production wells. In most oilfields the pressure gradients are maintained by injecting another fluid usually water but sometimes gas and termed 'water flooding' or 'gas flooding', respectively into the reservoir through injection wells" to reduce "the" depletion rate of "the" pressure difference. "The injected water displaces the oil and occupies the pore space that it'"s "originally occupied. By contrast, gas fields are normally exploited simply by reducing the pressure at the production well" the gas in "the" reservoir expands as "the pressure drops" using compressors "and thus flows to the production well" as a result of polytropic expansion nature of hydrocarbon gas.

"Enhanced oil recovery EOR involves injecting a fluid into an oil reservoir that increases oil recovery over that which would be achieved from just pressure maintenance" secondary recovery technique "by water or gas injection. For lighter oils, these processes include miscible gas injection, water alternating gas WAG injection, polymer flooding, flow diversion via polymer gels and the use of surfactants. For more viscous so called heavy oils these processes include steam injection and air injection leading to *in situ* combustion. The majority of EOR processes used today were first proposed in the early 1970s at a time of relatively high oil prices."

"Improved oil recovery IOR is a term that is sometimes used synonymously with EOR although it also applies to improvements in oil recovery achieved via better engineering and project management", it involve production enhancement such as "optimizing artificial lift, increasing well stimulation, reactivating idle wells, changing completion strategy, debottlenecking facilities", upgrading facilities e.t.c, "infill/step-out drilling" such as "recompletion, workover, infill drilling, step-out drilling, deepening" e.t.c, "secondary recovery" such as "water flooding, gas flooding immiscible, enhanced oil recovery" such as "gas flooding"-co_2, hydrocarbon rich "gas", water alternating "gas WAG, microbial MEOR, dilute surfactant/chemical, polymer" and "thermal". "Identifying volumes of oil that have been bypassed during water injection using seismic surveying and then drilling new wells to access those oil pockets. It was first introduced in the late 1980s when the oil price dropped and as a result there was less interest in EOR technologies. At this time there were significant improvements in computer processing speed, computer memory and seismic analysis. Improved computing power enabled engineers to build more complex geological models and thus estimate the effect of reservoir heterogeneity on flow. Improved seismic analysis algorithms combined with more powerful computers meant that engineers and geoscientists could use 'four-dimensional' seismic surveying, involving the comparison of seismic data taken at different times, in combination with reservoir simulation to identify bypassed volumes of oil on the scale of hundreds of" meters "horizontally and tens of" meters "vertically. Developments in measurement while drilling and a new capability to drill deviated, horizontal and multi-lateral wells meant that engineers could target these bypassed accumulations very accurately and drain them."

"Using combinations of traditional EOR and IOR technologies it has been possible to achieve RFs of between 50% and 70% for some oil fields but this is still less than the typical RF for a gas field" equivalent to very high irreducible oil saturation compare to that of gas . It is believed that much of this remaining oil is trapped or bypassed in volumes that cannot be accessed by IOR technologies, on length scales that cannot be resolved by seismic surveying or accessed by drilling new wells (Ann Muggeridge, 2014).

"New and improved EOR processes are needed to access this remaining oil and improve RFs further while maintaining economic oil production rates. This thesis will review existing and emerging EOR technologies, discussing the underlying science, its application and its limitations. In particular, it will focus on recent advances in our understanding of the nature of wettability in rocks and discuss the opportunities arising from the much wider range of polymers that is now available commercially. These two factors have driven a renewed interest in existing EOR technologies and the development of new methods. We will concentrate primarily on the recovery of conventional, light oil from fields that have already been or are about to be developed. The recovery of heavy and unconventional oils will not be discussed in any detail, nor will we consider the recovery of oil by IOR techniques except in passing.”

EOR PROCESS

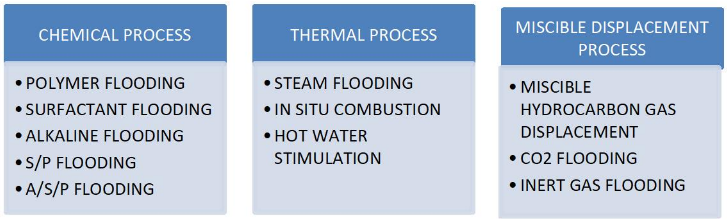


Figure 7: Classification of EOR PROCESS

“They can be classified into the following processes;”

“Miscible Gas Injection Processes”

“A fluid/solvent that dissolves the reservoir oil is introduced into the reservoir such as Alcohol, Refined hydrocarbons, LPG or exhaust gas. This process improve recovery efficiency by reducing Viscosity, condensing and vaporizing gas drive and displacing oil from pore spaces. This process Can further be grouped into” CO_2 “flooding, miscible hydrocarbon displacement and inert gas Flooding.”

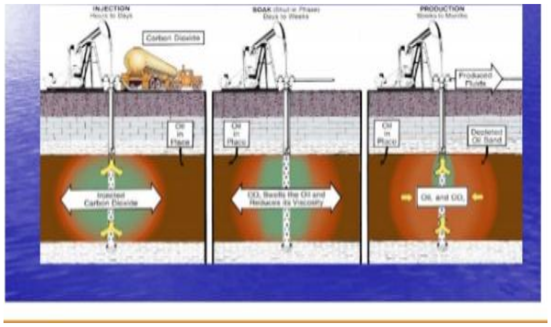


Figure 8: CO_2 flooding process.

CO_2 “flooding consists of injecting large quantities of” CO_2 (“15% or more hydrocarbon pore” volume) “in the reservoir to form a miscible flood”. “In the case of light oils” thermodynamics “miscibility may be achieved at pressure of the order of 140 to 210 bar (2000”-3000psi) “with” very “Viscous oils the miscibility pressure can never be reached”.

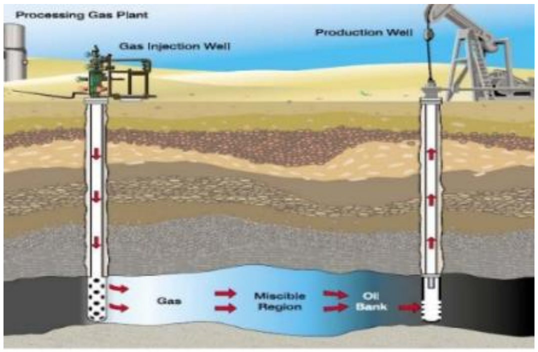


Figure 9: miscible gas injection.

However, the CO_2 “dissolved in the oil has a direct effect on the properties of the mixture, and the viscosity reduction thus obtained is obviously beneficial”. “During displacement of the” CO_2 "within the porous medium there is large contact area between gas and oil. A rapid mass transfer between the oil and” CO_2 "takes place” by fractionation of “the” oil.

Water- Alternating- Gas Injection (WAG): “Alternates slugs of miscible gas and water injection to mobilize the target oil”. Gas rises “and” waterfalls.

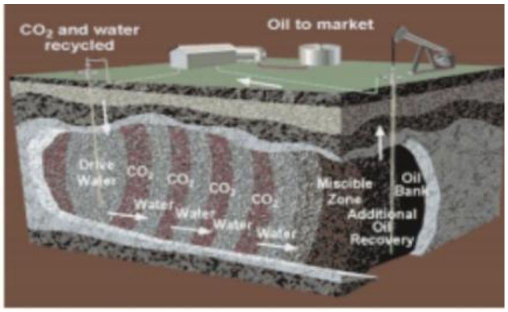


Figure 10: Water alternating Gas Injection.

Advantage of this process is less CO_2 is needed. The problem encountered is density differences between CO_2 and water/oil may causes gas to go up in the formation.

"Chemical Processes"

"Chemicals are injected into the reservoir to alter fluid or rock properties. Candidate reservoir should have adequate injectivity since the injected fluids have lower mobility. Active water drive reservoirs should be avoided as they have potential for low residual oil saturation. Reservoirs with gas caps may not be good candidates for this method because mobilized oil might re- saturate the gas cap. Reservoir formations rich in clay should be avoided because they increase adsorption of the injected chemical. Moderate salinity brine reservoirs are preferable because high salinity concentration interact unfavorably with the injected chemicals. Under this process, there are five different processes namely; Polymer flooding, Surfactant flooding, Alkaline flooding, Surfactant/ Polymer flooding and Alkaline/ Surfactant/ Polymer Flooding".

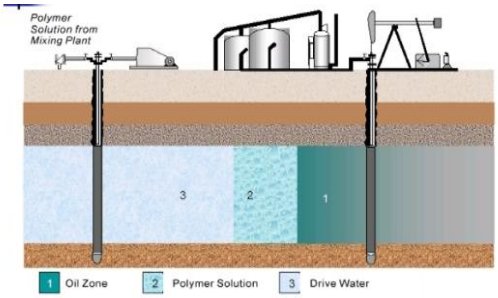


Figure 11: "Polymer flooding process."

“Polymer” flooding “consists of adding water soluble polymers to water” for polymer flooding recovery process carried out after the primary recovery or adding soluble polymer to water flooding in the case of a polymer flooding process carried out after secondary recovery process “before it is injected into the reservoir”. Examples “of” such “polymer” are polyacrylamide and polysaccharides. The function of “the polymer is to” control “the mobility” ratio mobility “of the” dispersing phase “to mobility of the” dispersed phase thereby improves volumetric sweep and smoothing permeability profile efficiency. The purpose of adding surfactant to the dispersing fluid is to reduce the “oil saturation by lowering” the “oil”-water “interfacial tension and” increase “volumetric sweep efficiency by reducing” oil “water mobility ratio.”

Lastly, “surfactant flooding is a well established method of chemical enhanced oil recovery” process . “This method has proven successful as it increases oil recovery through a combination of mechanisms. These include interfacial tension IFT reduction, wettability alteration, foam generation and emulsification”.

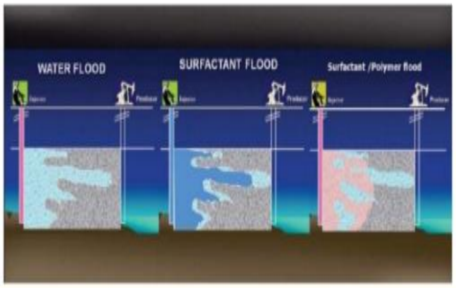


Figure 12: Surfactant polymer flooding.

Surfactant or “surface active agents are” chemicals “substances that adsorb on or concentrate at a surface or fluid/fluid interface when present at low concentrations in a system. They” affect “the interfacial properties” of other material such as the interfacial tension, surface tension and wettability significantly in a degenerative manner. In their chemical “form, surfactants consist of a” hydrocarbon “non” ionic non “polar portion called” the “hydrophobic tail” or lipophilic “tail” moieties which may be cyclic or straight chained in manner and a non hydrocarbon ionic polar portion called the hydrophilic head or lipophobic head moieties. “The most common” surfactant “used in” surfactant “flooding” process “are sulfonated hydrocarbons.”

“Thermal processes”

“Thermal EOR processes are defined to include all processes that supply heat energy” which is generated at the surface or in-situ “to the rocks and fluids contained in a reservoir thereby enhancing the ability of oil (including other fluids) to flow by primarily reducing its viscosity. The heat cause thermal expansion which affects the relative permeability and also sometimes” causes “the activation of solution gas drive”.

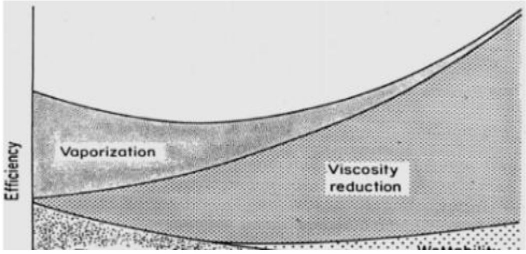


Figure 13: Contributions of the different mechanisms to the EOR by thermal recovery methods.

“The oil caused to flow by the supply of thermal” energy “is produced through” nearby “wells. There are three categories of thermal methods:”

“**Cyclic Steam” Injection (“Steam” Stimulation, “Steam Soak Huff” and “Puff”)**

In “this process, Steam is” injected down “a” producing “well to heat up the area” around “the well bore” and increase “recovery of the oil immediately” adjacent “to the well. After” injection “of short period, the well is placed back” on production.

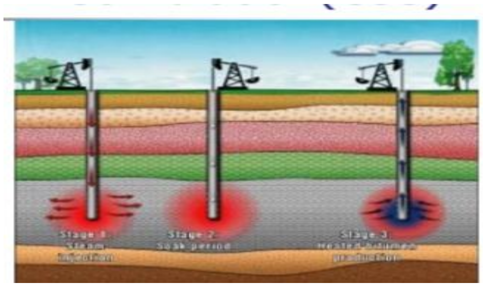


Figure 14: “Cyclic steam Stimulation (CSS) or Huff and Puff”.

“This is essentially a well bore stimulation technique, each well responding independently. This process is repeated until production falls below a profitable level”. “Cyclic steam stimulation is sometimes applied to heavy-oil reservoirs to boost recovery during the primary productions phase. During this time it assists natural reservoir energy by thinning the oil so it will more easily move through the formation to the injection/production wells. It” is divided into three stages: steam injection, steam soaking, Heated oil production. In the steam stimulation “process, steam is injected into the reservoir at rates of the order of” 1000B/“d for a period of weeks; the well is then allowed to flow back and is later pumped. In suitable applications, the production of oil is rapid and the process is efficient, at least in the early cycles”. Stimulation before flooding is almost essential in order “to achieve” flow “communication between” the “injection and production wells”. Communication can be established between pairs of wells by creating a fracture between them. This can be done by injecting steam at a sufficiently high pressure.

“Steam Drive (Steam Flooding, Continuous Steam Injection)”

“Steam is injected through injection wells” continuously “and the oil is displaced” or driven “to” separate “surrounding producing wells as in conventional fluid injection” operation.

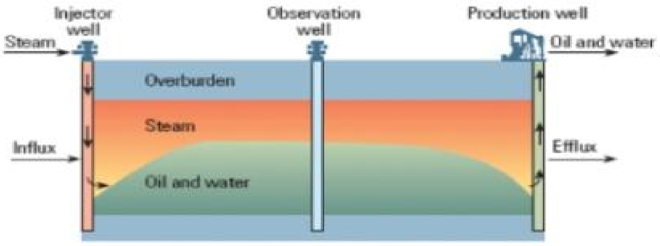


Figure 15: Steam injection process.

“Less viscous crude oils can be steam flooded if they don’t respond to water. This method reduces viscosity; bring about steam distillation and supplies pressure to drive oil to the producing well”.

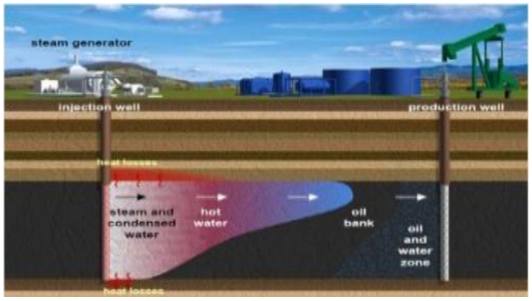


Figure 16: Steam flooding process.

“Steam flooding consists of injecting % quality steam to displace oil. Normal practice is to precede and accompany the steam drive by a cyclic steam stimulation of the producing wells called huff” n “puff”. “Applicable to viscous oils in massive, high permeability sandstones or unconsolidated sands. Oil saturations must be high, and pay zones should be” greater “than” 20ft “thick to minimize heat losses to adjacent formations”.

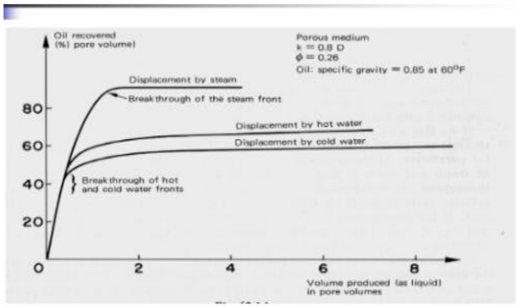


Figure 17: "A" comparison of "Displacement by cold water, Hot water, and steam".

"In-Situ Combustion (Fire-flood)"

"This process involves in-situ combustion of portions of the oil. There are two mechanisms involved namely forward and backward combustion".

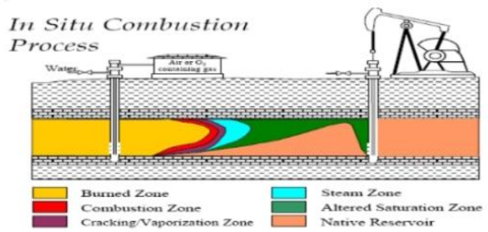


Figure 18: In situ Combustion process.

"Air is Pumped into the reservoir which either self-ignites or is ignited, depending on reservoir Temperature and composition. This heat produced is used to thin the oil and permit it to flow more easily towards the producing wells. Most of these processes are" modeled "due to its capital involvement thus the need for reservoir Simulation to help predict the outcome of such processes". In situ combustion is considered because of availability of air, Reduced water requirement compared to steam, "Applicable to a wide range of reservoirs and" fluid characteristics, No theoretical pressure limitation, can be applied to deep reservoirs where lifting costs make water flood unattractive, "can be" applied "as a follow-up to steam"-based "processes", lack of obvious alternatives.

“Heavy Oil recovery methods”

“These methods include:”

“Steam Assisted Gravity Drive SAGD”

“It’s an advanced form of steam Stimulation Involving a pair of horizontal wells, one 4 to 6 meters above the other. The upper one is used for injecting high pressure steam, which heats the oil and reduces its viscosity”.

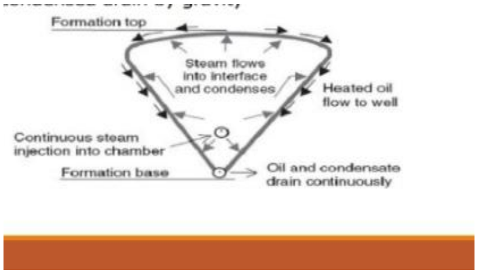


Figure 19: Diagram of flow movement of injected steam and heated oil.

“The Heated oil drains into the lower wellbore and is produced from there” as shown above. Using two parallel horizontal well steam injected into upper and form a steam chamber reduces oil viscosity. Steam condenses at interface, oil condenses drain by gravity.

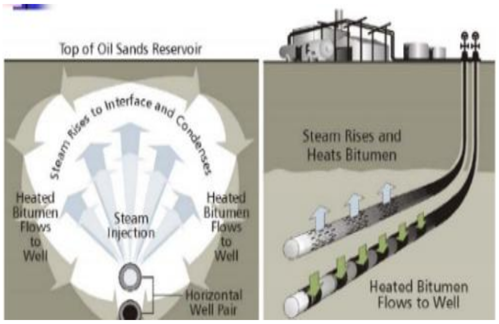


Figure 20: “Steam Assisted Gravity Drainage process.”

“The” use “of the” SAGD “process” can provide an increase in the recovery of about 50% or more which is significantly better than the recovery of 15% which is achieved using steam stimulation process.

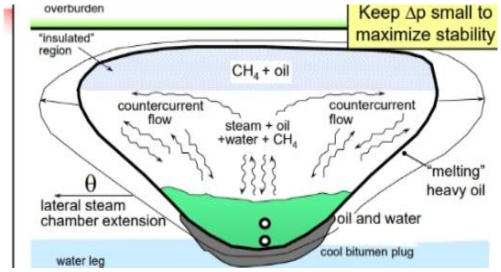


Figure 21: Steam Assisted Gravity Drainage physics.

Successful demonstration of the SAGD process has been carried out by AOSTRA in its Underground Test Facility in Athabasca. This pilot facility employs horizontal steam injectors located parallel to and closely above the horizontal producers. The use of horizontal wells is required for “the economic” application “of the” SAGD principle “to the” production “of heavy oil” and bitumen. This potential application that encouraged imperial oil to build the first Canadian “horizontal well in the cold lake oil sands” in 1978. When the process is used to produce conventional heavy oils as distinct from bitumen, there is more flexibility in locating the injector. As the steam chamber grows upwards, it usually encounters the top of the reservoir waiting a year or two and then the chamber spreads sideways.

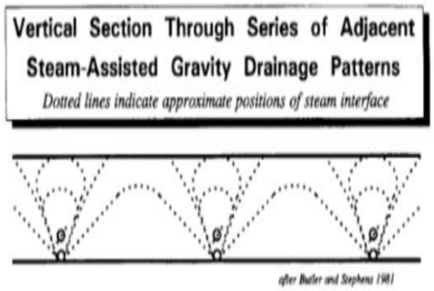


Figure 22: Steam Assisted Gravity Drainage patterns.

“Toe-to-Heel Air Injection THAI”

It “is a proposed method of recovery that combines a vertical air injection well with a horizontal production well”.

RECENT TECHNOLOGIES NON THERMAL METHOD

VAPEX process: VAPEX is an acronym of Vapour Extraction or Vapour Assisted Petroleum Extraction. A non- thermal and immiscible process “new emerging technology for extraction of heavy oil. Founded in 1989 by Butler and Mokrys, Just one field pilot in Northwest Alberta, DOVAP”.

In this new concept VAPEX, “light hydrocarbon (low molecular weight) vapors at a pressure close to their dew points are injected into the reservoir using” an “injection well.”

“Hydrocarbon vapor diffuses and dissolves in the bitumen or heavy oil and reduces the viscosity. The diluted and upgraded oil drains by” its “gravity to a production well”.

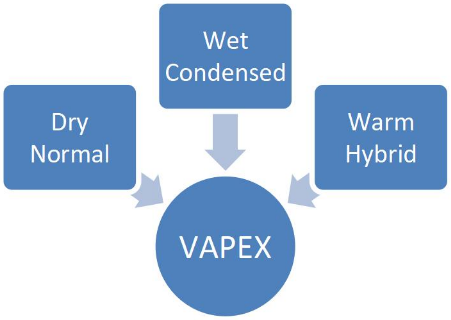


Figure 23: Vapex Methods.

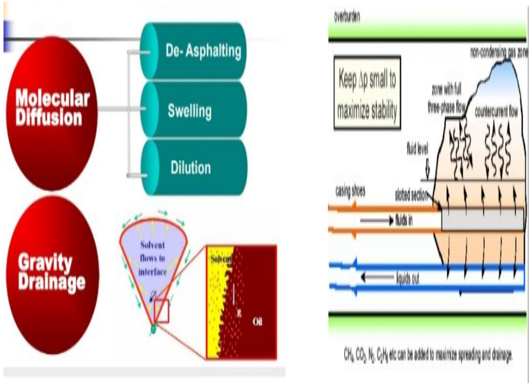


Figure 24: Vapex Mechanism.

"WHY IS RECOVERY LOW?"

"Water flooding is currently the preferred recovery technique for most reservoirs because of" its cheapness, "the higher sustained oil production rates, and the overall higher" Recovery Factor, "that are obtained compared with the case if water were not injected. Oil production without injection is often termed primary recovery. This is because the first wells drilled in a field development are typically production wells to enable oil production and thus the start of" production and "income from a field. Where reservoir pressure is well above the bubble point, primary production can be continued for some time before additional pressure support is required to prevent gas coming out of solution in the reservoir" (two phase zone).

"During depletion, oil flows" from "the" reservoir through the "production wells to the surface because the pressure at the base of the well exceeds that exerted by the hydrostatic head of the column of oil in the well" (pressure differential). "Initially, this occurs naturally but over time the oil rate tends to decrease as the reservoir pressure decreases. In the absence of water injection, pumping may be used to maintain oil" production "rate at economic levels" due to "pressure" maintenance. "If reservoir pressure falls below the oil bubble point pressure, gas that was initially dissolved in the oil will come out of solution and, because it has a much lower viscosity" and low density "will flow preferentially to the production well. At the same time the viscosity of the remaining oil increases" because the dissolved or internal gas are extracted out forming an external phase, “reducing its mobility” further “and increasing the” heaviness “of the oil.” “This will reduce the oil production rate further (although it may increase the total (oil plus gas) production rate through reducing the hydrostatic head in the well). Water (or gas) injection is usually applied before this happens so as to maintain reservoir pressure above the” two phase “bubble point” envelope. “For this reason, it is sometimes known as secondary recovery.”

“Water flooding is relatively cheap, especially for offshore fields because of the ready availability of seawater, although care has to be taken to ensure that the injected water does not result in unwanted, adverse reactions in the reservoir” because of presence of impurities or external materials. “In some cases injected brines may react with the naturally occurring water in the reservoir (termed connate water) to form scale while injecting very pure water rather than brine may result in clay swelling. Both of these may block the rock pores and reduce the rock permeability. The cost of drilling additional wells for injection is more than outweighed by the increased oil rates that result. Re-injection of gas (produced along with the oil) is used” in the same initial injection well “when there is no easy, economic way to export it for sale.”

“The factors affecting RF from water flooding (and gas flooding) can be understood by considering the following approximate relationship:”

“where (i) RF is the recovery factor which is defined as the” ratio “of” the volume of “oil recovered over the volume of oil initially in place (OIIP), both measured at surface conditions. (ii) E_{PS} is the microscopic displacement efficiency. This describes the fraction of oil displaced from the pores by the injected water, in those pores which are contacted by the water. (iii) E_S is the macroscopic sweep efficiency the proportion of the connected reservoir volume that is swept by the injected fluid(s). This is principally affected by heterogeneity in rock permeability and by gravitational segregation of the fluids. (iv) E_D is the connected volume factor the proportion of the reservoir volume connected to wells” to “the total reservoir volume. This represents the fact that sealing faults or other low-permeability barriers may result in compartments of oil that are not in pressure communication with the rest of the reservoir. (v) E_C is the economic efficiency factor, representing the physical and commercial constraints on field life such as facilities life”, surface facilities integrity, “capacity to deal with produced gas and water, reservoir energy (the reservoir pressure may become so low that fluids cannot be produced).”

“It can even be seen that if each of the efficiency factors”, “microscopic displacement efficiency”, area “sweep efficiency” and “vertical sweep efficiency” are 90%, 70% and 80% respectively then “the overall Recovery factor is only” 50% “which is” a products of the three efficiency factor. Increasing RF more significantly “therefore requires each of these factors to be increased to close to 100%.”

“EOR methods are targeted at increasing” microscopic displacement efficiency “E_{PS} and” macroscopic sweep efficiency “E_S while IOR methods also aim to increase” connected volume factor “E_D and to some extent” macroscopic sweep efficiency “E_S. Improving” economic efficiency factor “E_C is mainly the role of the production and facilities engineers but is also affected by EOR methods if these reduce the amount of water and gas produced alongside the oil, enabling oil to be produced for longer before economic limits are reached.”

“Factors influencing microscopic displacement efficiency”

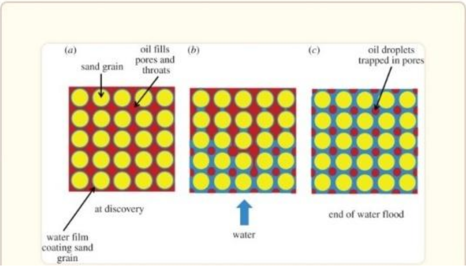
“The typical microscopic displacement efficiency from a water flood is 70% or less. This is mainly because oil ganglia become trapped in the pore space” by fluid rock “capillary effects” (surface effects) and fluid fluid capillary effects (interfacial effects) as “the fluid flow” from “the reservoir to the well” “but E_{PS} is also affected by the relative permeability characteristics of the rock, which control the relative mobility of the oil and water when moving through the pore space.”

“The importance of pore-scale capillary effects on a displacement can be quantified by the capillary number

C_a=”$\frac{v\mu}{\gamma}$………equation 1

“Where v is the interstitial velocity, μ is the fluid viscosity and γ is the interfacial tension (IFT) between the displaced and displacing fluid. When” capillary number “C_a is” less than “10^{-5} flow is dominated by capillary effects and, in particular, capillary trapping is likely to occur. The typical interstitial velocity in an oilfield displacement distant from the wells is approximately 10^{-5} m s^{-1} while the viscosity of typical light oil is similar to that of water” approximately “10^{-3} Pa s. The IFT between brine and oil is approximately 70 mN m^{-1} so for a typical water flood the capillary number is approximately 2.5×10^{-7}. It is generally not possible to apply a sufficiently large pressure gradient between wells to significantly increase the interstitial velocity or to maintain this velocity while injecting a high-viscosity fluid, thus the only way for a reservoir engineer to increase the capillary number is to reduce the IFT. Based on the above calculations this means that the IFT between the oil and the displacing fluid has to be less than approximately 0.1 mN m^{-1} in order to minimize capillary trapping.

Both capillary and relative permeability effects are also influenced by the wetting” behavior “of the rock in which the oil is found. As discussed in the previous paragraph, if the rock is water wet then there is higher residual oil saturation, the proportion of oil which remains permanently trapped by capillary effects” to “the” rock at the “pore scale is” high. This is “caused by the growth in the water film on the rock surface during water injection, which ultimately leads to water bridging at the pore throats so-called snap-off trapping. As a result, little oil is produced after water breakthrough at the production well unless a higher pressure drop is applied which is impractical in most cases.”



“Figure” 25: “Oil trapping in water wet rock.”

“ (a) At discovery the sand grains are coated with a thin water film and the pores are filled with oil; (b) as water flooding progresses the water films become thicker until (c) the water films join and oil continuity is lost”

“If the rock is oil wet, then the proportion of oil trapped by capillary effects is much lower, as oil continuity is maintained over the rock surfaces and through the pore throats, but water breakthrough is earlier and there is a long period of time during which oil and water are Produced simultaneously. The net result is that overall recovery is generally higher if the reservoir rock is oil wet but only after a very large throughput of water.”

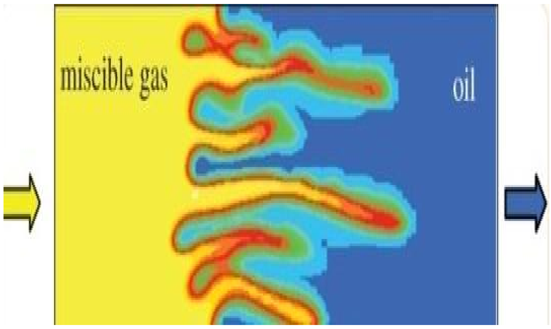
“Most oil reservoir rocks are thought to have a heterogeneous wettability, usually termed mixed wettability, in that larger pores and throats have both water and oil wet surfaces but smaller pores remain mainly water wet. It is believed that the reservoir rock changes from an initially water wet state to this mixed wettability state after the migration of oil into the reservoir.” Ionic “compounds in the oil alter the wettability of the rock by a range of interactions including precipitation of asphaltenes, acid base interactions and ion binding between charged sites on the pore walls and polar hydrocarbon moieties involving higher valency ions in the water that shares the” pores “space with the oil. The wettability of reservoir rock thus depends upon its mineralogy, the crude oil composition, the connate water composition and the pore size distribution.”

“During Water flooding in a mixed wettability rock oil and water drain simultaneously through the pore space, snap off is reduced as most throats have both oil and water wet surfaces and thus there is less capillary trapping of oil. This” simultaneously “drainage of water and oil through the pore space behind the water front combined with the lower residual oil saturation means that more oil is recovered than when the rock is either water or oil wet”

“Increasing microscopic displacement efficiency depends upon finding ways to reduce capillary effects, by reducing the oil water (or gas) IFT, and modify the rock wettability to the optimum mixed wettability state.”

“Factors influencing macroscopic sweep efficiency”

“The macroscopic sweep efficiency of” flooding process “is principally affected by the *geological heterogeneity* in the reservoir, which controls the spatial” or directional “distribution of” permeability “and” porosity. “Rock permeability is dependent on the number, size and connectivity of the pores in the rock. The permeability of a typical reservoir rock is approximately $10"^{-13}m^2$. “A very good reservoir rock might have permeability as high as 10^{-11} m^2 while a permeability of $10"^{-15}m^2$ “would be considered very poor. It is controlled by the size of the sediment grains from which the rock was formed, their packing and the subsequent” digenesis (“chemical alteration) and cementation (mineral deposition) around those grains. The patterns of grains forming a sedimentary rock depend upon the depositional environment in which the original sediments were formed. These result in permeability heterogeneities with” length scales “from” millimeters “to” kilometers.



“Figure” 26: Displacement of oil by miscible Gas Injection

“Higher permeability channels or layers often described as thief zones through the rock are one common adverse” effects “of geological heterogeneity. The injected water flows” prefentially “through these zones, bypassing volumes of oil contained in the lower permeability portions of the reservoir. This results in early water production along with the oil and reduced” Recovery factor.

“A particular problem is that the distribution of permeability in a reservoir is usually very uncertain” due “to” its anisotropy manner. “It is possible to infer the general” properties “of the heterogeneity from the depositional environment and sometimes to correlate specific rock layers between wells, but there is virtually no information about the detailed permeability distribution on smaller" length scales. "This means that statistical approaches, often based on limited numbers of realizations of the possible reservoir heterogeneity, are needed when attempting to predict reservoir performance."

"The effect of geological heterogeneity is exacerbated if the injected fluid has a much lower viscosity than the oil, as is the case when gas is injected instead of water. This effect is characterized by the mobility ratio M, which compares the mobility of the" "displaced fluid" or phase "and displacing fluid" or phase "in the porous medium".

$M = \frac{\mu_{s} k_{r\,D(S_{or})}}{\mu_D K_r D(S_{wc})}$ ……………………………………………………………………………………………………equation 2

Where $k_{rD}(s_{or})$ "is the relative permeability of the porous medium to the displacing phase at the residual oil saturation" s_{or}, $k_{rD}(s_{wc})$ "is the relative permeability of the oil to the displacing phase at the immovable" or connate "water saturation" s_{wc} "and μ is the viscosity of the fluid. This is derived from Darcy equation. The viscosity component of this equation is usually dominant. Even in a homogeneous reservoir, the macroscopic sweep" efficiency "will be reduced when" mobility ratio "M" is greater than 1 due to unstable viscous fingering "where fingers of the displacing fluid develop along the gas oil" interfaces, "rather than the more efficient even contact zone." A typical oil water viscosity ratio is about 2 while a typical gas oil viscosity ratio is about 20 (Muggeridge A, 2012). In most cases, it is channeling "caused by the reservoir heterogeneity rather than viscous fingering that dominates macroscopic sweep" efficiency.

"Macroscopic sweep" efficiency "may also be affected by gravitational segregation but this is more often observed in gas oil rather than water oil displacements because of the higher density" contrasts "between gas and oil. The gas tends to rise above the oil because of its low density and then flow rapidly along the top of the reservoir in an unstable gravity tongue because of its low viscosity. This can result in very early gas breakthrough" the "poor vertical sweep efficiency."

“Improving the macroscopic sweep efficiency depends upon finding techniques that minimizes the impact of geological heterogeneity” difference in the geological properties of the reservoir rock. “This is usually achieved by a mixture of viscosity modification of the injected fluid and/or flow diversion in which the water is diverted from the higher permeability zones in the reservoir into the lower permeability rock still containing displaceable oil.”

“In gas floods, it is also important to minimize gravitational segregation” that is “the difference between the” density of the “displacing fluid and the displaced fluid in the” case of gas flood which are gas and oil respectively. It also makes it possible to produce “heavy oil”, tar “and oil sands that cannot be produced by conventional methods. Applications of” heat energy enhanced oil recovery “methods such as thermal process, can improve oil recovery from these types of reservoirs”.

Numerical Methods

“Numerical methods are techniques by which mathematical problems are re-formulated so that they can be solved with arithmetic operations. Although there are many kinds of numerical methods, they have one common characteristic, they are invariably involving large numbers of tedious arithmetic calculations”.

There “are” “several reasons why you should study numerical methods:”

- “Numerical methods are extremely powerful” mathematical “problem solving tools. They are capable of handling large systems of equations” with more than one variable, non linear equation and expression, and complicated geometrics in engineering and non-“engineering practice and that are often impossible to solve analytically. As such, they greatly enhance your problem solving skills” faster and easier.
- “Many problems cannot be approached using canned programs. If you are conversant with numerical methods and” a good user of computer machines “at computer

programming, you can design your own programs to solve problems without having to buy or commission expensive software" with canned programs.

- "Numerical methods are an efficient vehicle for learning to use computers. It is well known that an effective way to learn programming is to actually write computer programs. Because numerical methods are for the most part designed for implementation on computers, they are ideal for this purpose. Further, they are especially well-suited to illustrate the power and the limitations of computers. When you successfully implement numerical methods on a computer and then apply them to solve otherwise intractable problems, you will be provided with a dramatic demonstration of how computers can serve your professional development. At the same time, you will also learn to acknowledge and control the errors of approximation that are part and parcel of large scale numerical calculations".
- "Numerical methods provide a vehicle for you to reinforce your understanding of mathematics. Because one function of numerical methods is to reduce higher mathematics to basic arithmetic operations, they get at the "nuts and bolts" of some otherwise obscure topics. Enhanced understanding and insight can result from this alternative perspective".
- Numerical methods use high-speed and high level language "computers to solve the mathematical equations describing the physical behavior of the processes in a reservoir to obtain a numerical solution to the reservoir behavior" in lieu to "field" integration and or differential equation to yield "analytical solution. However, when analytical solutions breakdown, simple approximate methods numerical solutions are employed. There are three methods available for discretization process of converting partial differential equation, PDE into algebraic equations; the Taylor series method, the integral method and the variation method (Abou-Kassem et al, 2006").
- Numerical analysis, one "of" the branches "of mathematics and computer science that creates, analyzes, and implements algorithms for obtaining numerical solutions to problems involving continuous variables. Such problems arise throughout the" engineering, "natural sciences, social sciences, medicine, and business. Since the mid

20th century, the” drastically increase “in availability of digital computers” and growth of power supply, “has led to an increasing use of realistic mathematical models in science and engineering, and numerical analysis of increasing sophistication is needed to solve these more detailed models of the world.”

- “The formal academic area of numerical analysis ranges from quite theoretical mathematical studies to computer science issues. With the increasing availability of computers, the new discipline of scientific computing, or computational science, emerged during the 1980s and 1990s. The discipline combines numerical analysis, symbolic mathematical computations”, symbolic mathematical interpretations, “computer” graphic designs, “and other areas of computer science to make it easier to set up”, look into “and” understand, solve, “interpret” and present “complicated” and difficult “mathematical models of the real world” phenomenon “in” a simpler, easier and faster ways.

“Common perspectives in numerical analysis”

“Numerical analysis is concerned with all” areas and “aspects of the numerical solution of a problem, from the theoretical development”, understanding “and” interpretation “of numerical methods to their practical implementation as reliable and efficient computer” canned and non canned “programs. Most numerical analysts specialize in small” sub fields, “but they share some common concerns, perspectives, and mathematical methods of analysis. These include the following:”

- “When presented with a problem that cannot be solved directly, they try to” reformulate “it” to “a “nearby problem” that can be solved more easily. Examples are the use of” numerical “interpolation in developing numerical integration methods and root-finding methods” to solve mathematical problems.

- “Widespread use of the language and results of linear algebra”, functional “analysis, and” real “analysis with its simplifying notation of norms, vector spaces and operators.”

- “There is a fundamental concern with error, its” magnitude, “and its analytic form. When a problem is” approximated, “it is” perceptive “to understand the nature of the error in the computed solution. Moreover, understanding the form of the error allows creation of extrapolation processes to improve the convergence” behavior “of the numerical method.”

- “Numerical analysts are concerned with stability, a concept referring to the sensitivity of the solution of a problem to small changes in the data or the parameters of the problem. Consider the following example. The polynomial $p(x) = (x - 1)(x - 2)(x - 3)(x - 4)(x - 5)(x - 6)(x - 7$”$)$……………………………………………………………………equation 3
  “or expanded, $p(x) = x^7 - 28x^6 + 322x^5 - 1{,}960x^4 - 6{,}769x^3 - 13{,}132x^2 + 13{,}068x - 5$”,040………………………………………………………………………………………………….equation 4
  has “roots that are very sensitive to small changes in the coefficients. If the coefficient of x^6 is changed to −28.002, then the original roots 5 and 6 are perturbed to the complex numbers 5.459 0.540i a very significant change in values. Such a polynomial $p(x)$ is called unstable or ill-conditioned with respect to the root-finding problem. Numerical methods for solving problems should be no more sensitive to changes in the data than the original problem to be solved. Moreover, the formulation of the original problem should be stable or well-conditioned.

- Numerical analysts are very interested in the effects of using finite precision computer arithmetic. This is especially important in numerical linear algebra, as large problems contain many rounding errors.”

- “Numerical analysts are generally interested in measuring the efficiency or “cost” of an algorithm. For example, the use of Gaussian elimination to solve a linear system $Ax = b$”…………………………………………………………………………………………equation 5
  “containing n equations will require approximately $2n^3/3$ arithmetic operations.

Numerical analysts would want to know how this method compares with other methods for solving the problem. The following is a rough categorization of the mathematical theory underlying numerical analysis, keeping in mind that there is often a great deal of overlap between the listed areas."

"Numerical linear and nonlinear algebra"

"Many problems in applied" and industrial "mathematics" involves "solving systems of linear equations, with the linear system occurring naturally in some cases and as a part of the solution process in other cases. Linear systems are usually written using matrix vector notation" such as: "*Ax* = *b* with *A* the matrix of coefficients for the system" example $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{33} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$, *x* the "column vector" quantity "of the unknown variables $x_1,\ldots, x_n$" for the system example $x_{1i}+x_{1j}+x_{1k}$, and *b* a given column vector which implies that the product of a matrix and vector give rise to a "vector . Solving linear systems with up to a" thousand "variables is now considered relatively straightforward in most cases. For small to" moderately "sized linear systems, *n"* is less than or equal to one thousand, the most widely used "numerical method is Gaussian elimination and its variants; this is simply a precisely stated algorithmic variant of the method of elimination of variables that is introduced in elementary algebra. For larger linear systems, there is a variety of approaches depending on the structure of the coefficient matrix *A*. Direct methods" leads "to a theoretically exact solution *x* in a finite number of steps" or discrete sizes, "with Gaussian elimination the best known example. In practice, there are errors in the computed value of *x* due to rounding" and approximation "errors in the computation, arising from the finite length of numbers in standard computer arithmetic. Iterative methods are approximate methods that create a sequence of approximating solutions of increasing accuracy".

Non linear "problems are" always solved "numerically by reducing them to a sequence of linear problems. As a simple but important example, consider the problem of solving a nonlinear

equation $f(x) = 0$”……………………………………………………………………………………………………equation 6
“Approximate the graph of $y = f(x$”)………………………………………………………………………equation 7
“by the tangent line at a point $x^{(0)}$ near the desired root use of parentheses is a common notational convention to distinguish successive iterations from exponentiation, and use the root of the tangent line to approximate the root of the original nonlinear function $f(x)$. This leads to Newton’s iterative method for finding successively better approximations to the desired root: $x^{(k+1)} = x^{(k)} - f(x^{(k)})/f'(x^{(k)}$”)……………………………………………………………………….equation 8
“ $k = 0, 1, 2$”, ……………………………………………………………………………………………………equation 9
“where $f'(x)$ indicates the first derivative of the original function.”

“This generalizes to handling systems of nonlinear equations. Let $\mathbf{f}(\mathbf{x}) = 0$ denote a system of n nonlinear equations in n unknowns $x = (x_1,\ldots, x_n)$. Newton’s method for solving this system is given by $x^{(k+1)} = x^{(k)} + \delta^{(k)} f'(x^{(k)})\delta^{(k)} = -f(x^{(k)}$ ”)…………………………………………………………equation 10
“k $= 0, 1, 2,\ldots$ In this, $f'(x)$ is a generalization of the derivative known as the Jacobian matrix of $f(x)$, and the second equation is a linear system of order n. There are numerous other approaches to solving” non linear “systems, most based on using some type of approximation involving linear functions.”

“An important class of problems occurs under optimization” as “a” case study. “Given a real valued function $f(x)$ with x” as “a vector” parameter “of” an unknown’s quantity, “a value of x that minimizes $f(x)$ is sought. In some cases x is allowed to vary freely, and in other cases there are constraints on x. Such problems occur frequently in” engineering and technology, “business applications.”

“**Approximation theory”**

“This category includes the approximation of functions with simpler or more tractable functions and methods based on using such approximations. When evaluating a function $f(x)$ with x a real or complex number, it must be kept in mind that a computer or calculator can only do a finite number of operations. Moreover, these operations are the basic arithmetic operations

of addition, subtraction, multiplication, and division, together with comparison operations such as determining whether x" variable "is" greater than y variable "is true or false. With the four basic arithmetic operations, it is possible to evaluate" higher order "polynomials $p(x)$
$= a_0 + a_1x + a_2x^2 + \cdots + a_nx^n$ "……………………………………………………………………………………equation 11
"as well as rational functions polynomials divided by polynomials. By including the comparison operations, it is possible to evaluate different polynomials or rational functions on different sets of real numbers x. The evaluation of all other functions e.g. $f(x)$ = Square root of" 2 √"x or 2^x must be reduced to the evaluation of a polynomial or rational function that approximates the given function with sufficient accuracy. All function evaluations on calculators and computers are accomplished in this manner. One common method of approximation is known as interpolation. Consider a set of points (x_i,y_i) where $i = 0, 1, \ldots, n$, and then find a polynomial that satisfies $p(x_i) = y_i$ for all $i = 0, 1, \ldots, n$."

"The polynomial $p(x)$ is said to interpolate the given data points. Interpolation can be performed with functions other than polynomials although" they "are" mostly performed on polynomials equation which is "most common, with important cases being" trigonometric polynomials, "rational functions" and spine "functions made by connecting several polynomial functions at their" end points "they are commonly used in statistics and computer" graphic designs.

"Interpolation has a number of applications. If a function $f(x)$ is known only at a discrete set of data points $x_0, \ldots, x_n$, with $y_i = f(x_i)$, then interpolation can be used to extend the definition to nearby points x. If n is at all large" that is greater than one thousands, spine "functions are generally preferable to simple polynomials."

"Most numerical methods" used "for" approximating "the" value "of" an "integrals and derivatives of a given function $f(x)$ are based on interpolation" and changing the form the equation. For example begin by constructing, re formulating and/or "interpolating function $p(x)$ often a polynomial that approximates" the exact value of the function "$f(x)$ and then integrate or differentiate $p(x)$ to approximate the corresponding integral or derivative of $f(x)$."

“Solving differential and integral equations”

“Most mathematical models used in the natural sciences and engineering are based on ordinary differential equations differential” equation with just one variable, partial differential equations with two or more variables, “and integral equations. Numerical methods for solving these equations are primarily of two” categories. “The first type approximates the unknown function in the equation by a simpler function, often a polynomial or piecewise polynomial” spine “function, chosen to closely follow the original equation. The finite element method discussed above is the best known approach of this type. The second type of numerical method approximates the equation of interest, usually by approximating the derivatives or integrals in the equation. The approximating equation has a solution at a discrete set of points, and this solution approximates that of the original equation. Such numerical procedures are often called finite difference methods. Most initial value problems for ordinary differential equations and partial differential equations are solved in this way. Numerical methods for solving differential and integral equations often involve both approximation theory and the solution quit large linear and nonlinear systems of equations.”

CHAPTER THREE

METHODS USED

Trapezoidal “Rule”

“Many applications of calculus involve definite integrals” those integrals with boundary conditions upper and lower boundary. If we can solve for the anti derivative of an “integrand, then we can evaluate the integral fairly easily” in a quick manner. When this process is impossible, “we turn to numerical methods. The numerical method we will discuss here is called the trapezoid rule. Although we often can carry out the calculations by hand, the method is most effective” and more accurate “with the use of a computer or programmable calculator. But at the moment let us not concern ourselves with these details”. Firstly, “we will describe” this “method, and then consider ways to” make use of it and implement it to determine an approximate value called the ideal value close to the exact value of the integral under consideration.

“The trapezoidal is the first of Newton-Cotes integration formulae. Trapezoidal rule is an already established” and proofed “numerical integration formula which involves the use of the integral under a straight line to approximate the integral under a curve with two or more points. The respective mathematical model for the rule is clearly outlined below.”

“I” $=\int_a^b f(x)dx$……………………………………………………………………………………equation 12

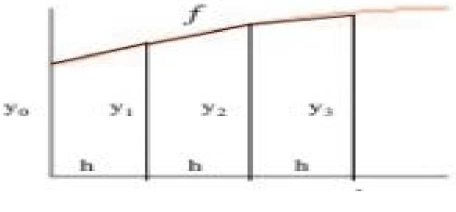
For a single strip (n=two points)

$I = \frac{h}{2}\{f(x_0)+f(x_1)\} - \frac{h^3}{12} \mathrm{n} f'' x$…………………………………………………………………equation 13

For multiple strips (n > two points)

$I =$“ $\frac{h}{2}\{f\ (x_0) + 2\ (f\ (x1) + f\ (x2) + f\ (x3) + \cdots + f($” $xn-1)$ +“f” $(xn)\} - \frac{h_3}{12}$ “f” ” x…………equation 14

$Error = -\frac{h^3}{12} nf'' x$……………………………………………………………………………………....equation 15



a =x_o x_1 x_2 x_3=b

Figure 27: Variation of a non linear function f(x)

Where,

$f(x_0)$=y_0……………………………………………………………………………………………………equation 16

$f(x_1)$ = y_1……………………………………………………………………………………………………equation 17

$f(x_2) = y_2$……………………………………………………………………………………………………equation 18

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$f(x_n) = y_n$……………………………………………………………………………………………………equation 19

I = Area under the graph ………………………………………………………………………………equation 20

“The general idea is to use trapezoids” consisting “of rectangles” and trapezium which can be further divided into triangles and rectangles “instead of” ordinary “rectangles to approximate the area under the graph of a function. Working on the interval [a, b], we subdivide it into n subintervals of equal width h” =$\frac{b-a}{n}$…………………………………………………………………equation 21
“This gives rise to the partition a = $x_0 \leq x_1 \leq x_2$”≤ ⋯ ≤ x_n = “b”………………………………equation 22
“where for each j”, x_j = “a” +j_h………………………………………………………………………equation 23
0 ≤ j ≤ n. Moreover, we let y_j = f (x_j)…………………………………………………………………equation 24
“0 ≤ j ≤ n. That is, the vertical edges go from the x-axis to the graph of the” integral equation 12. From “the sketch above where we have shown a finite number of” sub intervals. “If we are going

to use trapezoids instead of rectangles as our basic area elements, then we have to have a formula for the area of a trapezoid".

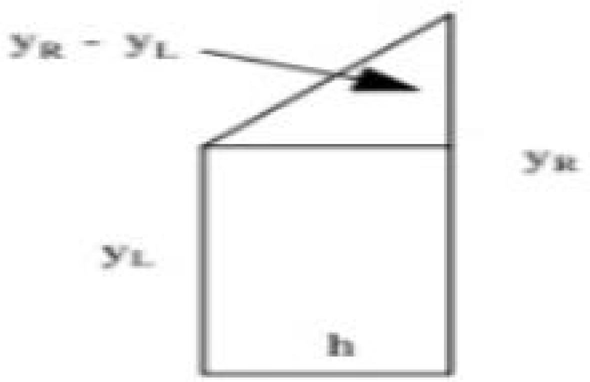


Figure 28: Trapezoid consists of Rectangle and Triangle

"With reference to the sketch above, the area of a trapezoid consists of the area of the rectangle plus the area of the triangle, or h" $(y_L) + \frac{h}{2}(y_R - y_L) = \frac{h}{2}(y_L + y_R)$……equation 25

"So, the area is h times the average of the lengths of the two vertical edges. Now, we return to the original problem of" solving for "the definite integral of a function f"(x) bounded "on the interval [a, b].Trapezoid rule" is defined "as follows:"

The n-subinterval trapezoid approximation of $\int_a^b f(x)$dx is given by,

$T_n = \frac{h}{2}(y_o + 2y_1 + 2y_2 + 2y_3 + \cdots + 2y_{n-1} + \text{yn}) = \frac{h}{2}\left(y_o + y_n + 2\sum_{j=1}^{j=n} y_j\right) = I$………………………equation 26

to prove the formula correctly; "let's carry out the process of adding the areas of the trapezoids. Refer to" figure 28 and figure 29, "and use the formula we derived for the area of a trapezoid. Note that when we add the areas of the trapezoids starting on the left, the area of the first, second, and third" portions "are":

$\frac{h}{2}(y_o + y_1)$

$\frac{h}{2}(y_1 + y_2)$

$\frac{h}{2}(y_2 + y_3)$

So, y_0 and y_3, “the first and the last, each appear once; and all the other yj’s appear exactly twice. We can see from this example that there will be a similar pattern no matter the number of trapezoids: The first and the last vertical edge” appear “once, and all other vertical edges appear two times when we sum the areas of the trapezoids. This is exactly what the Trapezoid Rule entails in the formula above”.

“One drawback of the trapezoidal rule is that its error is related to the second derivation. Therefore more complicated approximations formulas can improve the accuracy for curves, these includes using” 2nd “and” 3rd “order polynomial”.

“Simpson’s Rule”

“Another technique for approximating the value of a definite integral is called Simpson’s Rule. Whereas the main advantage of the Trapezoid rule is its rather easy conceptualization and derivation, Simpson’s rule approximations usually achieve a given level of accuracy faster. Moreover, the derivation of Simpson’s rule is only marginally more difficult. Both rules are examples of what we refer to as numerical methods. In the Trapezoid rule method, we start with rectangular area-elements and replace their horizontal-line tops with slanted lines. The area-elements used to approximate, say, the area under the graph of a function and above a closed interval then become trapezoids. Simpson’s method replaces the slanted-line tops with parabolas. Though two points determine the equation of a line, three are required for a parabola. We also need to develop a formula for the area of a parabolic-top area-element if the sum of such areas is to become the Simpson approximation”.

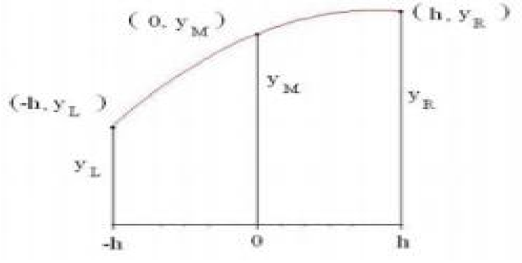


Figure 29: Trapezoid with a parabolic top.

“Suppose we consider a parabola y = Ax^2”+ By + “C”……………………………………………equation 27 “with its axis parallel to the y-axis and passing through three equally spaced points (−h”,y_l), (“0”,y_m), “and (h”,y_R). “Substituting the three” coordinates of a point “into the equation gives three equations in the three” constants which are “unknowns A, B, C.”

“Y_L=Ah^2−Bh+C”……………………………………………………………………………………………equation 28

Y_M = “C”…………………………………………………………………………………………………….equation 29

“ Y_R = Ah^2 +Bh + C”…………………………………………………………………………………..equation 30

“Solving these three equations by adding the first to the last, and then by subtracting the last from the first, yields:”

“2Ah^2 = Y_L” + Y_M –“2Y_M”………………………………………………………………………………equation 31

“B” = $\frac{1}{\text{"}h\text{"}}$ $\frac{Y_{R}-\text{"}Y_{L}\text{"}}{\text{"}2\text{"}}$ ……………………………………………………………………………………equation 32

“C” = Y_M …………………………………………………………………………………………………equation 33

“Next, we compute the area under the parabola” with equation 27 “and above the interval [−h,h] for the values of A, B, and C we just found:”

“$\int_{-h}^{h}$" A x^2 + “Bx” + C dx = (A$\frac{x_3}{3}$+ B$\frac{x_2}{2}$ + “Cx”)…………………………………………………………equation 34

= $\frac{1}{3}$ 2Ah^3 +2Ch………………………………………………………………………….equation 35

= h ($\frac{1}{3}$ 2Ah^2 +2C)………………………………………………………………………equation 36

= h ($\frac{1}{3}$ ("y_L +y_R −2y_M) + 2y_M"……………………………………………………equation 37

= “$\frac{h}{3}$ (y_L + y_R” −2 y_M + “6y_M")…………………………………………………equation 38

= $\frac{h}{3}$ ("y_L+ y_R + 4y_M")…………………………………………………………………equation 39

“The above formula holds for the area of a parabolic topped area element with base of length 2h and vertical edges of length” $y_L$on “the left and” y_R “on the right. The height at the midpoint is" y_M. “Now, let n be an even positive integer, and suppose we divide an interval [a,b] into n equal parts each of length h

=$\frac{b-"a}{n}$………equation 40

“Suppose f”(X) “is a function defined on [a, b]. As before we label the resulting partition” as in equation 22, where for each j, equation 23, 0 ≤ j ≤ n. And again, we let equation 24,”0≤ j ≤ n. That is, the vertical edges go from the x-axis to the graph of” function f(x). “Next, start at the left endpoint a of the interval and erect a parabolic-top area-element on the first two subintervals. The base of this area-element goes from x_0 to x_2, and we use as vertical sides the lines that intersect the graph at (x_0",y_0) “on the left and (x_2", y_2) “on the right. The point (x_1", y_1) “on the graph of f at the midpoint of the interval gives the third point we need to determine the parabola that forms the top of the area-element. From the formula we developed above, the area of this area element is equal to” $\frac{h}{3}$ ("Y_0 + Y_2 +4Y_1). If we repeat this process using the next two subintervals that go from x_2 to x_4, then the area of the resulting parabolic-top element will be from an application of the formula above” $\frac{h}{3}$("Y_2 +Y_4+4Y_3). Thus, the sum of the areas of the two parabolic top elements equals $\frac{h}{3}$ (Y_0+4Y_1+2Y_2+4Y_3+Y_4). We continue in this way until we have calculated the areas of the $\frac{n}{2}$ parabolic-top area elements and added them together. A pattern begins to emerge in the form of the sum of the areas of the $\frac{n}{2}$ parabolic-top area-elements. The sum will equal” $\frac{h}{3}$ multiplied by: “Y_0"+Y_n , “i.e. the sum of the heights of the leftmost and rightmost vertical edges; plus 4 times the sum of the odd-indexed heights; plus 2 times the sum of the even-indexed heights because these edges belong to two successive area-elements, one on the left and the other on the right. This explains the form of the Simpson’s Rule approximation which we now state” below.

“Let n be even. The n-subinterval Simpson approximation to” $\int_a^b$ "$f(x)$dx is given by”

S_n= “$\frac{h}{3}$($y_0 + 4y_1$+2y_2 +4y_3 +2y_4 +⋯+2y_{n-2}+4$y_{n-1} + y_n$)"……………………………………equation 41

= $\frac{h}{3}$ (y_0+ y_n+4$\sum y_{odd}$ +2$\sum y_{even}$)………………………………………………………………………………equation 42

“The Simpson’s 1”/3th “Rule In this rule, the function is approximated by a second-order degree polynomial between successive points. It corresponds to using second-order Lagrange polynomials in” modeling “data points.”

“Mathematically, $\boldsymbol{I}$” = $\frac{h}{3}$ $[(\mathrm{f}("x_0") + 4\mathrm{f}("x_1") + \mathrm{f}("x_3") + \mathrm{f}\ (x_{n-"1"}) + 2\mathrm{f}("x_2") + \mathrm{f}("x_4") + \mathrm{f}("x_{2n-2}") + \mathrm{f}("x_{n-5}")] + \frac{-nh^5}{90f''''(x)}$ ……………………………………………………………………………equation 43

“The error associated with this rule is given by the model”; $\boldsymbol{Error}$ = $\frac{-nh^5}{90f''''(x)}$ …………equation 44

“The Simpson’s Three-Eight Rules this rule corresponds to using third-order polynomial to fit four points. Integrating over the four points gives; the model is given as shown below.”

“$\boldsymbol{I}$ = $\frac{3h}{8}$" $\left[f("x_0") + 3f\left("x_1"\right) + \ldots + f("x_{3n-1}") + 2f("x_3")\ldots + f("x_{3n-3}") + f("x_{3n}")\right]$ + $\frac{-"nh^5"}{80f''''\ (x)}$………equation 45

“The error associated with this rule is given by the model”; $\boldsymbol{Error}$ = $\frac{-nh^5}{80f''''\ (x)}$ ………equation 46

“Numerical Error Past works on these” quadrature “have shown that they have tendencies to under or over predict sometimes, depending on the data architecture. Therefore, in view of this, it becomes pertinent that the differentials for the respective quadrature be accounted for in order to mitigate any of these cases occurring and consequently ensuring a more accurate incremental oil result from displacement mechanisms. Since laboratory rate –time values mimicking that obtainable from the field were considered, this paper utilizes the numerical differentiation technique, known as the Finite difference to account for the error as shown below” $\frac{dy}{dx}$= $\frac{\triangle \mathrm{f_i}}{\triangle \mathrm{x}}$ =$\frac{f_{i+1-}f_i}{\triangle \mathrm{x}}$ ……………………………………………………………………………………………equation 47

“This can be extended to the second and fourth differentials in order to fully account for the error in the Trapezoidal and Simpson’s rules respectively”.

$\frac{d^2y}{dx^2}$= $\frac{f_{i+2} - 2f_{i+1} + f_i}{\triangle x^2}$ ……………………………………………………………………………………………equation 48

“Error Comparisons”

“As we found to be true in the examples, Simpson’s rule is indeed much better than the Trapezoid rule. As n” tends to infinity “it generally converges much more rapidly to the value of the definite integral than does the Trapezoid rule.”

“We can get a sense of the differences in the rates of convergence of the two methods from the” following “two theorems”:

Theorem one: “Suppose the second derivative of” function “f”(x) “is continuous and hence” “necessarily bounded by a” positive number M_2on [“a, b].

If |error” T_n|=$\int_a^b$ "$f(x)$dx”–T_n……………………………………………………………………………………equation 49

then |error T_n |≤ $\frac{M_2(b-a)^3}{12n^2}$ ……………………………………………………………………………….equation 50

Theorem two: “Suppose the fourth derivative of f is continuous and hence necessarily bounded by a positive number” M_4 "on [a, b]. If |error” S_n|=$\int_a^b$ "$f(x)$"dx –S_n………………………equation 51

then |error S_n|≤ $\frac{M_{4\,(b-a)^5}}{180n^4}$ ………………………………………………………………………………….equation 52

“These theorems imply that in many situations, as n” tends to infinity, |errorT_n | tends to zero like $\frac{1}{n^2}$ and |errorS_n| tends to zero like$\frac{1}{n^4}$. “This explains why in general we are not surprised to find that Simpson’s rule converges to the value of the integral much faster than the” Trapezoidal “rule”.

The “Importance of the” Trapezoidal “and Simpson Rules is that they are simple to use and give excellent results, surprisingly so even for small n. A little arithmetic can yield a good estimate of a definite integral with only modest effort” in this age of computers.

“Microsoft Excel” Office

“Microsoft Excel is a spreadsheet editor developed by Microsoft for windows, macOS, Android, iOS, and ipadOS. It features calculation or computation capabilities, graphing tools, pivot tables, and a macro programming language called Visual Basic for Applications VBA”. It use “a grid of cells arranged in numbered rows and letter-named columns to organize data manipulations like arithmetic operations. It” also allows for “a” number “of” “optional command-line switches to control the manner in which Excel starts”.

“Microsoft Excel is one of the most versatile and popular spreadsheet programs. It serves as an electronic pad for accountants. It can easily perform simple as well as complex mathematical operations. Excel also provides the facility to convert the spreadsheet data into various charts like Bar, Pie” etc. Out of the most spreadsheet programs available. Excel is the most popular program. Excel is popular because it is versatile; it performs numerical calculations, and is very useful for non-numerical applications.

“Spreadsheet is a simple worksheet consisting of rows and columns in which any data can be entered. Spreadsheets are used” for doing various tasks, such as “performing calculations, recalculating results (if any data stored in them is changed), creating financial reports” and “comparing reports”. The spreadsheet “is” akin “to” a sheet of paper split into many thousands of lines and many hundreds of columns. Each column is labelled with A, B, C….AA, AZ….etc. and the rows numbered 1, 2, 3….. “Each cell has a unique address” or “cell reference” number such as A1, B14, Z5, which means column A row 1, column B row 14, and column Z row 5 respectively etc.

In Microsoft Excel, a worksheet is also called a spreadsheet.

“A workbook is the file in which you work and store your data. Because each workbook can contain many worksheets, you can organize various kinds of related information in a single file.”

Steps “to create a bar chart using Microsoft excel” office.

- “Add a bar chart right on a form”.
- “In the ribbon, select Create > Form Design”.

- “Select Insert Chart >Bar >Clustered Bar”.
- “Click on the Form Design grid in the location where you want to place the chart. Resize the chart for better readability.”
- “In the Chart settings pane, select Queries, and then select the query you want. example, select QuarterlyExpensesQry.”
- “To configure the chart, select options under the following sections. Set: Axis category” (vertical and horizontal axis)
- “Select Format > Sumofprojected> Display Data Label.”
- “To add a data label to a bar, select the Format tab in the Chart settings pane, select “Sumofprojected” under Data series, and then select Display Data Label.”
- “To format the numbers, press F4 to open the property Sheet, and then set: Primary Values Axis format, Primary Values Axis font color, Primary Values Axis Display Units.”
- “To see the completed chart, right-click on the Form tab select or Form View”.

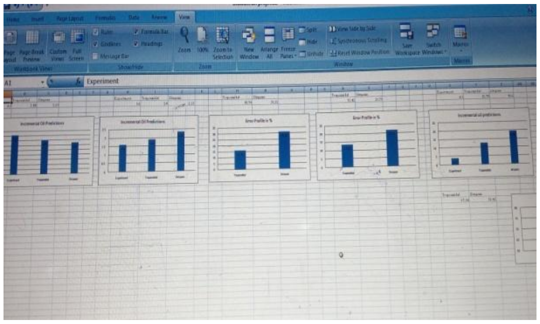
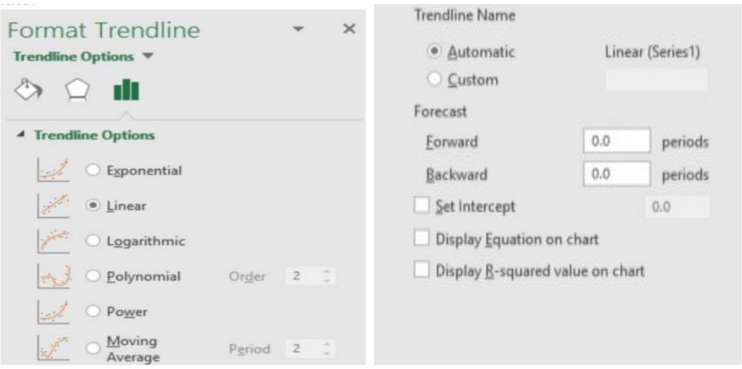


Figure 30: Bar chart creation on a Microsoft Excel spreadsheet.

<u>Steps to add trendline feature to display equation on a chart.</u>

- Select a “chart.”
- “Select the top right of the chart.”
- “Select Trendline. Note: Excel displays the Trendline option only if you select a chart that has more than one data series without selecting a data series.”
- “In the Add Trendline dialog box, select any data series options you want, and click OK.”



Format Trendline

Trendline Options

Trendline Options

○ Exponential

● Linear

○ Logarithmic

○ Polynomial Order 2

○ Power

○ Moving Average Period 2

Trendline Name

● Automatic Linear (Series1)

○ Custom

Forecast

Forward 0.0 periods

Backward 0.0 periods

☐ Set Intercept 0.0

☐ Display Equation on chart

☐ Display R-squared value on chart

Figure 31: Microsoft excel trendline options.

Estimate the area under the trendline by taking the bound integration of the trendline equation, $\int_a^b £(t)$, where £(t) is the equation of the trendline.

“CHAPTER FOUR”

“RESULTS AND” DISCUSSIONS

“This chapter focuses on the” plot “of” volumetric flow rate, time that is the graph showing “the relationship between the” volumetric “flow rate” against time “and” bar charts displaying “the” volume “of the” incremental oil recovered from the reservoir and the error profile for different case study as a result of using numerical method which are trapezoidal and Simpson rule to analyzed.

“This paper seeks to obtain accurate incremental oil prediction from successful EOR processes. The” Simpson Rule along with Trapezoidal rule were adapted to rate time data for incremental oil prediction. Multiple data points were used for each case study to replicate what is obtainable in field data. The equations for each projected declines, assuming the EOR process had not been initiated were obtained using Excel trend line function for the case studies Therefore, Incremental oil as shown in below, is defined mathematically as given.

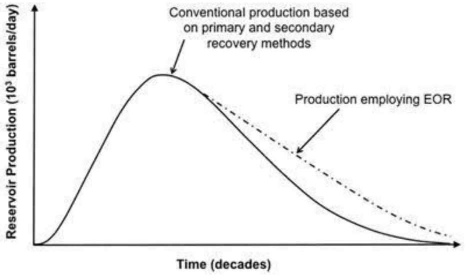


Figure 32: Profile of oil recovery rate against time.

The curve of the conventional production based on primary and secondary recovery methods that is the recovery methods initiated before EOR is as a result of the function *f*(t) while the curve of production employing Enhanced Oil Recovery that “is when EOR process is” initiated is “as a result” of “the” function *g*(t). Therefore, Incremental oil =$\int_a^b f(t)$ dt - $\int_a^b g(t)dt$ ……………………………………………………………………..equation 53

Where,

$\int_a^b f(t)$ dt = Integral of the composite function……………………………………………………..equation 54

$\int_a^b g(t)\mathrm{d}t$= Integral of the projected decline equation…………………………………………..equation 55

NOTE: FINAL RESULT is equal to 0.5* experiment result + 0.5*average numerical result.

BASE CASE SCENERIO (A)

LINEAR CORE FLOOD USING CYLINDRICAL CORE (PHYSICAL MODEL)

The rate time curve of the cases considered in this analysis are shown below:

(A) POLYMER FLOODING

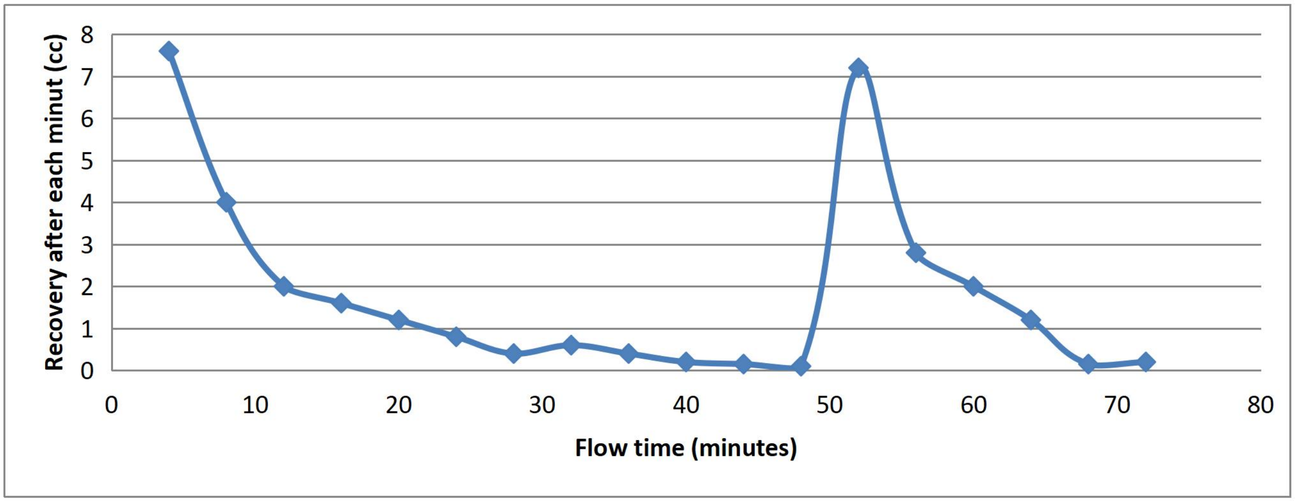


Figure 33: Rate-time profile for a polymer flooding.

Table 2: Table showing the recovery per minute and cumulative recovery for polymer flooding.

Flow time (minutes)	Recovery after each minute(cc/min)	Cumulative recovery(cc)
4	7.6	30.4
8	4	16
12	2	8
16	1.6	6.4
20	1.2	4.8
24	0.8	3.2
28	0.4	1.6
32	0.6	2.4
36	0.4	1.6
40	0.2	0.8
44	0.15	0.6
48	0.1	0.4
52	7.2	28.8
56	2.8	11.2
60	2.0	8.0
64	1.2	4.8
68	0.15	0.6
72	0.2	0.8

INCREMENTAL OIL PREDICTIONS:

EXPERIMENTAL RESULT: 130.4cc

SIMPSON RULE RESULT: 426.7cc

TRAPEZOIDAL RULE RESULT: 459.2cc

FINAL RESULT: 286.675cc

(B) POLYMER FLOODING

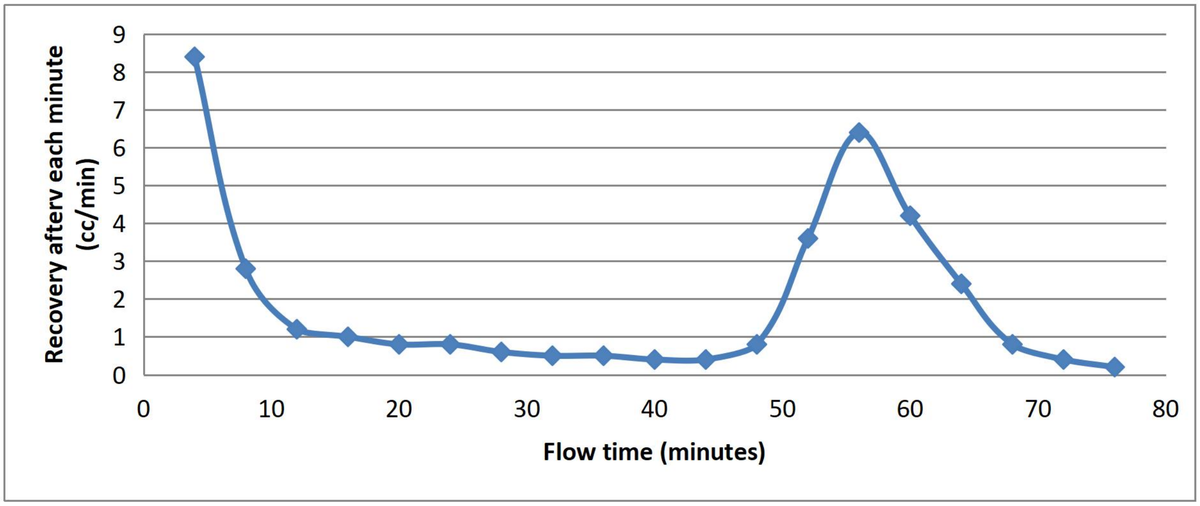


Figure 34: Rate-time profile for a polymer flooding.

Table 3: Table showing the recovery per minute and cumulative recovery for polymer flooding.

Flow time (minutes)	**Recovery after each minute(cc/min)**	**Cumulative recovery(cc)**
4	**8.4**	**33.6**
8	**2.8**	**11.2**
12	**1.2**	**4.8**
16	**1.0**	**4.0**

20	**0.8**	**3.2**
24	**0.8**	**3.2**
28	**0.6**	**2.4**
32	**0.5**	**2.0**
36	**0.5**	**2.0**
40	**0.4**	**1.6**
44	**0.4**	**1.6**
48	**0.8**	**3.2**
52	**3.6**	**14.4**
56	**6.4**	**25.6**
60	**4.2**	**16.8**
64	**2.4**	**9.6**
68	**0.8**	**3.2**
72	**0.4**	**1.6**
76	**0.2**	**0.8**

INCREMENTAL OIL PREDICTIONS:

EXPERIMENTAL RESULT: 144.8cc

SIMPSON RULE RESULT: 505.6cc

TRAPEZOIDAL RULE RESULT: 510.4cc

FINAL RESULT= 326.4cc

(C) ALKALINE SURFACTANT FLOODING

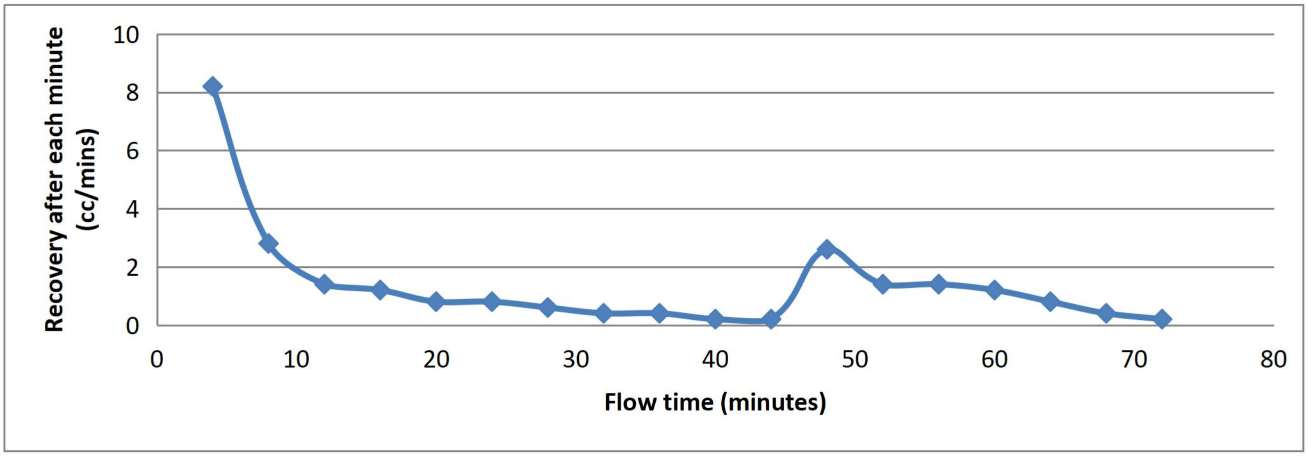


Figure 35: Rate-time profile for an alkaline surfactant polymer flooding.

Table 4: Table showing the recovery per minute and cumulative recovery for alkaline surfactant polymer flooding.

Flow time (minutes)	Recovery after each minute(cc/min)	Cumulative recovery(cc)
4	8.2	32.8
8	2.8	11.2
12	1.4	5.6
16	1.2	4.8
20	0.8	3.2
24	0.8	3.2
28	0.6	2.4
32	0.4	1.6
36	0.4	1.6
40	0.2	0.8
44	0.2	0.8

48	**2.6**	**10.4**
52	**1.4**	**5.6**
56	**1.4**	**5.6**
60	**1.2**	**4.8**
64	**0.8**	**3.2**
68	**0.4**	**1.6**
72	**0.2**	**0.8**

INCREMENTAL OIL PREDICTIONS:

EXPERIMENTAL RESULT: 100cc

SIMPSON RULE RESULT: 330.7cc

TRAPEZOIDAL RULE RESULT: 332.8cc

FINAL RESULT: 215.875cc

The incremental oil prediction from each case study using the respective models Simpson rule and trapezoidal rule has been outlined below clearly using bar chart. Also, the error profile was also included to show clearly the accuracy of each method in percentage clearly with the aid of bar chart.

CASE ONE:

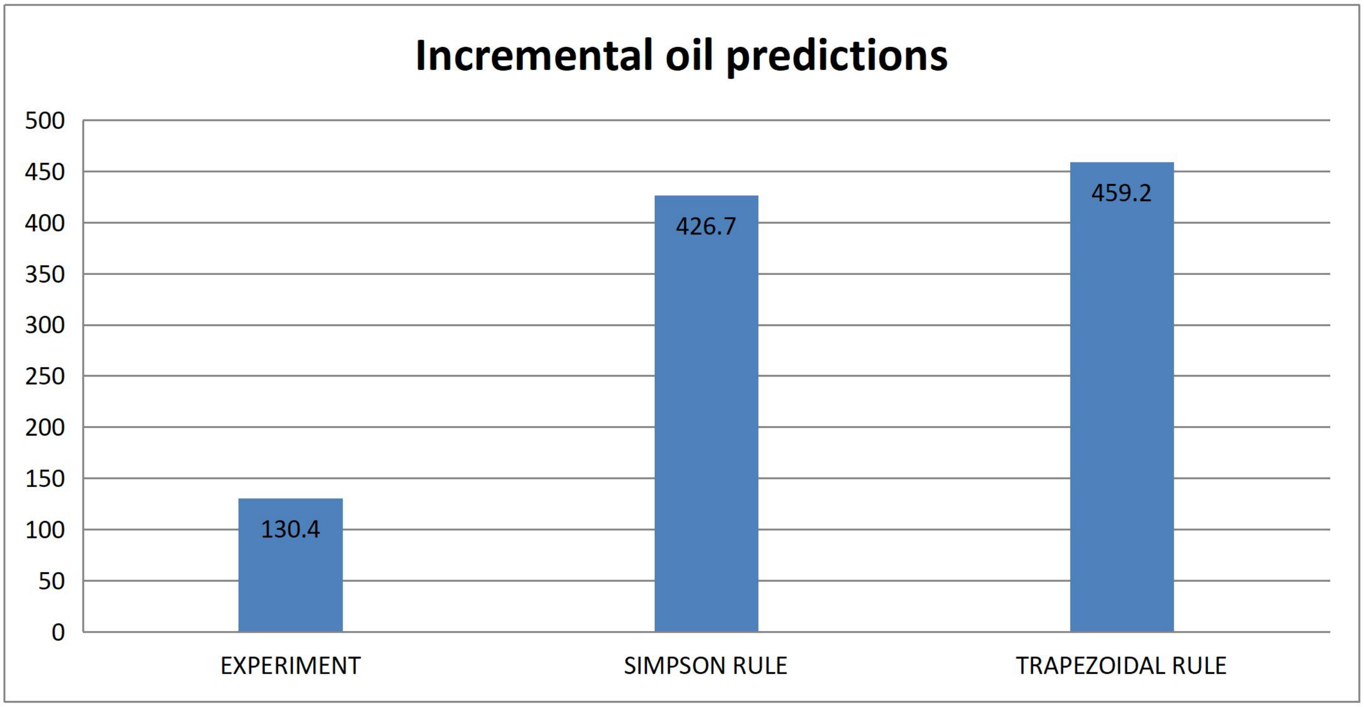


Figure 36: Comparison of incremental oil predictions for a polymer flooding.

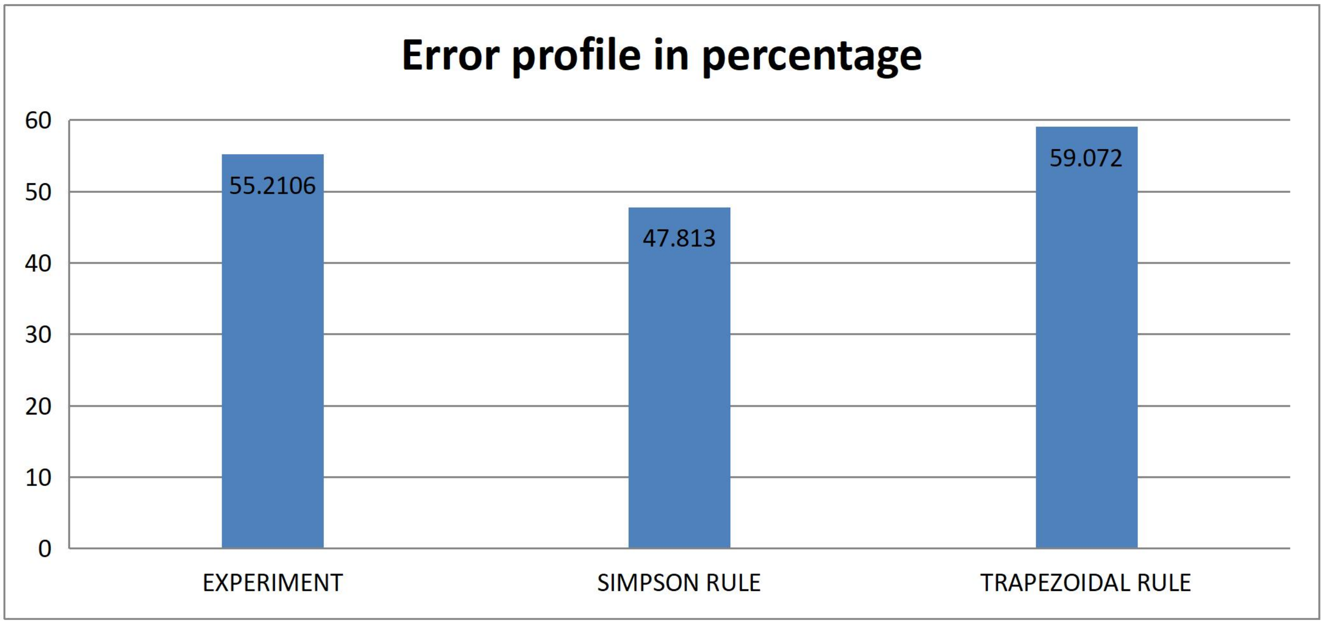


Figure 37: Error incurred by the prediction methods for a polymer flooding.

CASE TWO:

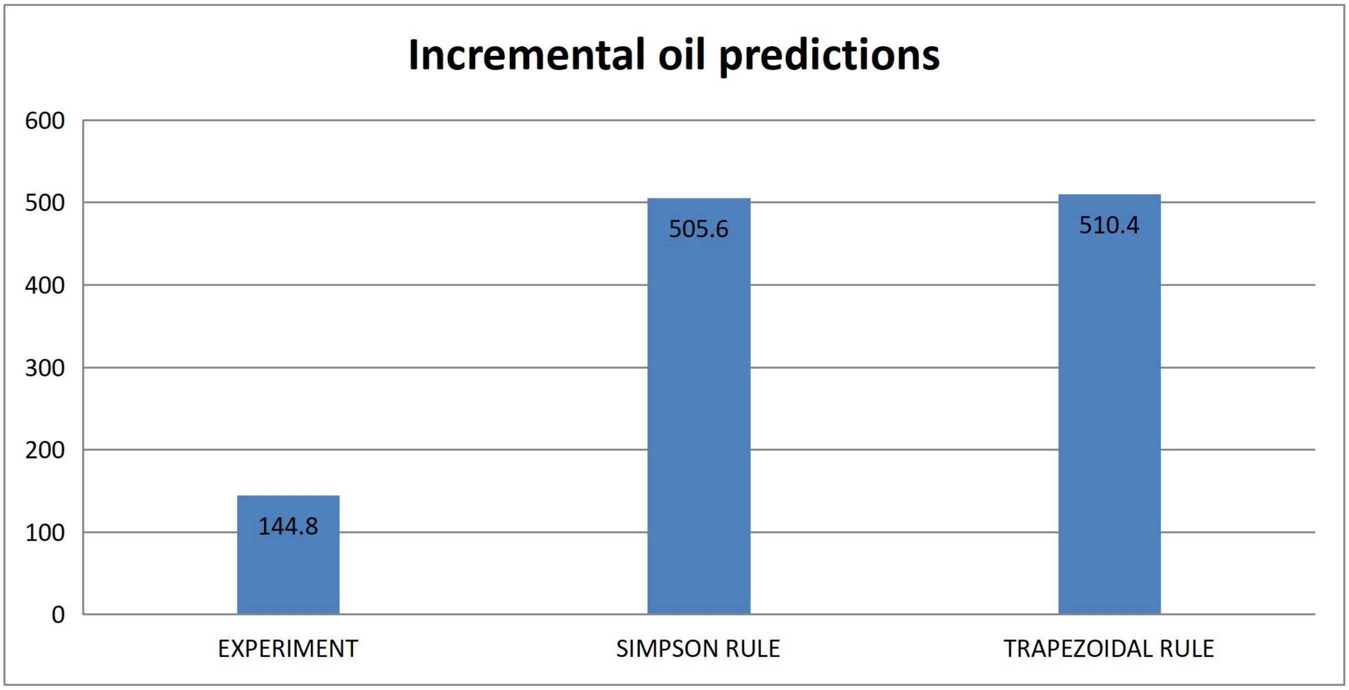


Figure 38: Comparison of incremental oil prediction for a polymer flooding.

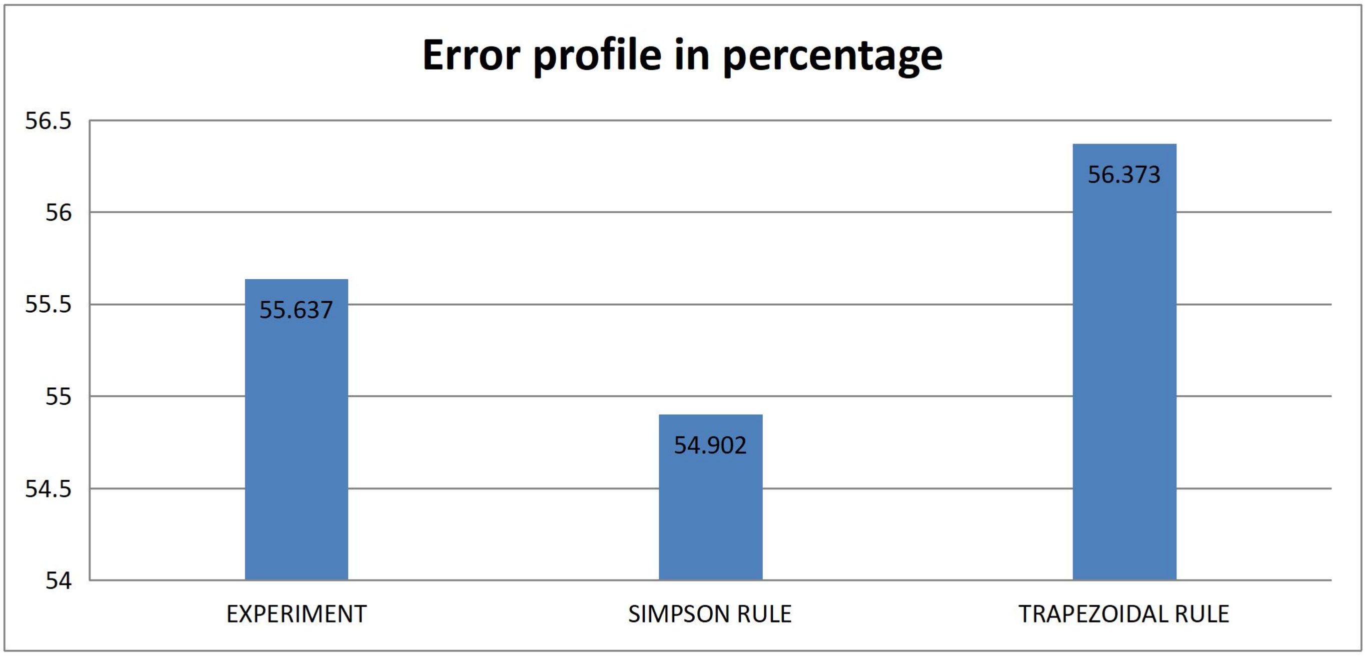


Figure 39: Error incurred by the prediction methods a polymer flooding.

CASE THREE:

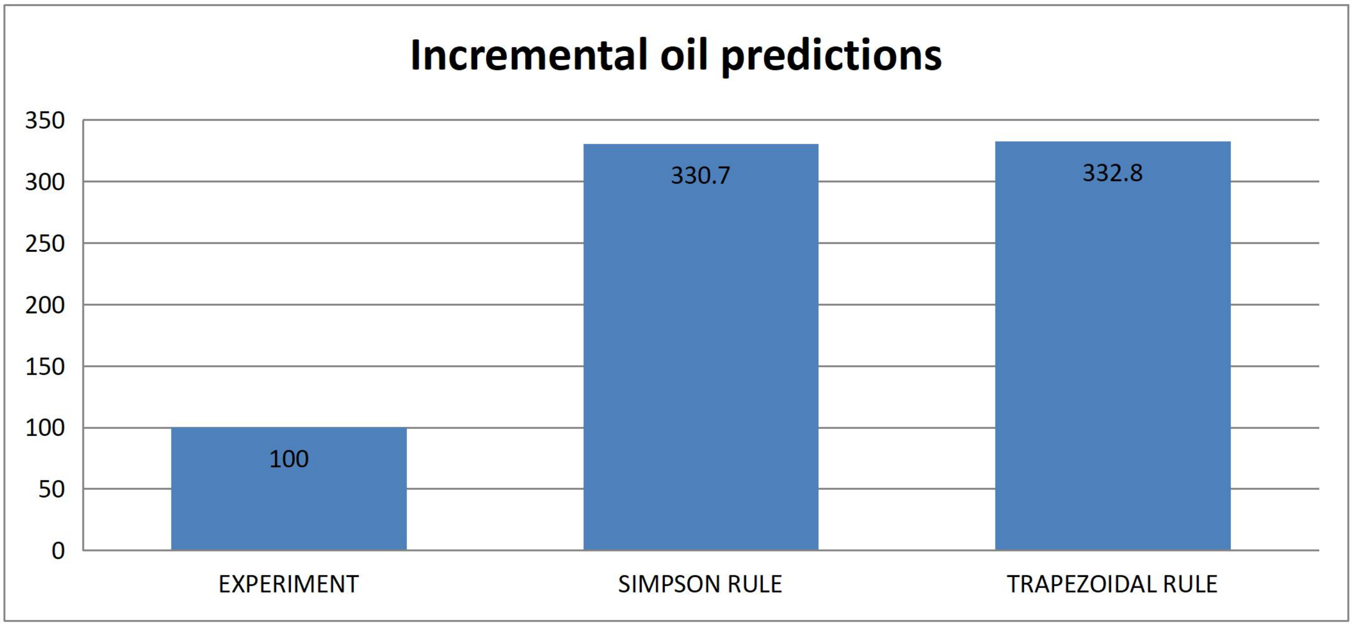


Figure 40: Comparison of "incremental oil" predictions "for" an "alkaline surfactant polymer flooding".

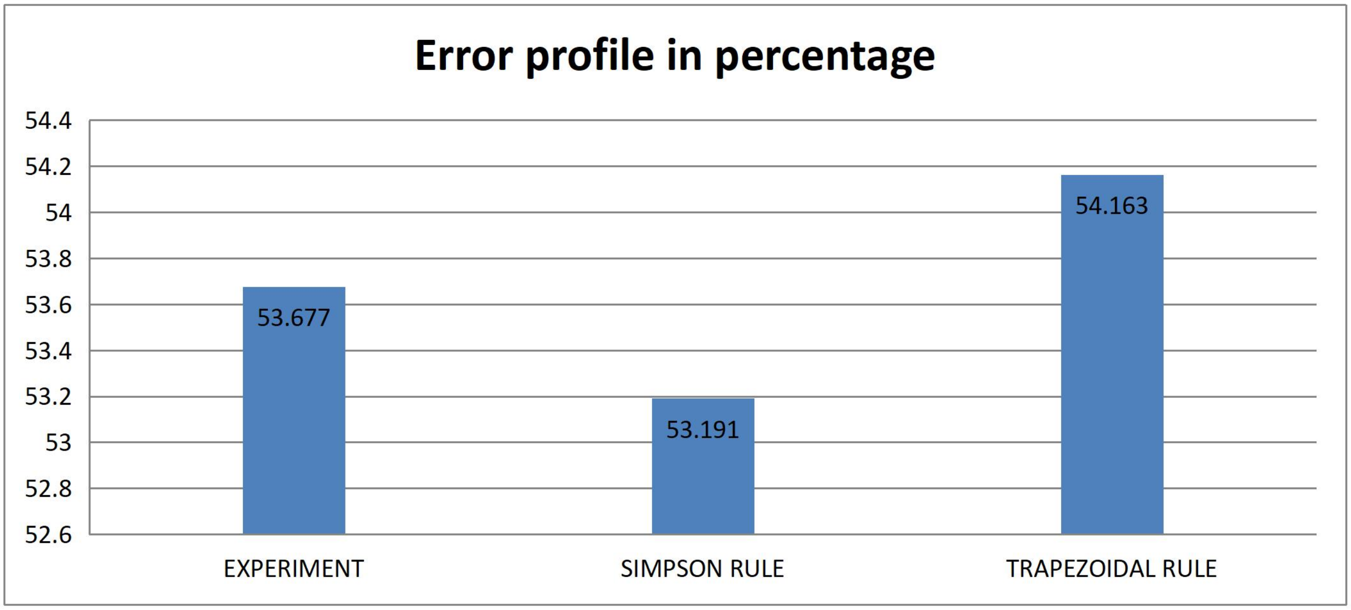


Figure 41: Error incurred by the prediction methods amount for an alkaline surfactant polymer flooding.

The results from the analyses as presented in figures 36 to 41 shows the superiority of the Simpson rule "in predicting incremental oil from successful EOR" process compared "to the"

Trapezoidal rule, which has previously been one of the most popular tools used in the past. This is shown clearly in the “error profiles” plots for each EOR process considered as the S rule return closer incremental oil values to those measured in the laboratory “One of the reasons for this” impressive performance by “the” Simpson rule “can be attributed to” its “adoption of higher polynomials” in modeling “data points” as compared to the straight line used in Trapezoidal rule. This ensures that rate data were well modeled and defined with appropriate functions. It is worthy to note that our “introduction of Finite difference in this research work has” helped “effectively in accounting for the error term in both the Simpson’s rule and trapezoidal rule”. In fact, it “has” further refined “the” performance “of” the Trapezoidal “when compare” the past observation when this was not taken in cognizance. This, which has been left unaccounted for in past works, shows truly that these rules sometimes can over/under estimate depending on the data we faced with and therefore should not be neglected or with contempt.

This thesis “seeks to obtain accurate incremental oil prediction from successful EOR processes. Shown below is the outcome of our applied models” Trapezoidal “and Simpson rule and” there “comparative performance with” existing “methodologies” such as Natural Cubic spine and Decline curve.

POLYMER FLOODING

EXPERIMENT RESULT= 20.4cc

CUBIC SPLINE RESULT= 19.298cc

SIMPSON RULE RESULT= 19.469cc

TRAPEZOIDAL RULE RESULT= 18.29cc

DECLINE RESULT= 13.5cc

FINAL RESULT= 19.02cc

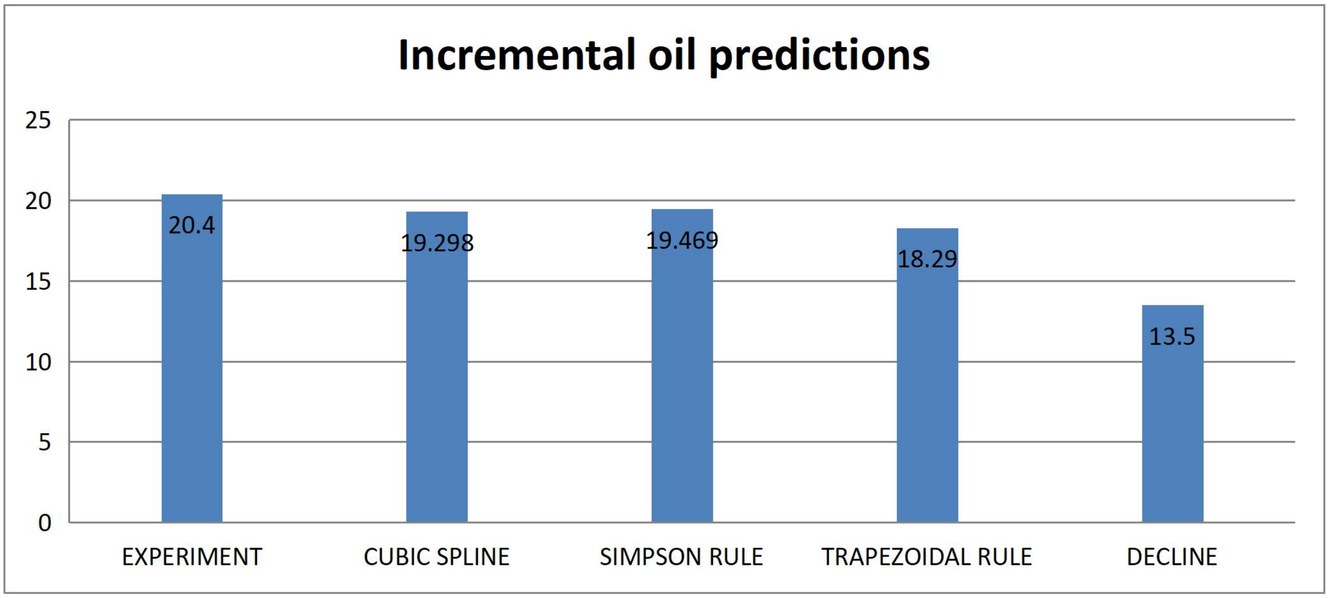


Figure 42: Incremental oil predictions for a polymer flooding.

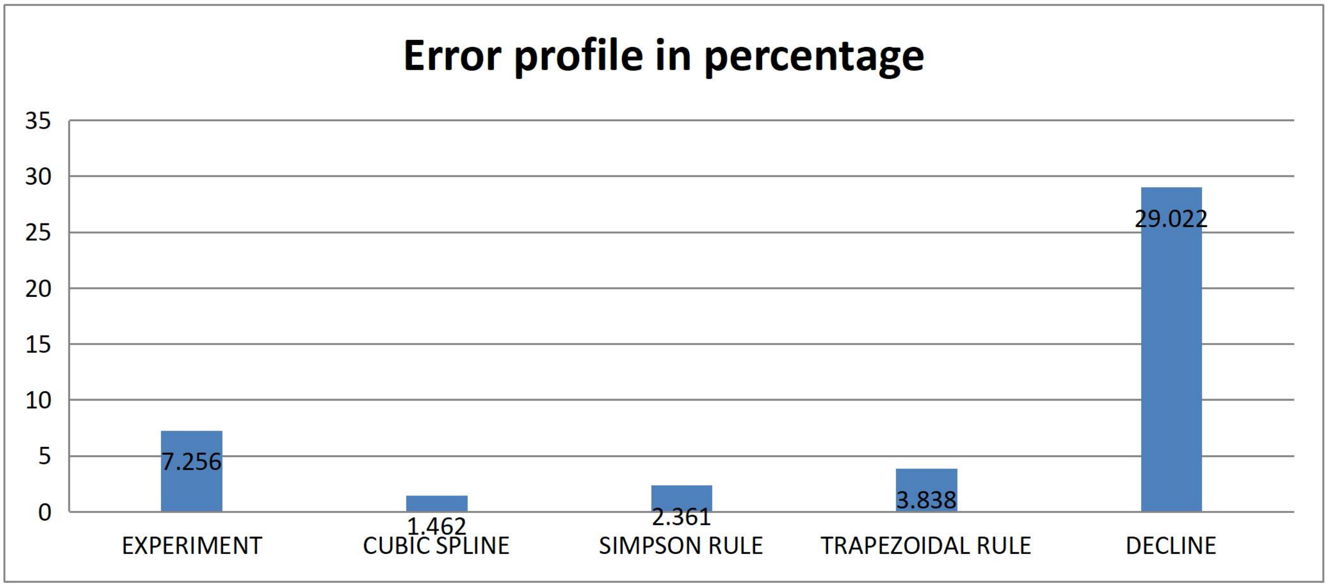


Figure 43: Error profile for a polymer flooding.

SURFACTANT POLYMER FLOODING

EXPERIMENT RESULT= 18.9cc

CUBIC SPLINE RESULT= 18.4cc

SIMPSON RULE RESULT= 18.363cc

TRAPEZOIDAL RULE RESULT= 17.91cc

DECLINE RESULT= 11.7857cc

FINAL RESULT= 17.757cc

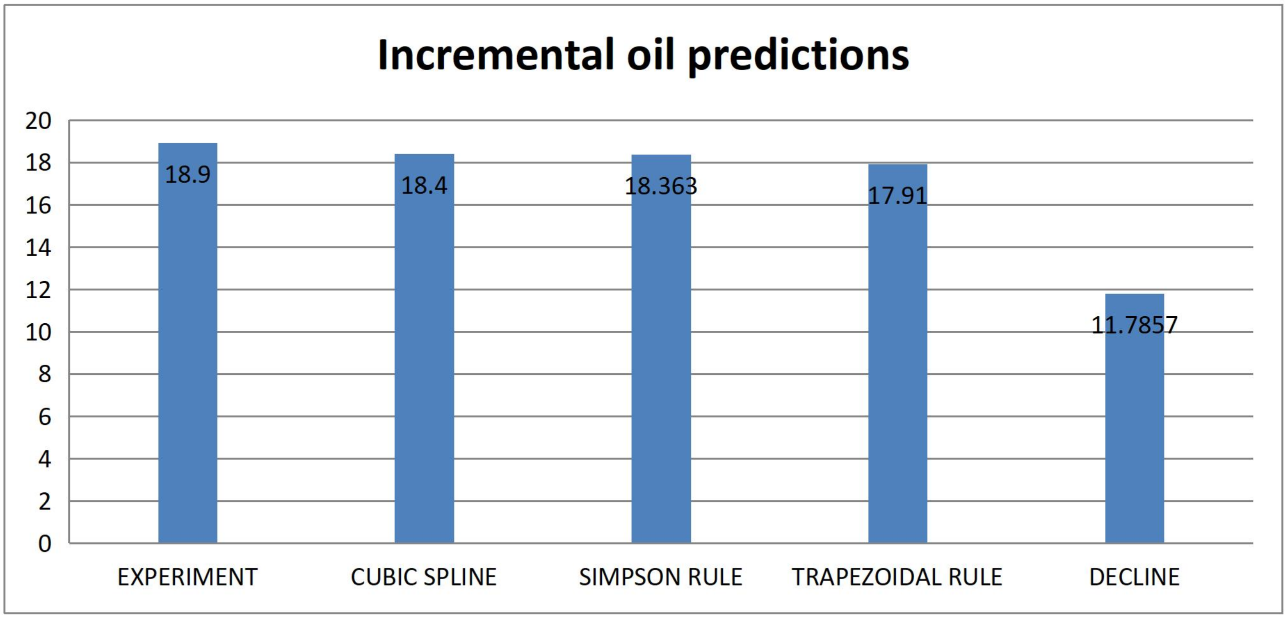


Figure 44: "Incremental oil prediction for" a "polymer flooding".

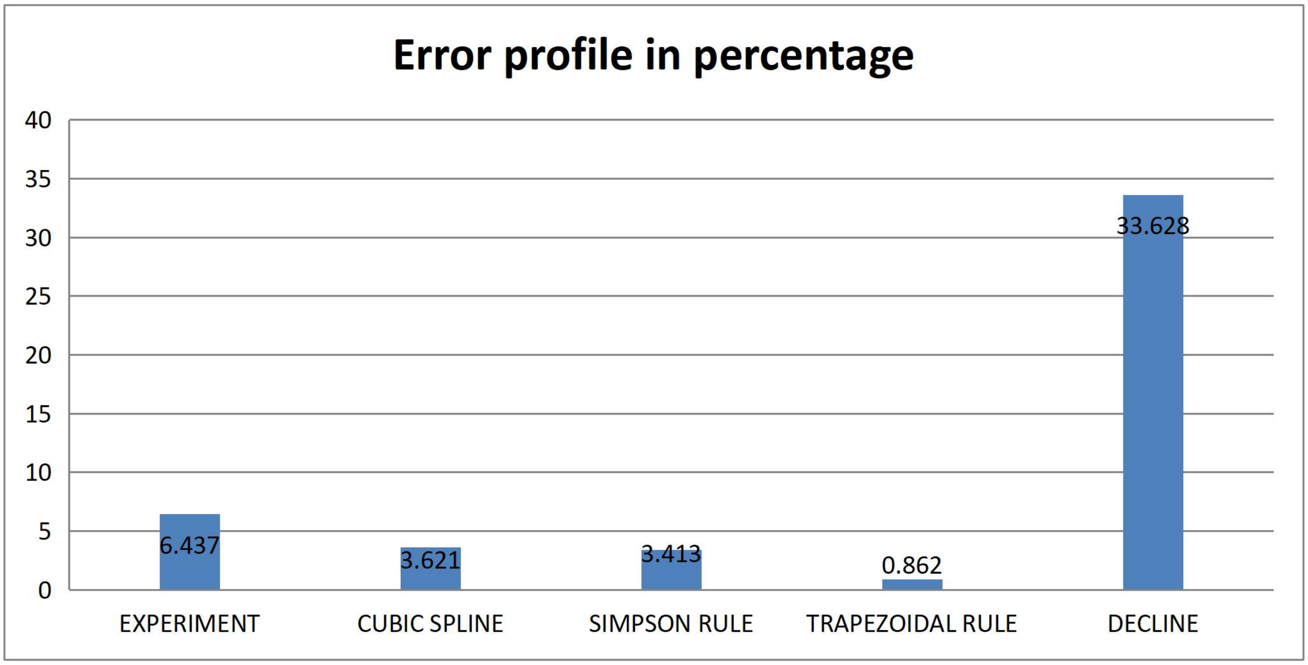


Figure 45: Error profile for a polymer flooding.

ALKALINE SURFACTANT POLYMER FLOODING

EXPERIMENT RESULT= 15.5cc

CUBIC SPLINE RESULT= 13.3cc

SIMPSON RULE RESULT= 15cc

TRAPEZOIDAL RULE RESULT= 12.9cc

DECLINE RESULT= 8.9cc

FINAL RESULT= 14.0125cc

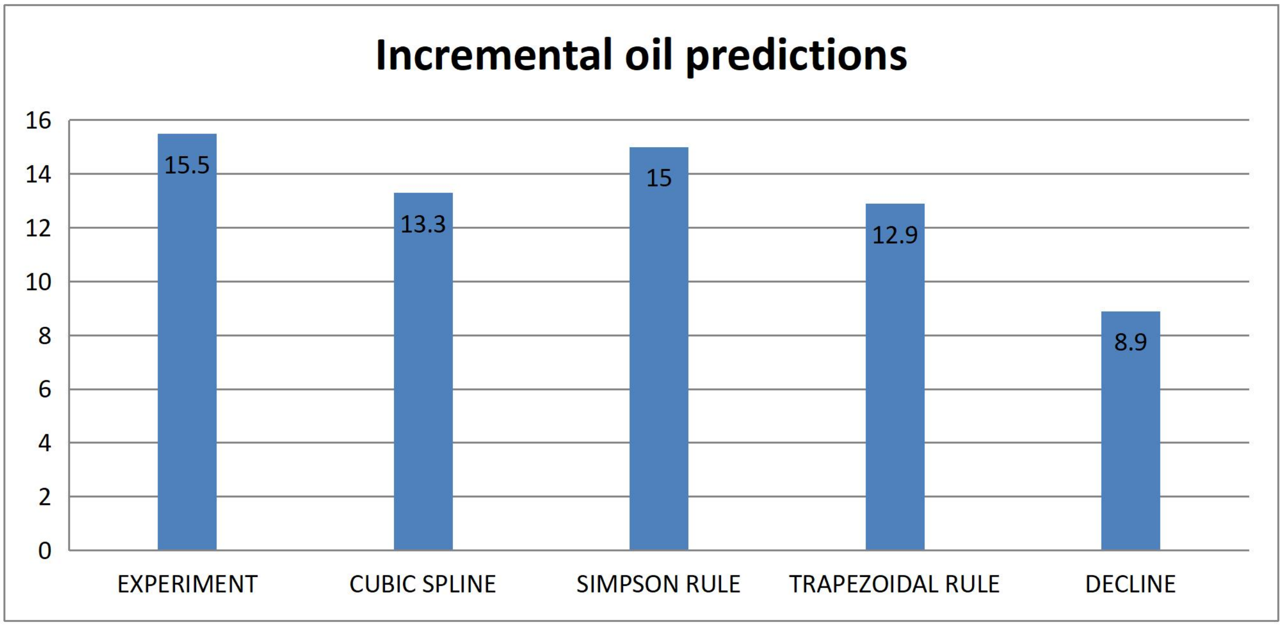


Figure 46: "Incremental oil" predictions "for" an "alkaline surfactant polymer flooding".

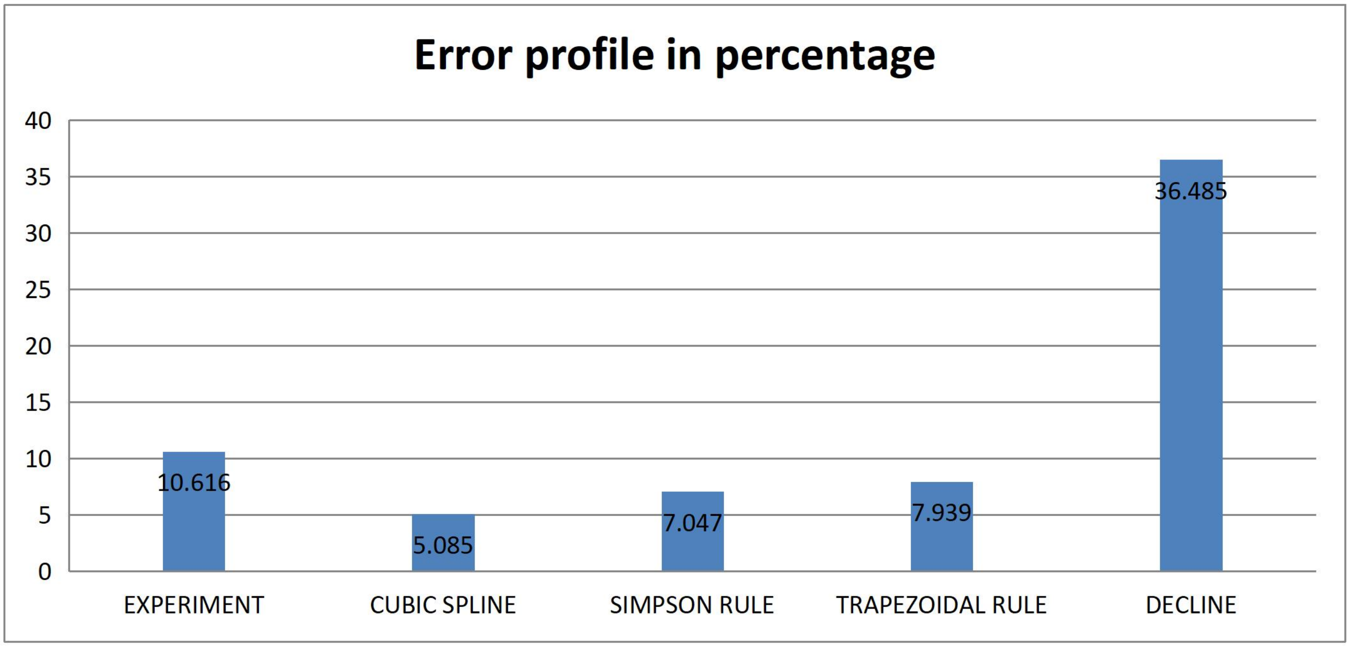


Figure 47: "Error profile for" an "alkaline surfactant polymer flooding".

"The figures" 42, 43, 44, 45, 46, 47 "clearly shows the superiority of the Natural cubic spline and Simpson quadrature in predicting incremental oil from successful EOR" processes compared “to the existing methodologies. This is shown clearly in the “error profile”” plots where “the Natural cubic” spine “and Simpson rule out-performed the Decline curve and the Trapezoidal rule. One of the reasons for this better” performance “of the Numerical methods can be attributed to the adoption of higher degree polynomials in” modeling “data points. In Natural cubic spline, a cubic polynomial is used to numerically model between two knot points while the Simpson’s has been shown to be exactly accurate for some cubic polynomials. This better model data points compare with Trapezoidal rule and totally different from the concept Decline curve adopts It is also worthy” of note that “the introduction of Finite difference, in this research work has effectively helped in accounting for the error term in both the Simpson’s rule and trapezoidal rule and has improved the result of both when compare to earlier work especially for the Trapezoidal rule”.

BASE CASE SCENERIO (B1)

AREAL CORE FLOOD USING HELE SHAW CELL SYSTEM

Step sizes of 1mins

The rate time curve of the cases considered in this analysis are shown below:

(A) WATER FLOODING

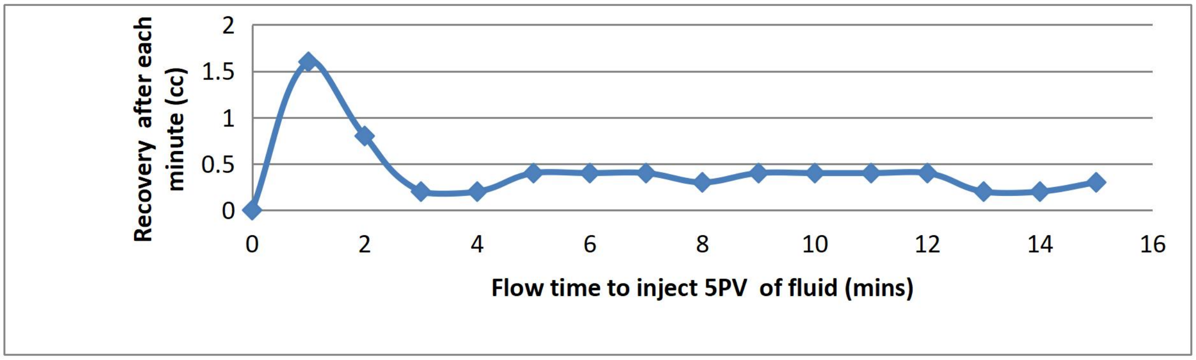


Figure 48: Rate-time profile for water flooding.

Table 5: Table showing the recovery per minute and cumulative recovery for water flooding.

Flow time (minutes)	Recovery after each minute(cc/min)	Cumulative recovery(cc)
0	0.0	0.0
1	1.6	1.6
2	0.8	0.8
3	0.2	0.2
4	0.2	0.2
5	0.4	0.4
6	0.4	0.4
7	0.4	0.4
8	0.3	0.3
9	0.4	0.4
10	0.4	0.4
11	0.4	0.4
12	0.4	0.4
13	0.2	0.2
14	0.2	0.2
15	0.3	0.3

OIL PREDICTIONS:

EXPERIMENT RESULT= 6.6cc

SIMPSON RULE RESULT= 5.57cc

TRAPEZOIDAL RULE RESULT= 5.65cc

FINAL RESULT: 6.105cc

(B) SURFACTANT FLOODING.

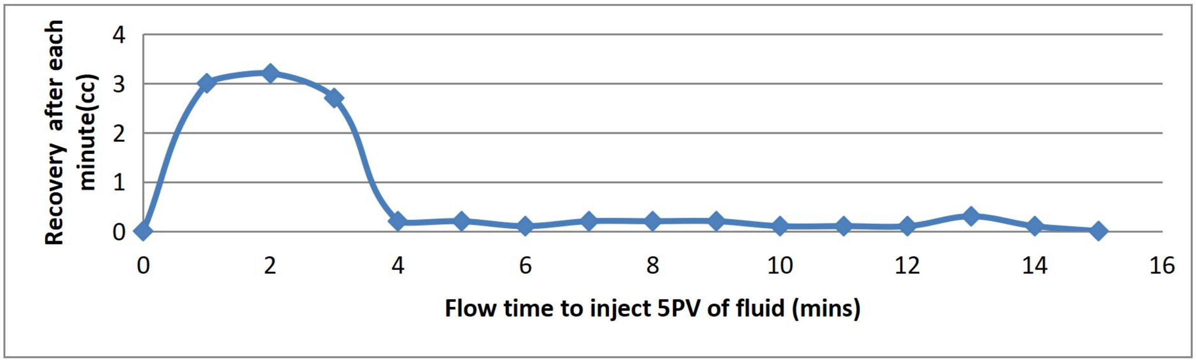


Figure 49: Rate-time profile for a surfactant flooding.

Table 6: Table showing the recovery per minute and cumulative recovery for surfactant flooding.

Flow time (minutes)	Recovery after each minute(cc/min)	Cumulative recovery(cc)
0	0.0	0.0
1	3.0	3.0
2	3.2	3.2
3	2.7	2.7
4	0.2	0.2
5	0.2	0.2
6	0.1	0.1
7	0.2	0.2
8	0.2	0.2
9	0.2	0.2
10	0.1	0.1
11	0.1	0.1
12	0.1	0.1

13	**0.3**	**0.3**
14	**0.1**	**0.1**
15	**0**	**0.0**

OIL PREDICTIONS:

EXPERIMENTAL RESULT= 10.7cc

SIMPSON RULE RESULT= 8.8cc

TRAPEZOIDAL RULE RESULT= 9.3cc

FINAL RESULT: 9.875cc

INCREMENTAL OIL PREDICTIONS:

EXPERIMENT RESULT: 4.1cc

SIMPSON RULE RESULT= 3.23cc

TRAPEZOIDAL RULE= 3.65cc

FINAL RESULT= 3.77cc

(C) SURFACTANT POLYMER FLOODING

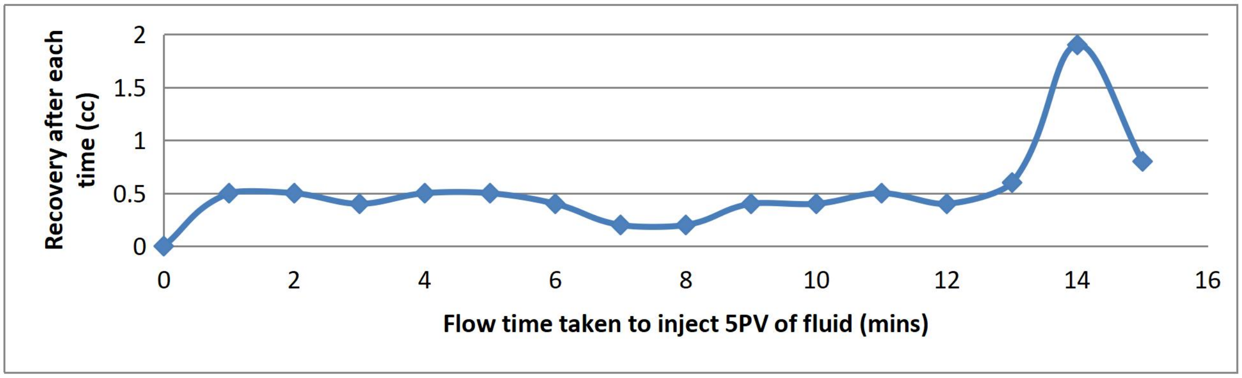


Figure 50: Rate-time profile for a surfactant polymer flooding.

Table 7: Table showing the recovery per minute and cumulative recovery for surfactant polymer flooding.

Flow time (minutes)	Recovery after each minute(cc/min)	Cumulative recovery(cc)
0	0.0	0.0
1	0.5	0.5
2	0.5	0.5
3	0.4	0.4
4	0.5	0.5
5	0.5	0.5
6	0.4	0.4
7	0.2	0.2
8	0.2	0.2
9	0.4	0.4
10	0.4	0.4
11	0.5	0.5
12	0.4	0.4
13	0.6	0.6
14	1.9	1.9
15	0.8	0.8

OIL PREDICTIONS:

EXPERIMENTAL RESULT= 8.2cc

SIMPSON RULE RESULT= 7.9cc

TRAPEZOIDAL RULE RESULT= 7.55cc

FINAL RESULT: 7.9625cc

INCREMENTAL OIL PREDICTIONS:

EXPERIMENT RESULT= 1.6cc

SIMPSON RULE RESULT= 2.33cc

TRAPEZOIDAL RULE RESULT= 1.90cc

FINAL RESULT= 1.8575cc

The incremental oil prediction from each case study using the respective models has been outlined below clearly using bar chart. Also the error profile was also included to show clearly the accuracy in percentage with the use of a bar chart of each method.

CASE ONE:

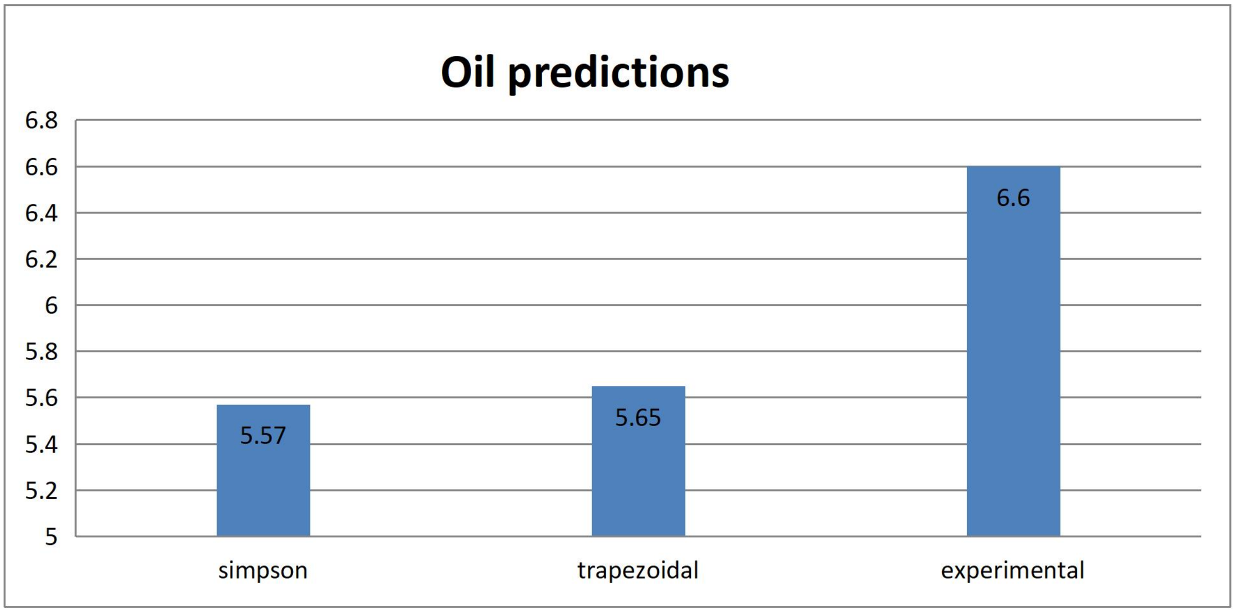


Figure 51: Comparison of oil predictions for a water flooding.

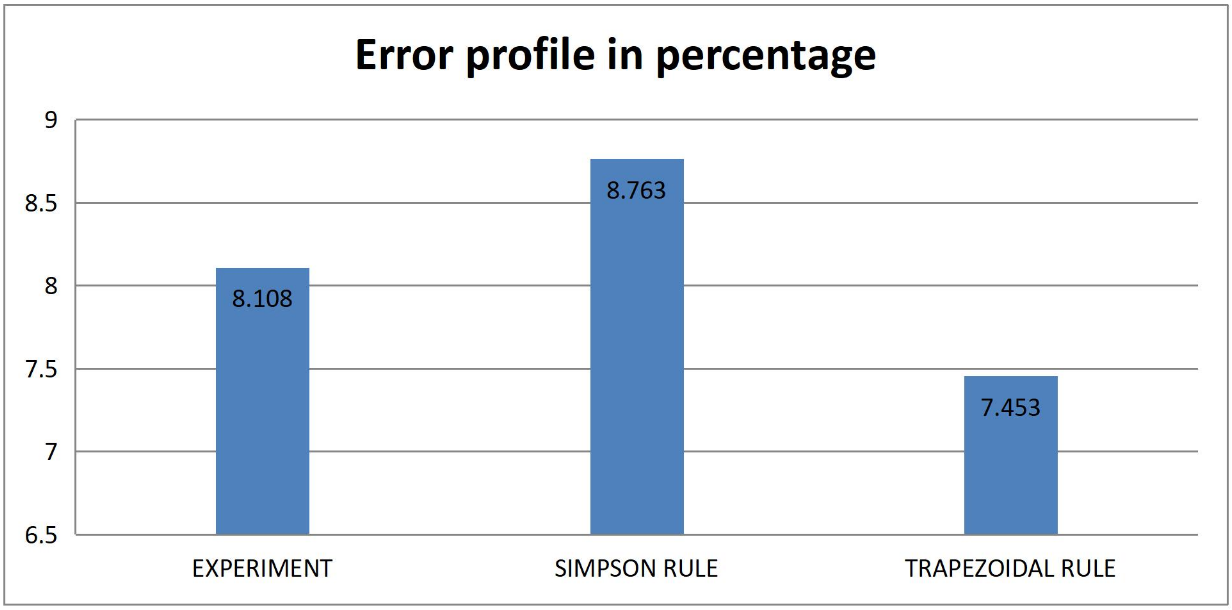


Figure 52: Error incurred by the prediction methods for a water flooding.

CASE TWO:

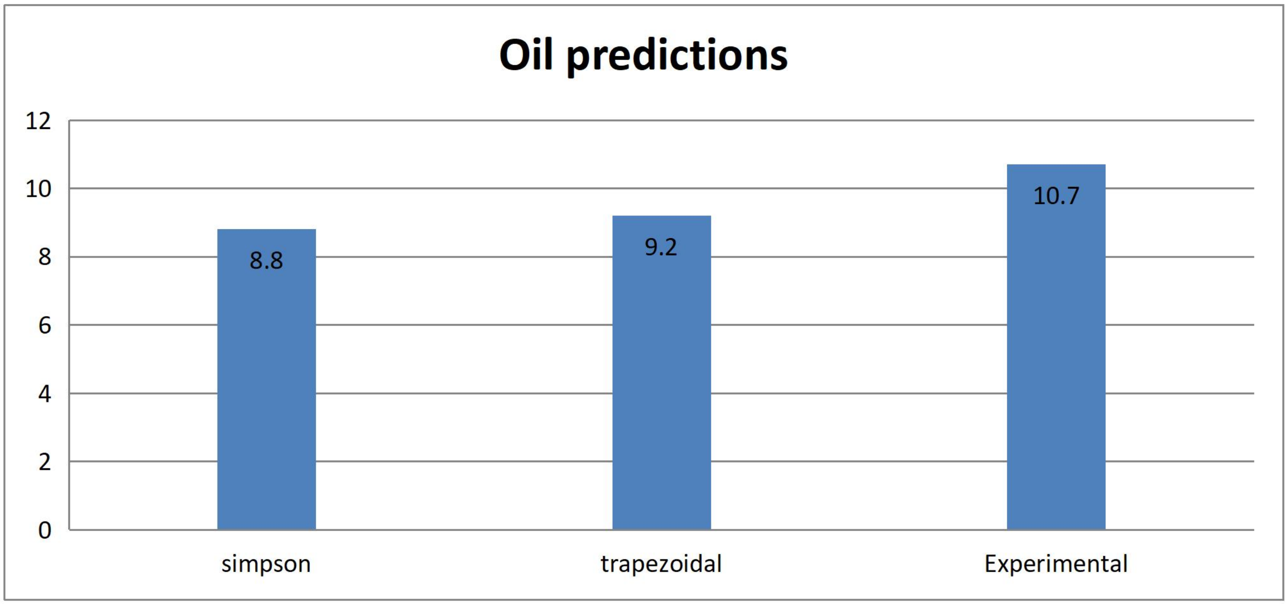


Figure 53: Comparison of oil predictions for a surfactant flooding.

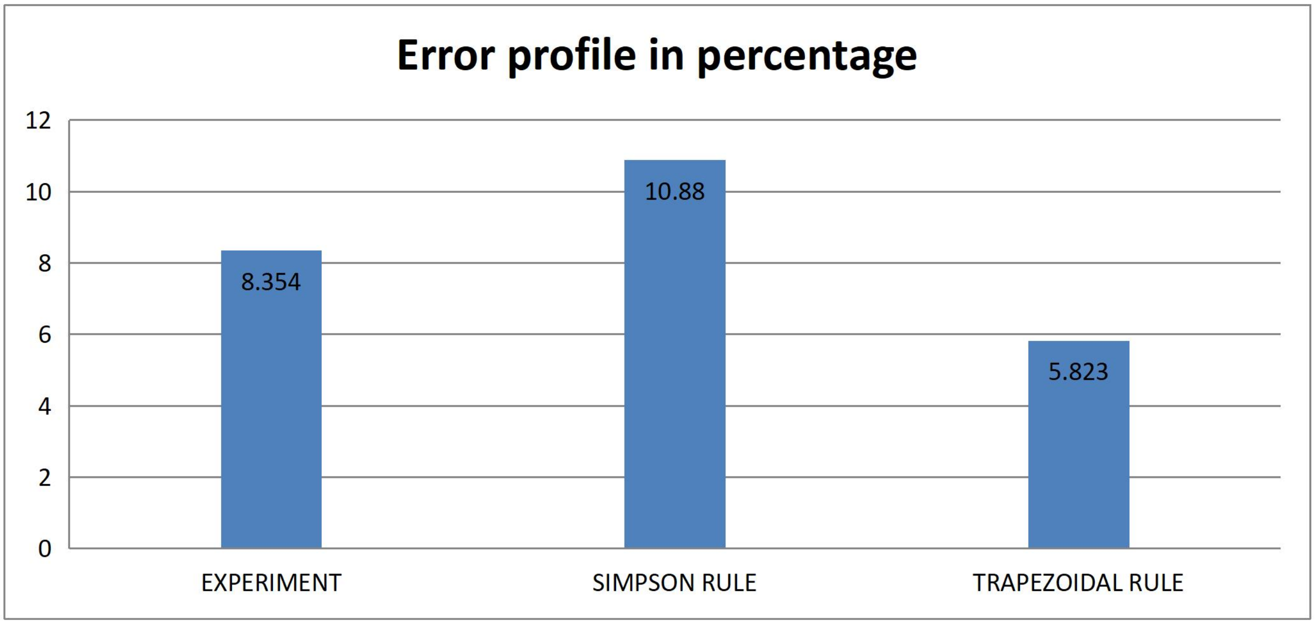


Figure 54: Error incurred by the prediction methods for a surfactant flooding.

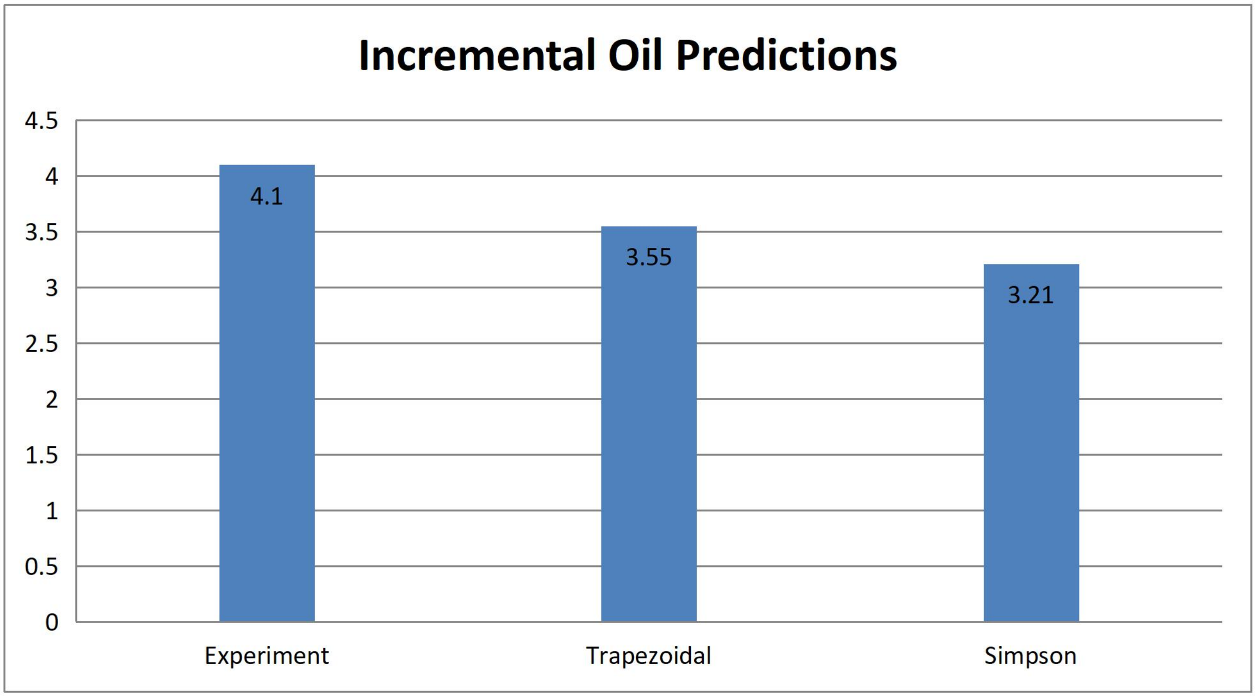


Figure 55: Comparison of Incremental oil predictions for a surfactant flooding.

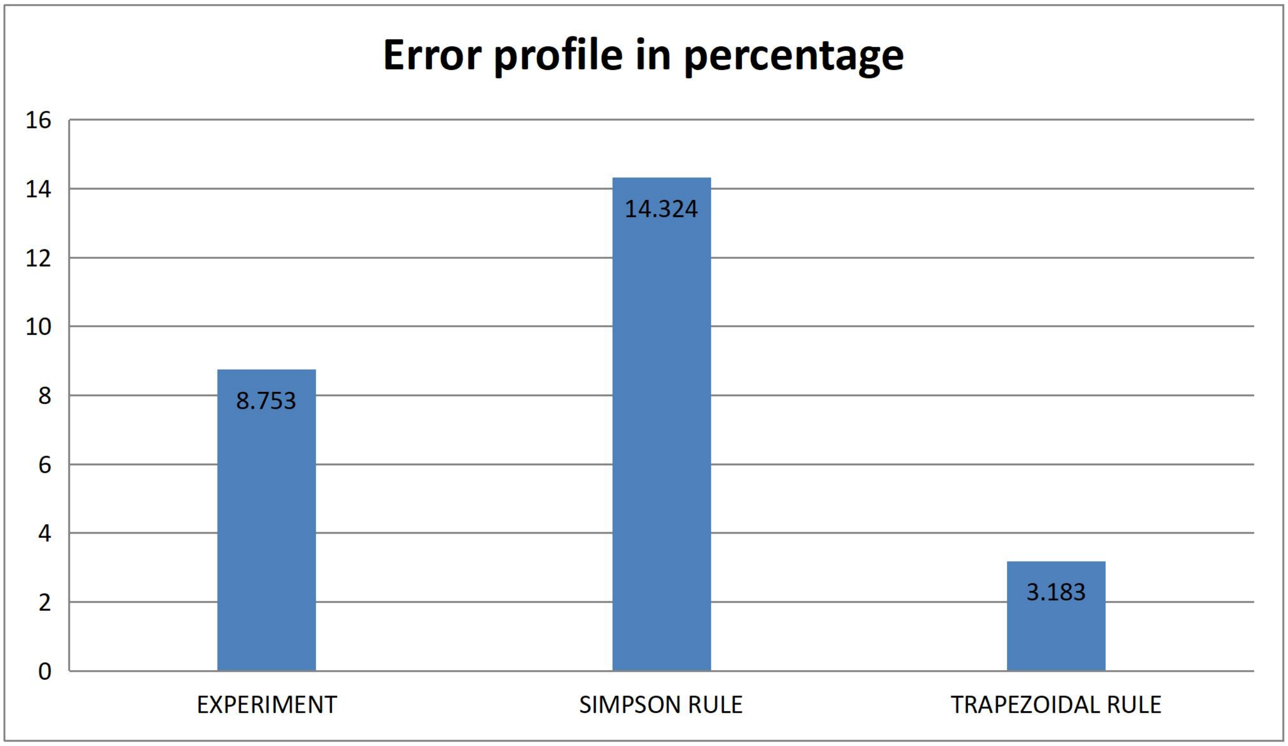
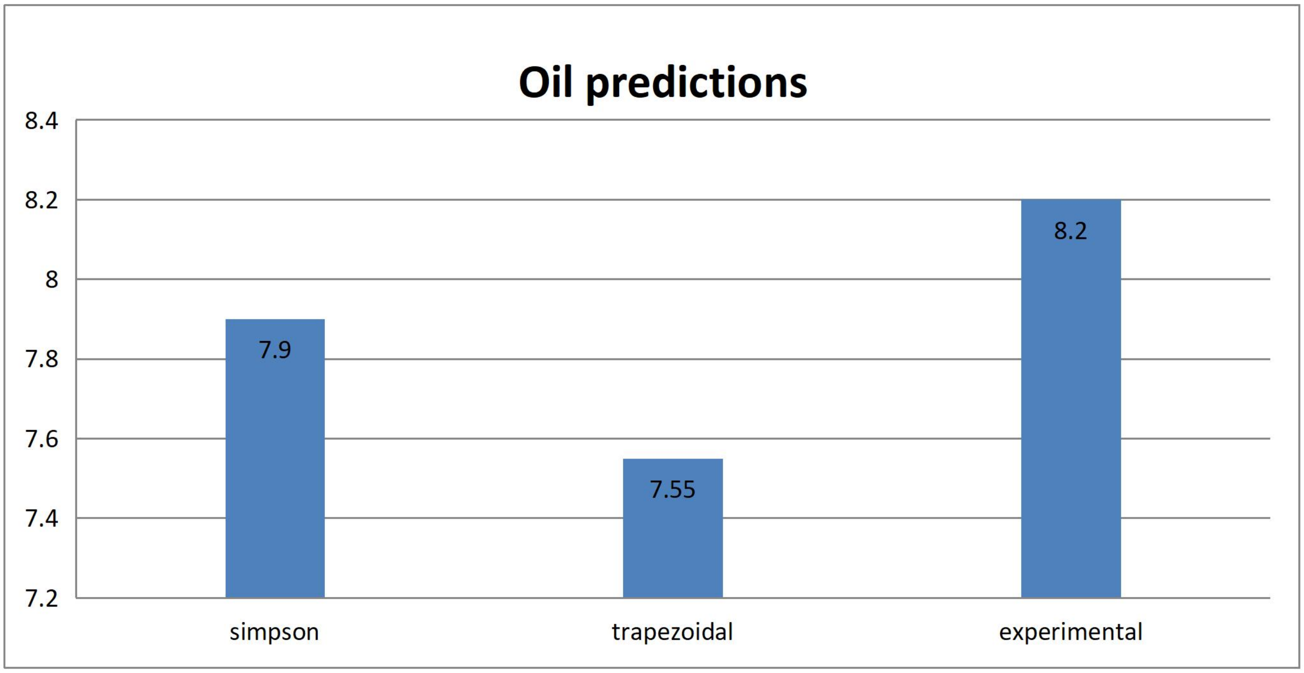


Figure 56: Comparison of Error incurred by prediction methods for a surfactant flooding.

CASE THREE:



“Figure 57: Comparison of Oil” predictions “for a” surfactant polymer “flooding”.

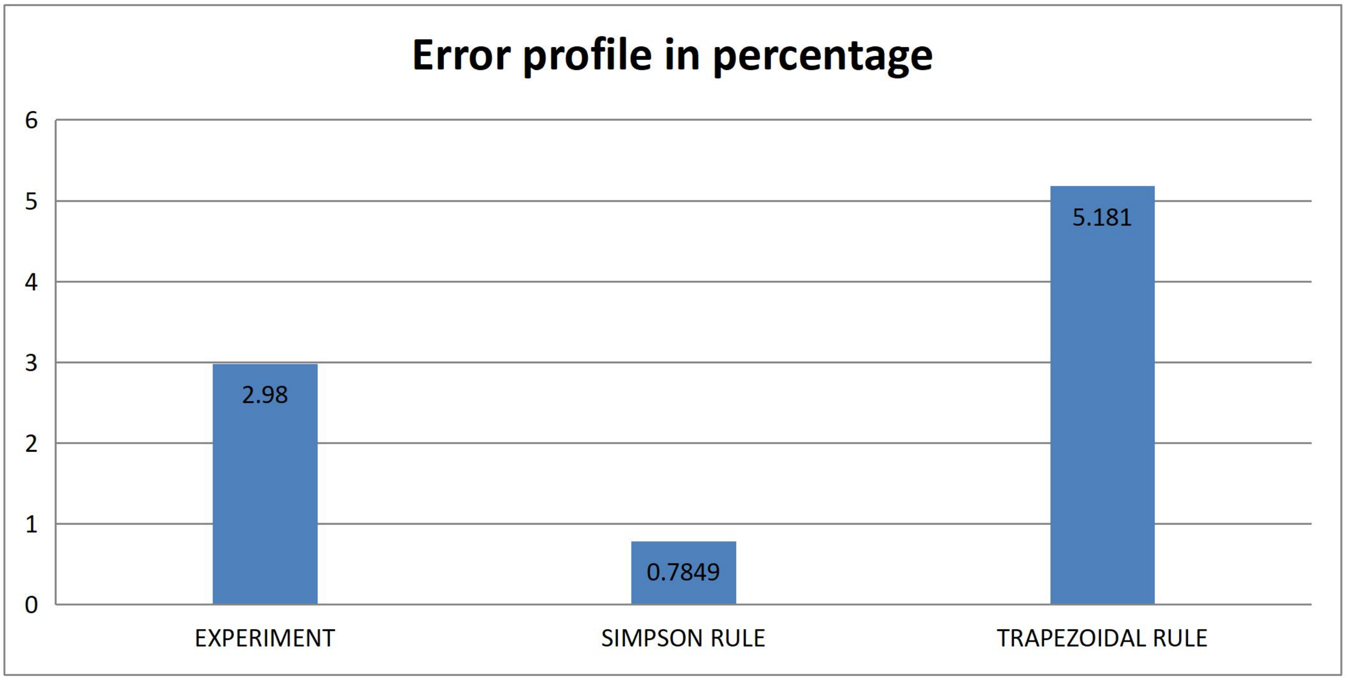


Figure 58: Error incurred by the prediction methods for a surfactant polymer flooding.

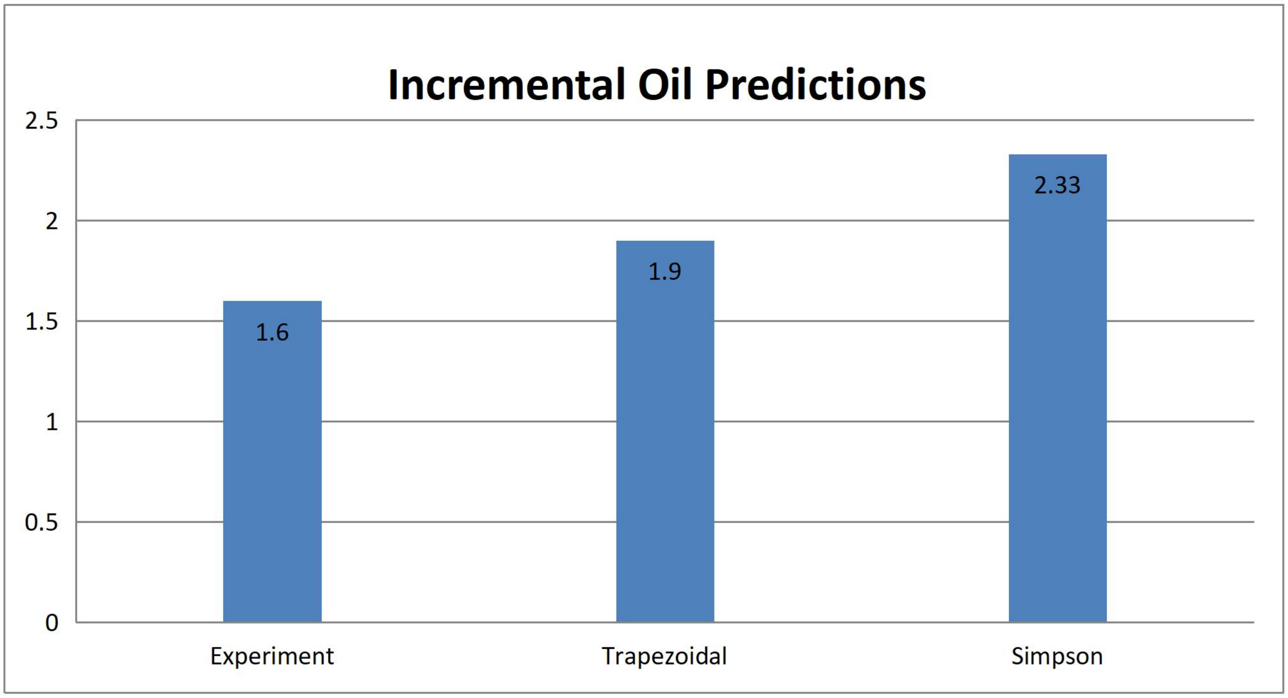


Figure 59: Comparison of incremental oil predictions for polymer surfactant flooding.

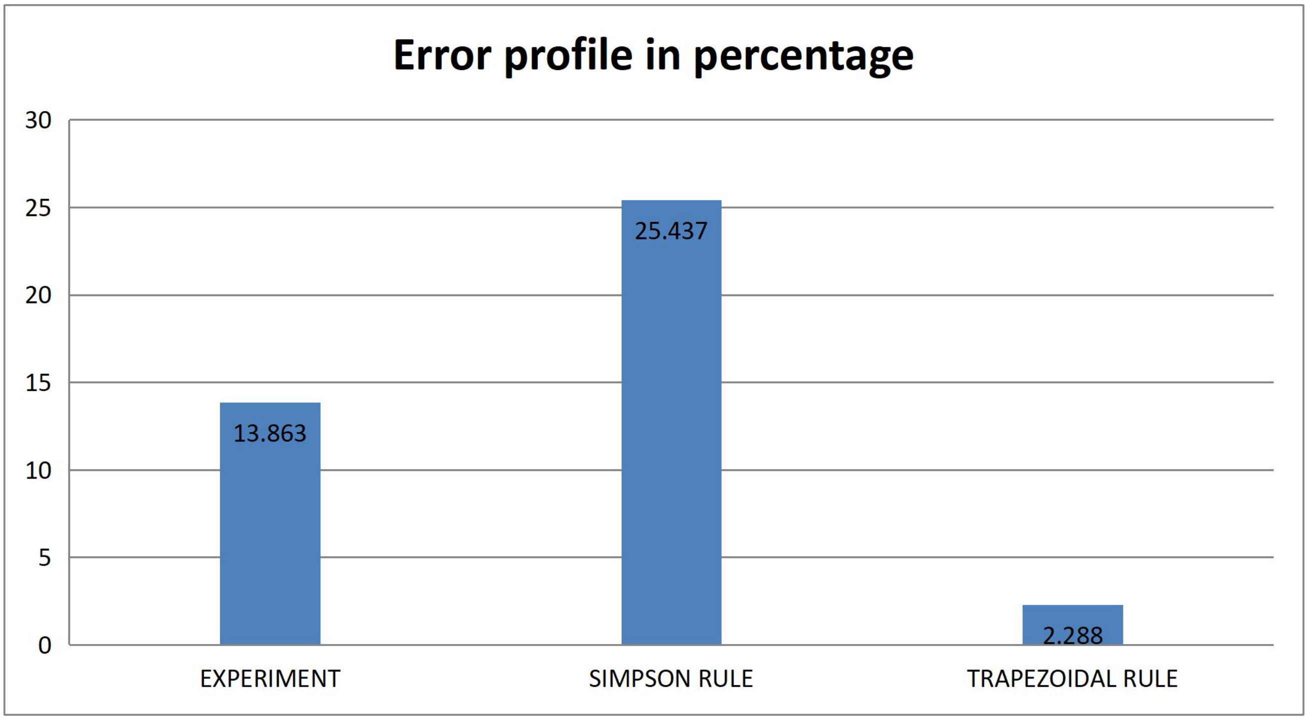


Figure 60: Error incurred by the prediction methods for surfactant polymer flooding.

BASE CASE SCENERIO (B2)

Step sizes of 3mins

The rate time curve of the cases considered in this analysis are shown below:

(A) WATER FLOODING

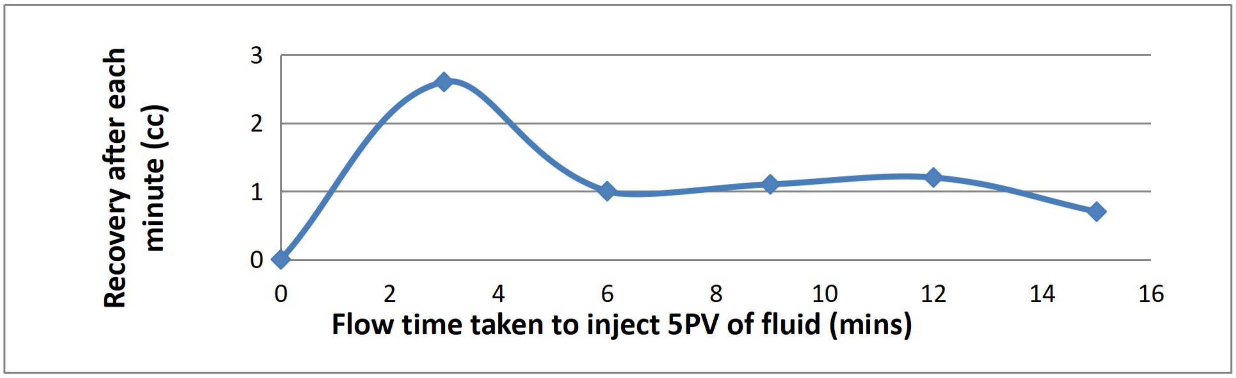


Figure 61: Rate-time profile for a Water flooding.

Table 8: Table showing the recovery per minute and cumulative recovery for water flooding.

Flow time (minutes)	Recovery after each minute(cc/min)	Cumulative recovery(cc)
0	0.0	0.0
3	2.6	7.8
6	1.0	3.0
9	1.1	3.3
12	1.2	3.6
15	0.7	2.1

OIL PREDICTIONS:

EXPERIMENT RESULT= 19.8cc

SIMPSON RULE= 19.2cc

TRAPEZOIDAL RULE= 18.75cc

FINAL RESULT= 19.3875cc

(B) SURFACTANT FLOODING

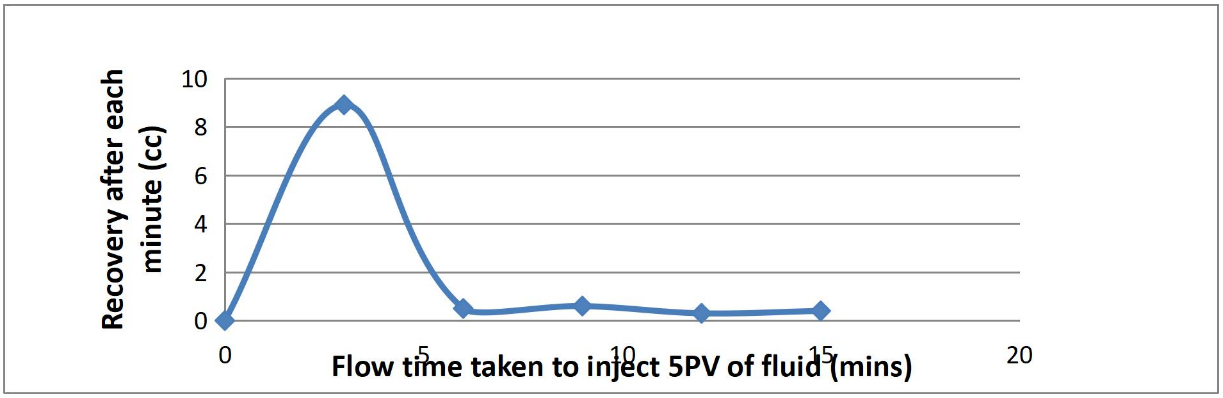


Figure 62: Rate-time profile for a surfactant flooding case study.

Table 8: Table showing the recovery per minute and cumulative recovery for surfactant flooding.

Flow time (minutes)	Recovery after each minute(cc/min)	Cumulative recovery(cc)
0	0.0	0.0
3	8.9	26.7
6	0.5	1.5
9	0.6	1.8
12	0.3	0.9
15	0.4	1.2

OIL PREDICTIONS:

EXPERIMENT RESULT= 32.1cc

SIMPSON RULE RESULT= 40.0cc

TRAPEZOIDAL RULE RESULT= 31.5cc

FINAL RESULT= 33.925cc

INCREMENTAL OIL PREDICTIONS:

EXPERIMENT RESULT= 12.3cc

SIMPSON RULE RESULT= 20.8cc

TRAPEZOIDAL RULE RESULT= 12.75cc

FINAL RESULT= 14.5375cc

(C) SURFACTANT POLYMER FLOODING

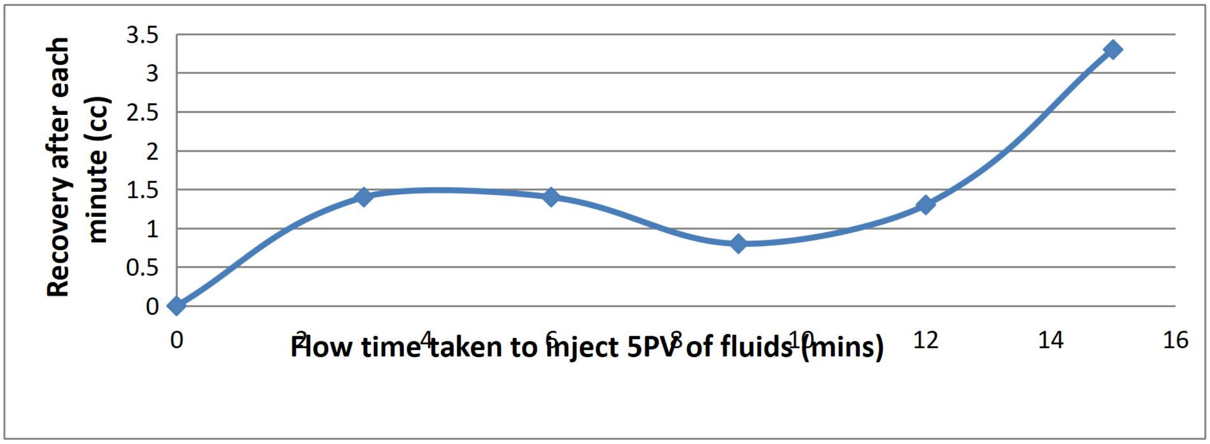


Figure 63: Rate-time profile for a Surfactant Polymer flooding.

Table 8: Table showing the recovery per minute and cumulative recovery for surfactant polymer flooding.

Flow time (minutes)	**Recovery after each minute(cc/min)**	**Cumulative recovery(cc)**
0	**0.0**	**0.0**
3	**1.4**	**4.2**
6	**1.4**	**4.2**
9	**0.8**	**2.4**
12	**1.3**	**3.9**
15	**3.3**	**9.9**

OIL PREDICTIONS:

EXPERIMENT RESULT= 24.6cc

SIMPSON RULE RESULT= 17.5cc

TRAPEZOIDAL RULE RESULT= 19.65cc

FINAL RESULT= 21.5875cc

INCREMENTAL OIL PREDICTIONS:

EXPERIMENT RESULT= 4.8cc

SIMPSON RULE RESULT= * cc**

TRAPEZOIDAL RULE RESULT= 0.9cc

FINAL RESULT= 2.85cc

CASE ONE:

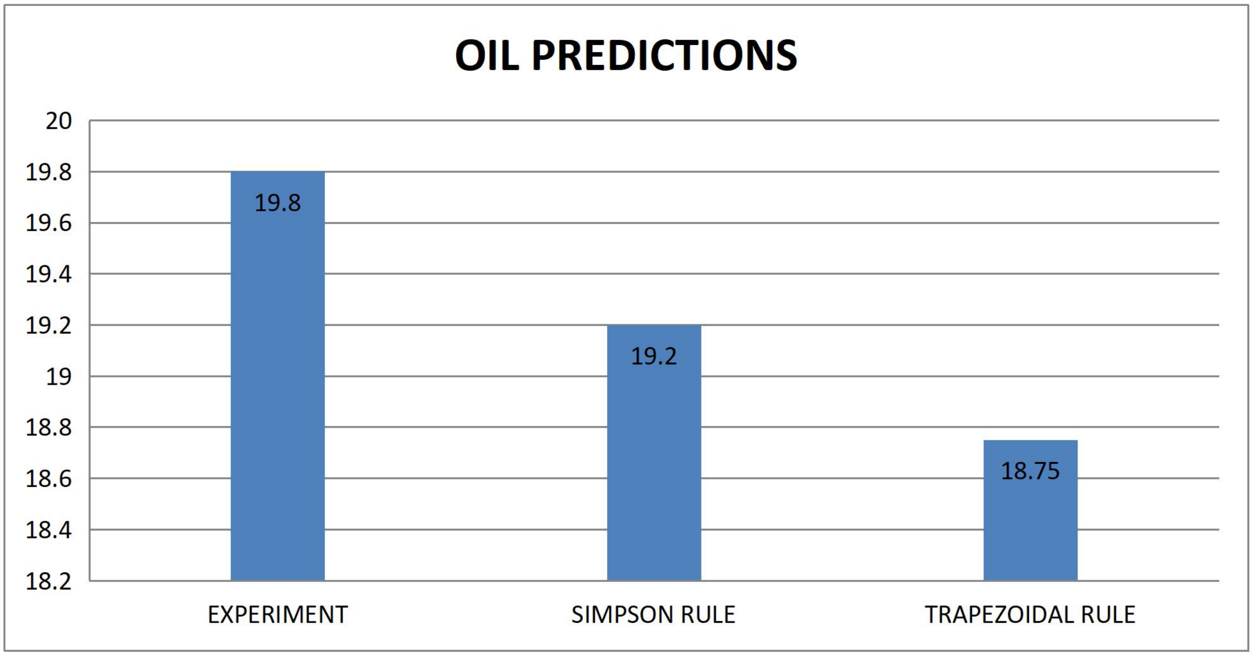


Figure 64: Comparison of oil predictions for a water flooding.

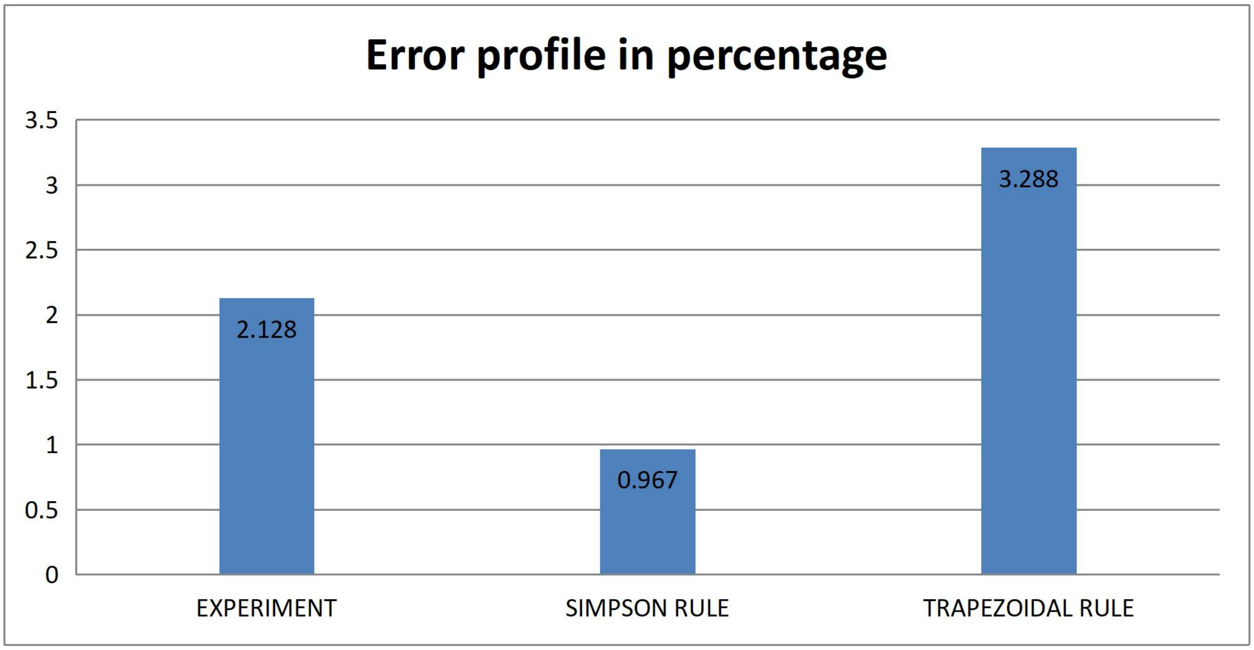


Figure 65: Error incurred by the prediction methods for a water flooding.

CASE TWO:

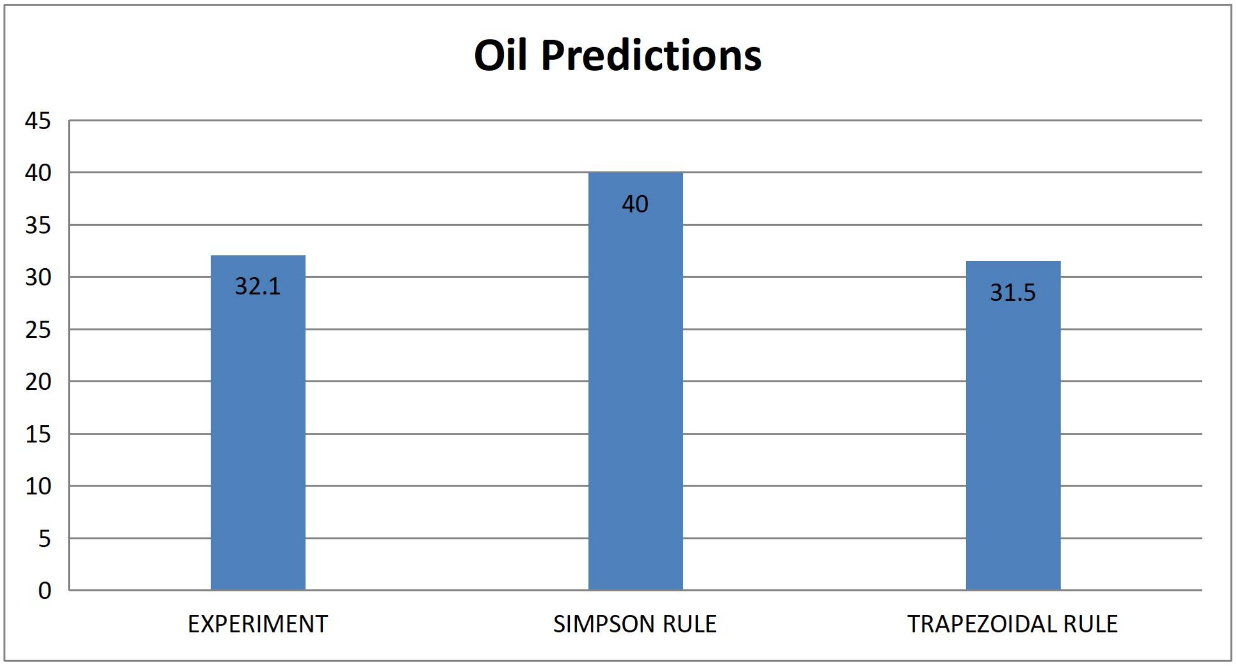


Figure 66: Comparison of Oil predictions for a surfactant flooding.

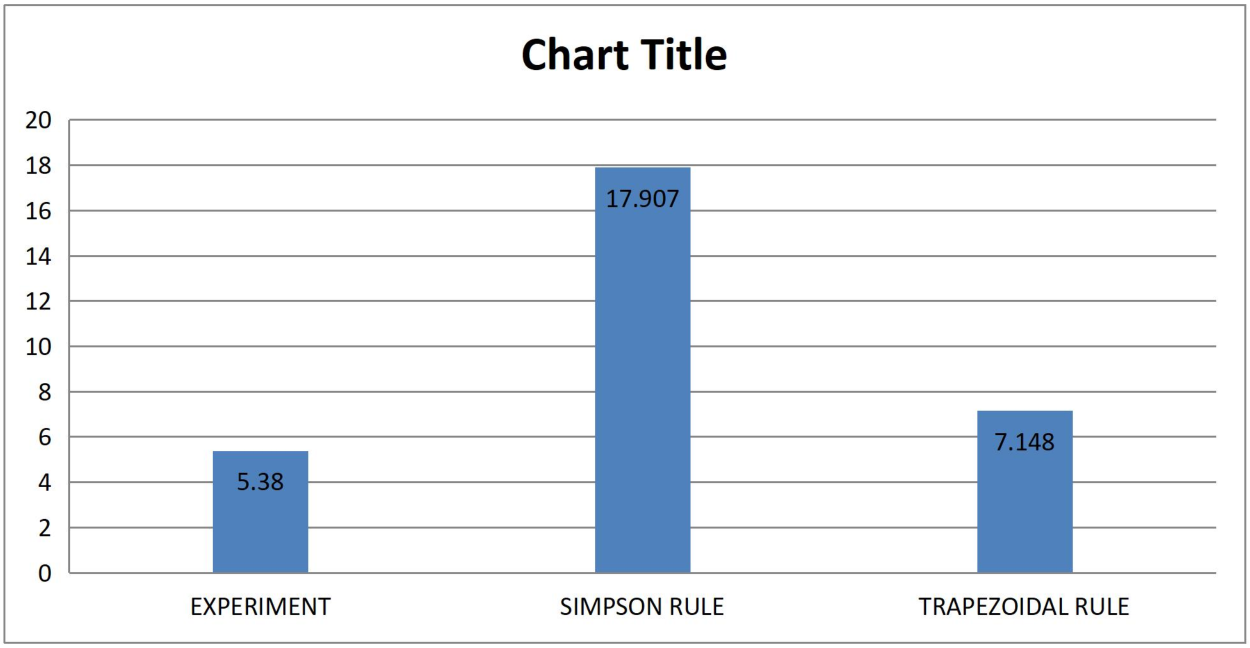


Figure 67: Error incurred by the prediction methods for a surfactant flooding.

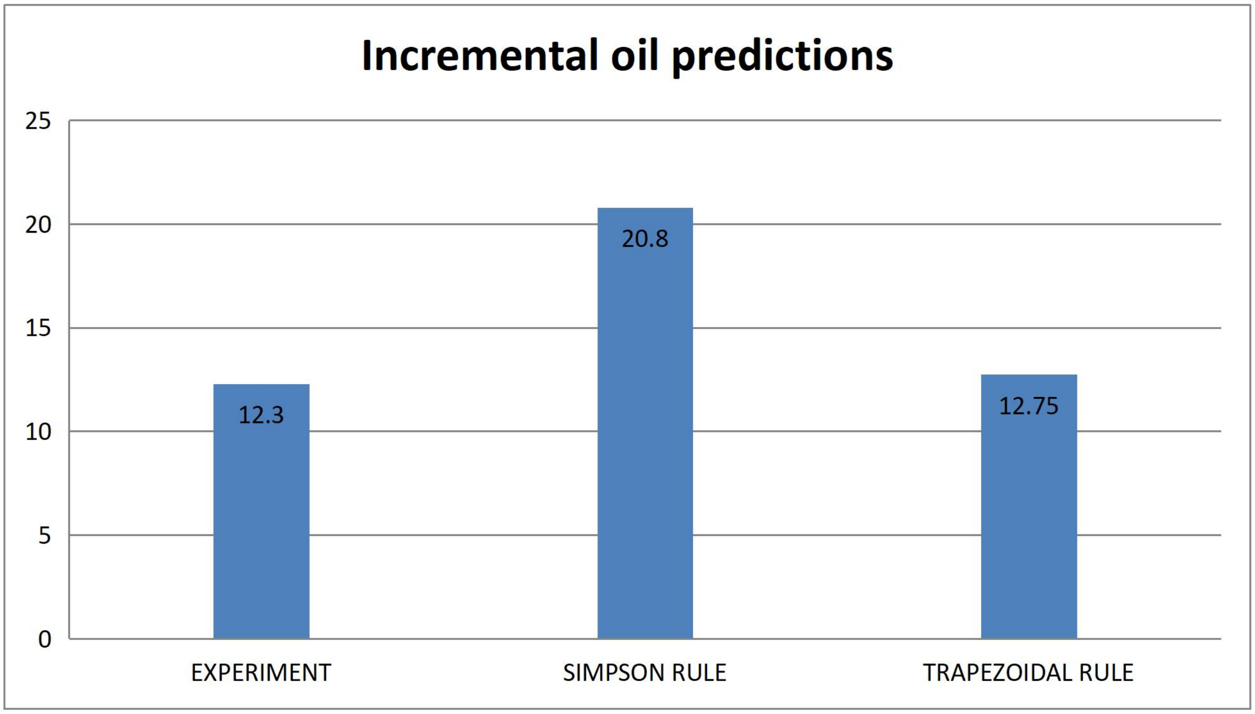


Figure 68: Comparison of incremental oil predictions for a surfactant flooding.

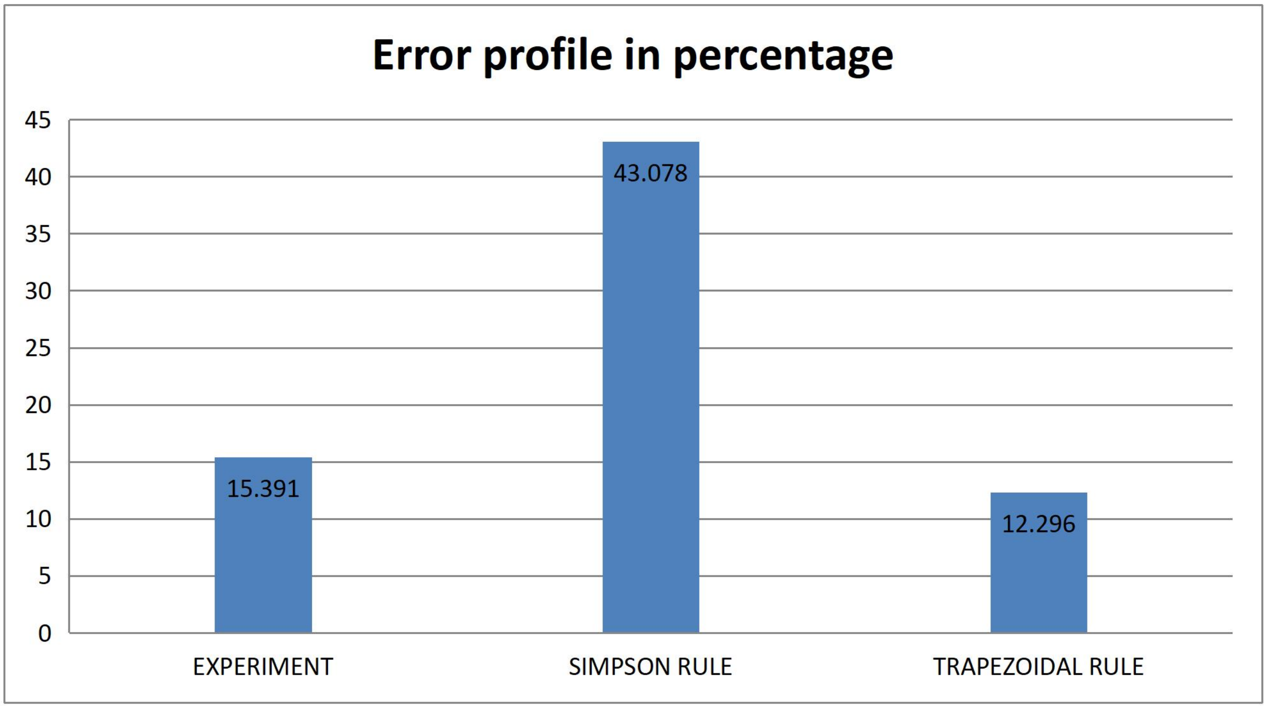


Figure 69: Error incurred by the prediction methods for a surfactant flooding.

CASE THREE:

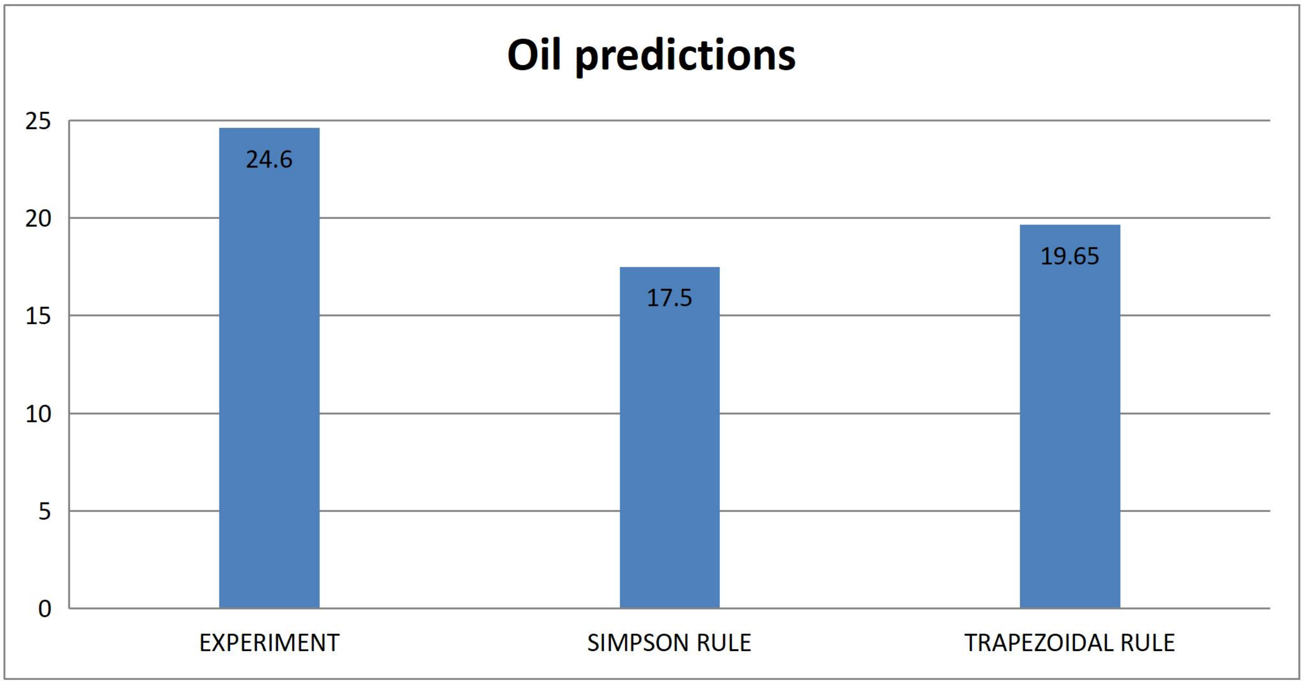


Figure 70: Comparison of Oil predictions for a surfactant polymer flooding.

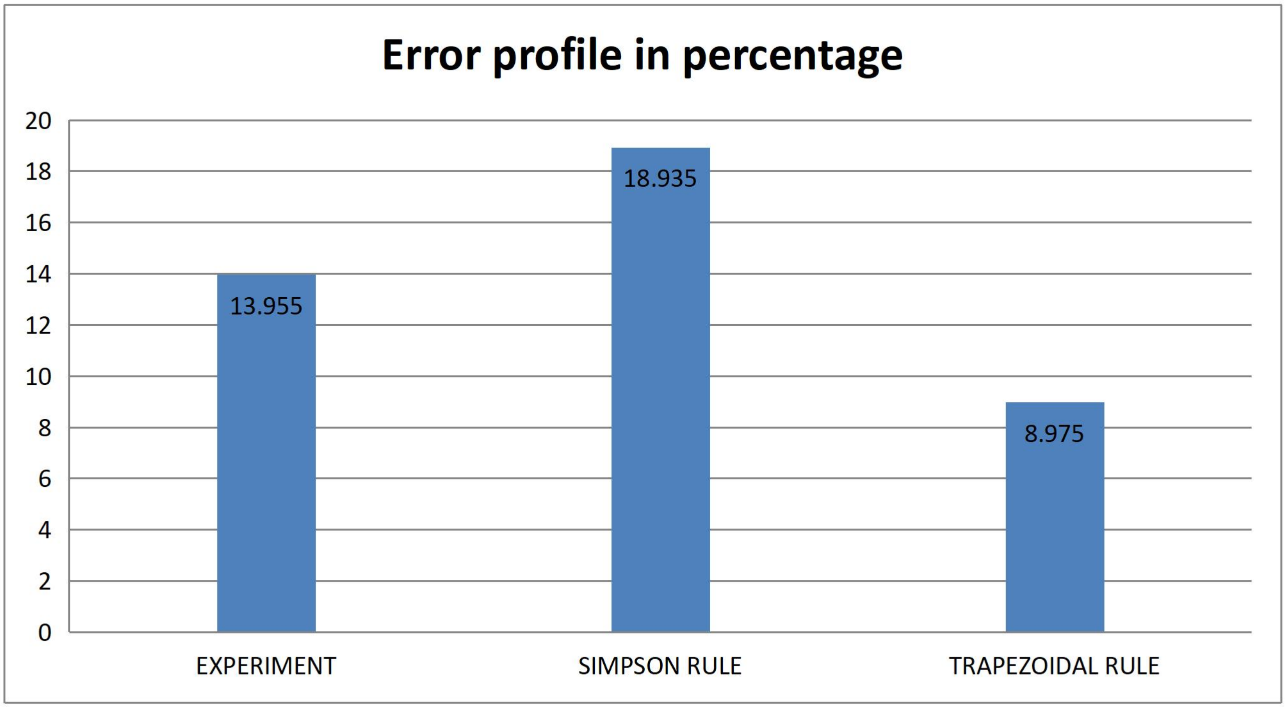


Figure 71: Error incurred by the prediction methods for a surfactant polymer flooding.

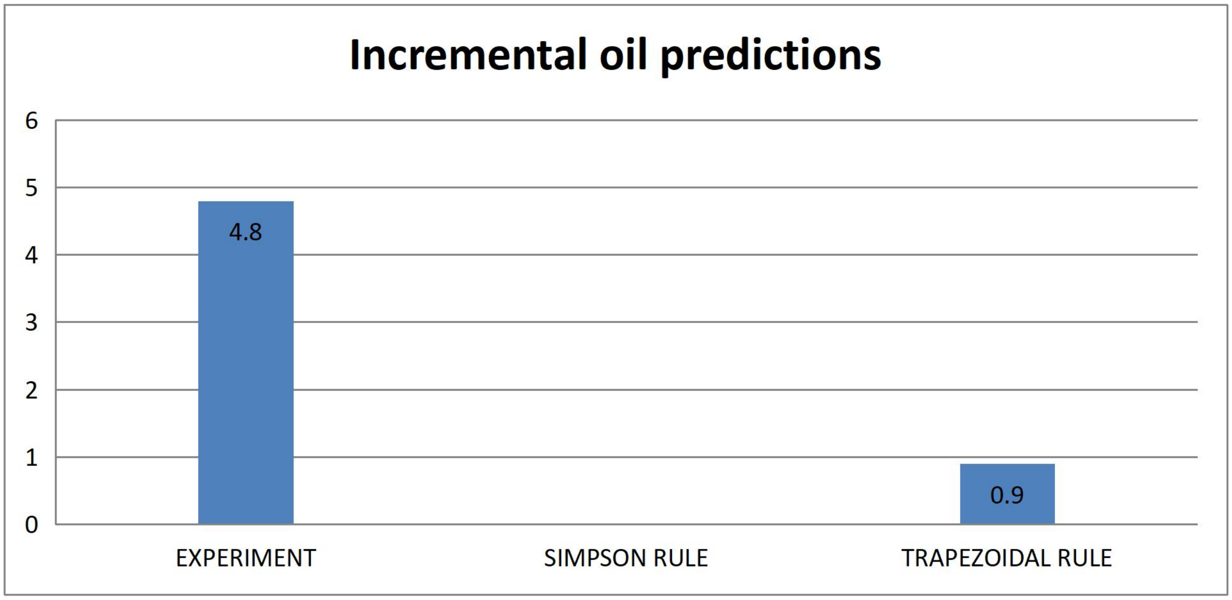


Figure 72: Comparison of Incremental oil predictions for surfactant polymer flooding.

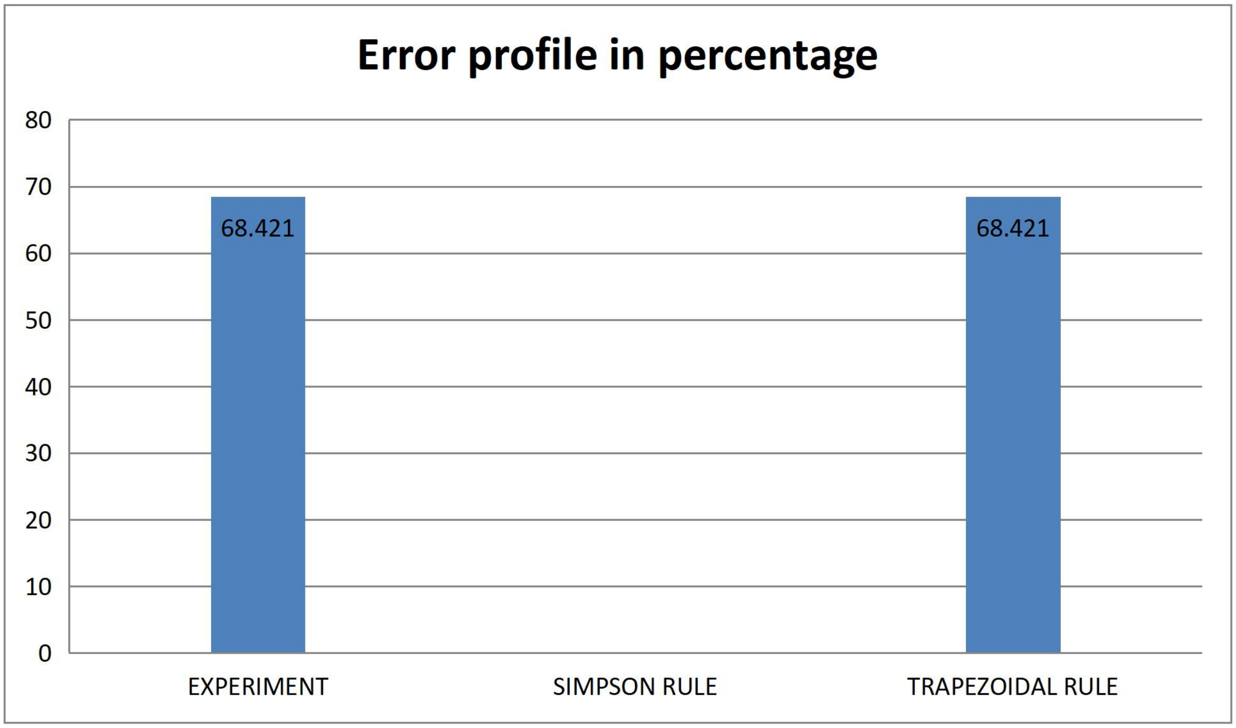


Figure 73: Error incurred by the prediction methods for a surfactant polymer flooding.

TRENDLINE EQUATION FOR A BASE CASE SCENERIO.

(A) WATER FLOODING.

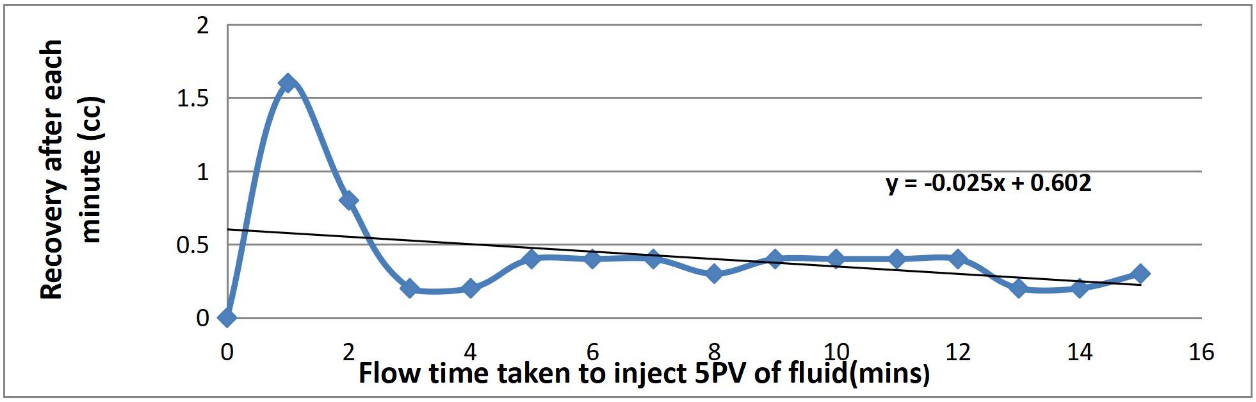


Figure 74: Rate time profile of water flooding.

OIL PREDICTIONS:

EXPERIMENT RESULT= 6.6cc

TRENDLINE EQUATION RESULT= 6.2175cc

FINAL RESULT= 6.40875cc

(B) SURFACTANT FLOODING

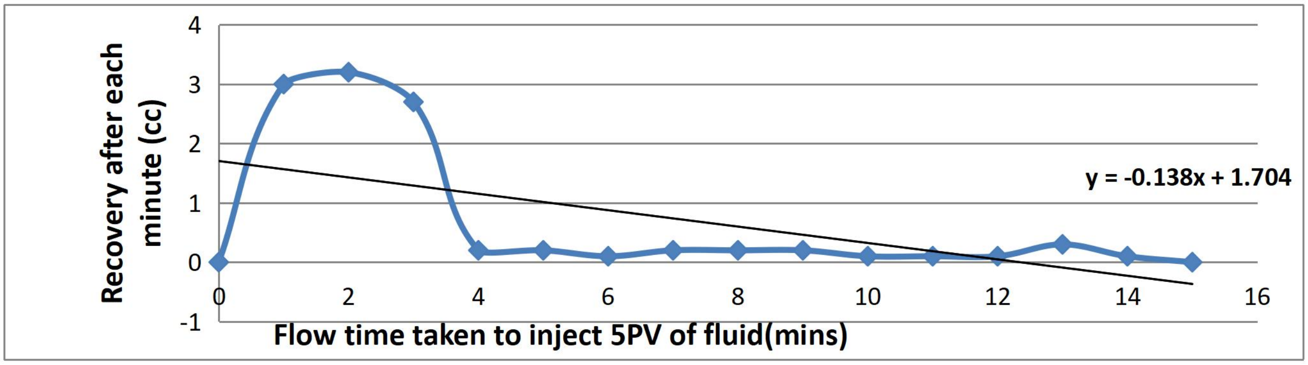


Figure 75: Rate time profile of surfactant flooding.

OIL PREDICTIONS:

EXPERIMENT RESULT= 10.7cc

TRENDLINE EQUATION RESULT= 15.147cc

FINAL RESULT= 12.9235cc

INCREMENTAL OIL PREDICTIONS:

EXPERIMENT RESULT= 4.1cc

TRENDLINE EQUATION RESULT=8.932cc

FINAL RESULT: 6.516cc

(C) SURFACTANT POLYMER FLOODING

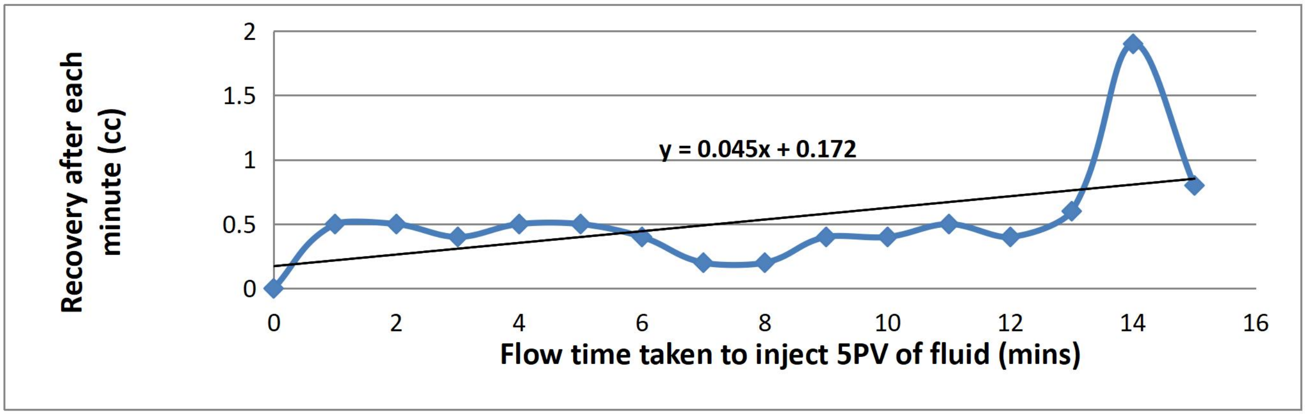


Figure 76: Rate time profile of a surfactant polymer flooding.

OIL PREDICTIONS:

EXPERIMENT RESULT= 8.2cc

TRENDLINE EQUATION RESULT= 7.6425cc

FINAL RESULT= 7.92125cc

INCREMENTAL OIL PREDICTION

EXPERIMENT RESULT= 1.6cc

TRENDLINE EQUATION RESULT= 1.425cc

FINAL RESULT= 1.5125cc

CASE A: WATER FLOODING

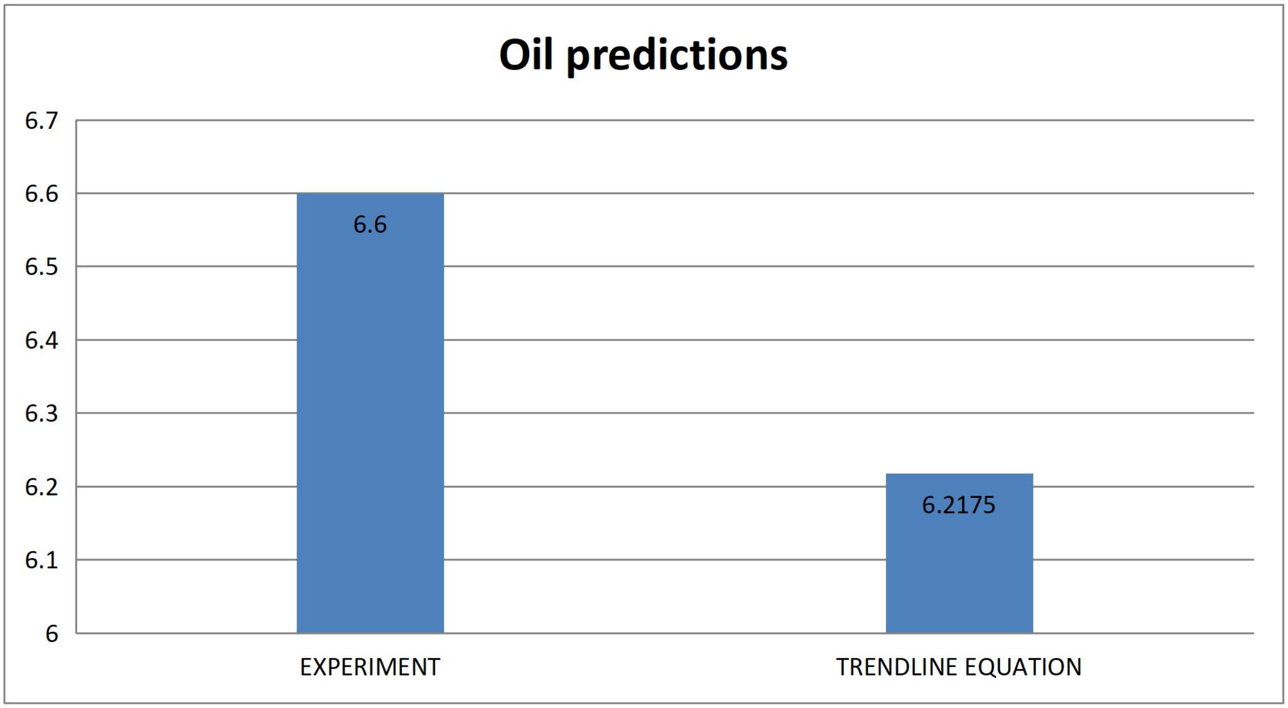


Figure 77: Oil prediction for a water flooding.

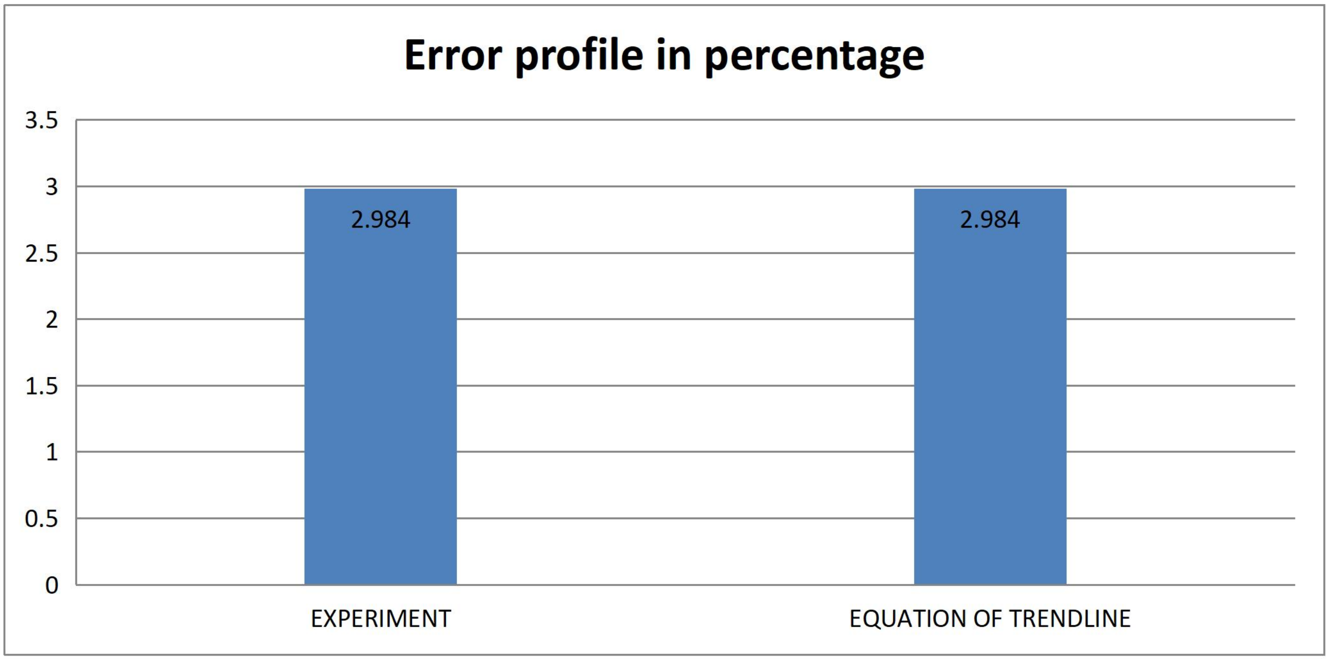


Figure 78: Error incurred by the prediction methods for a water flooding.

CASE B: SURFACTANT FLOODING

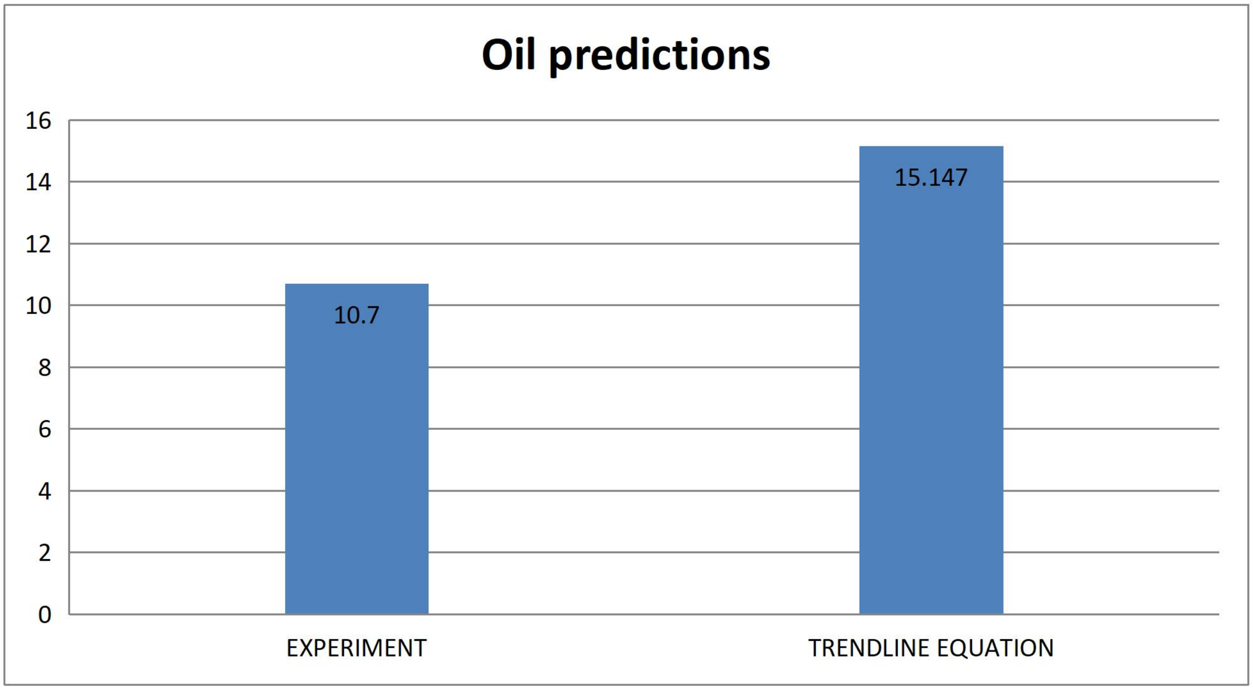


Figure 79: Oil prediction for a surfactant flooding.

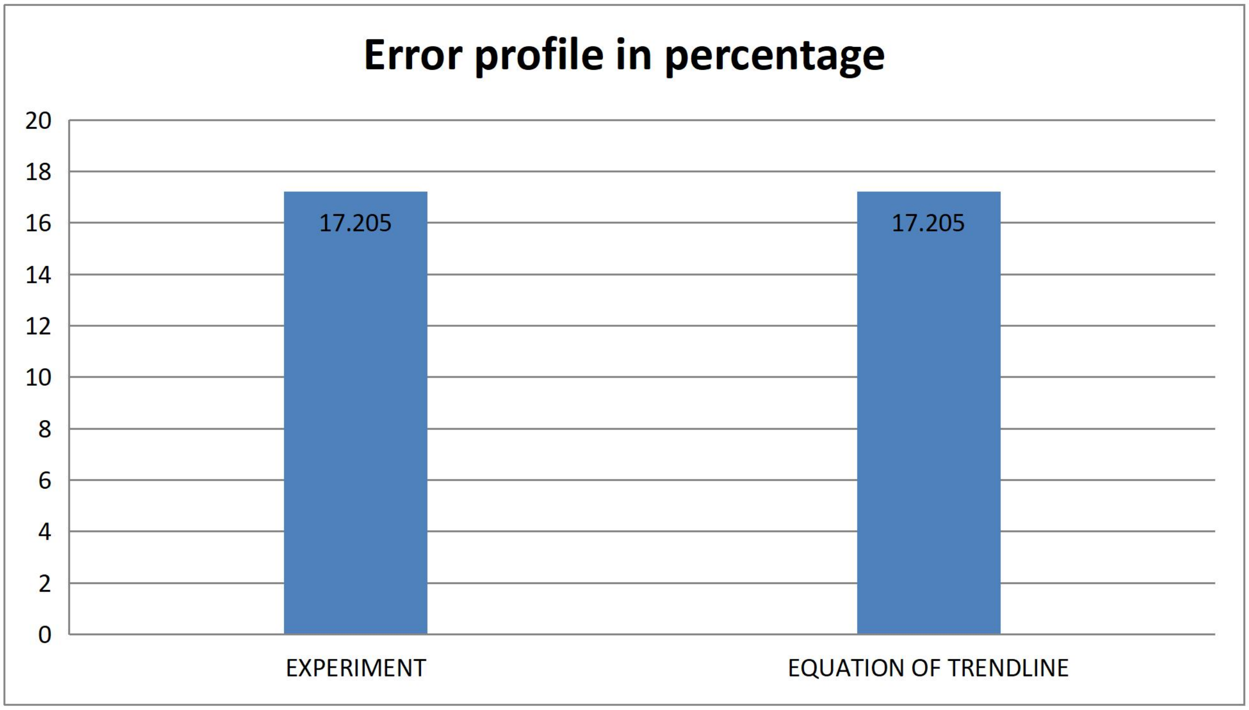


Figure 80: Error incurred by prediction methods for a surfactant flooding.

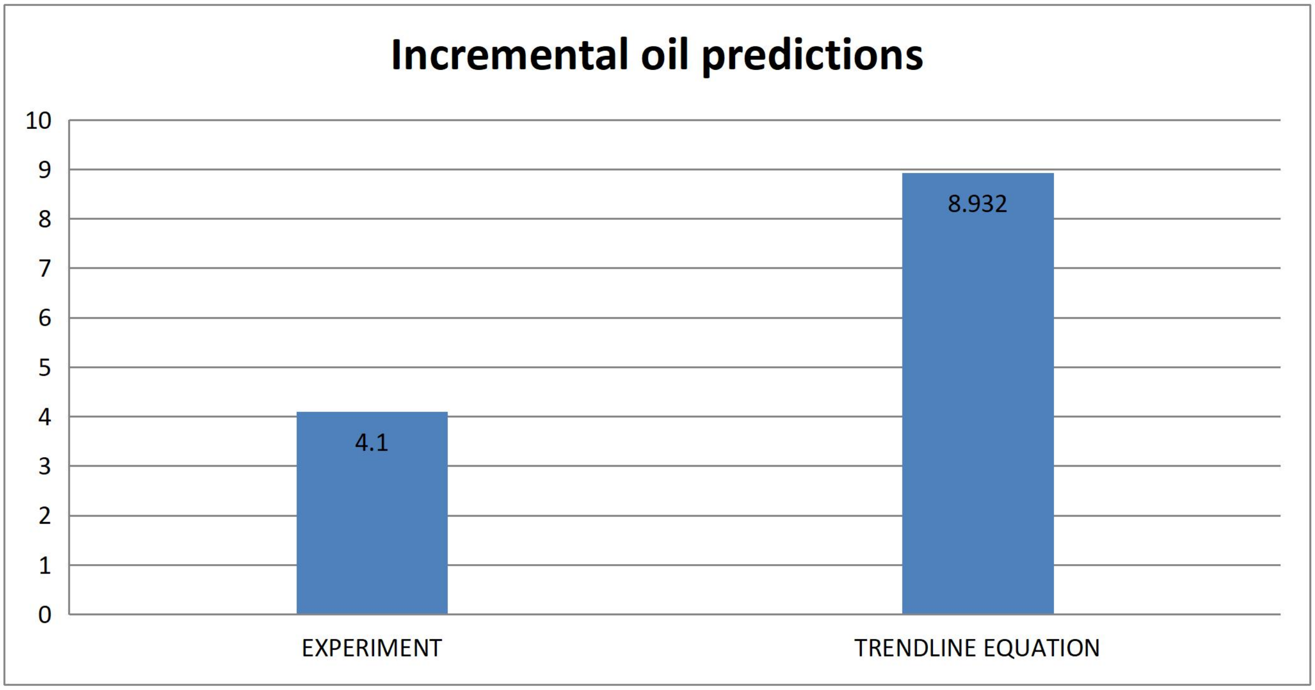


Figure 81: Incremental oil prediction for a surfactant flooding.

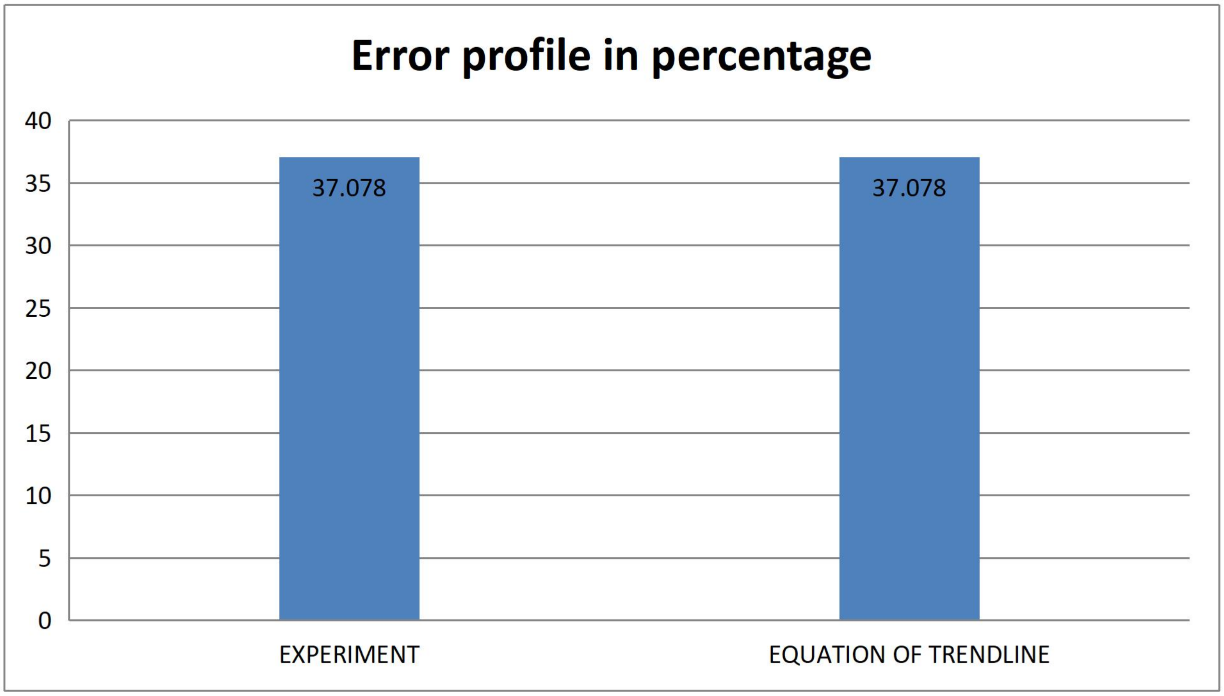


Figure 82: Error incurred by the prediction methods for a surfactant flooding.

CASE C: SURFACTANT POLYMER FLOODING.

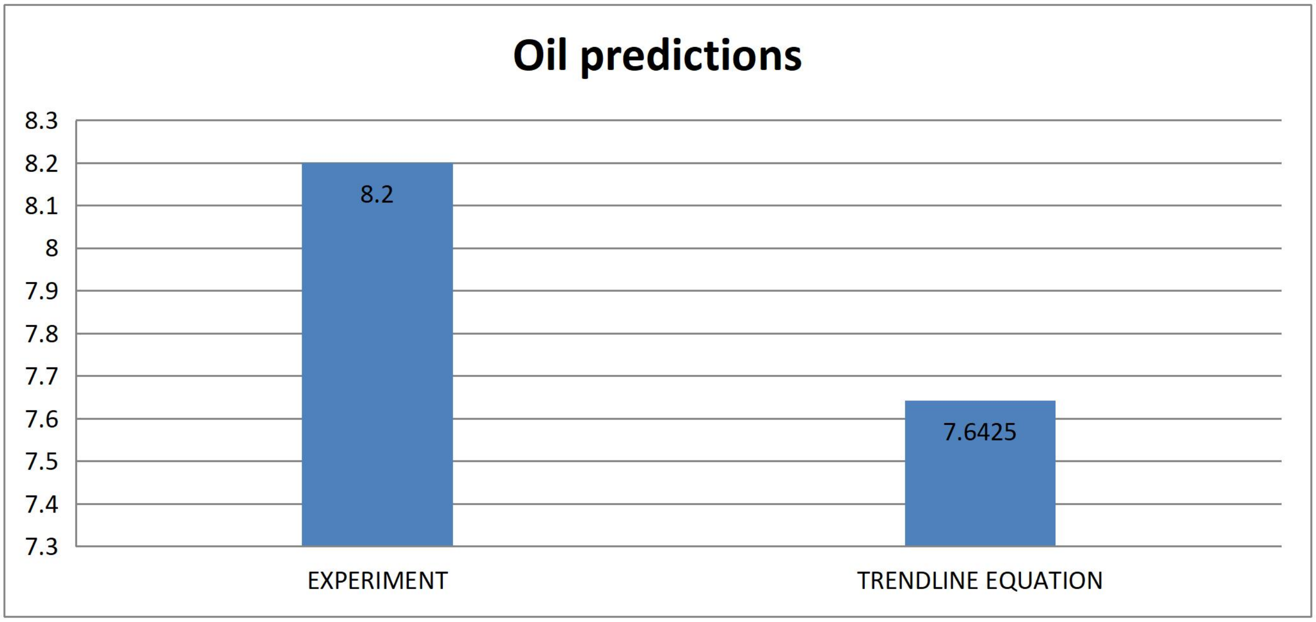


Figure 83: Oil prediction for a surfactant polymer flooding.

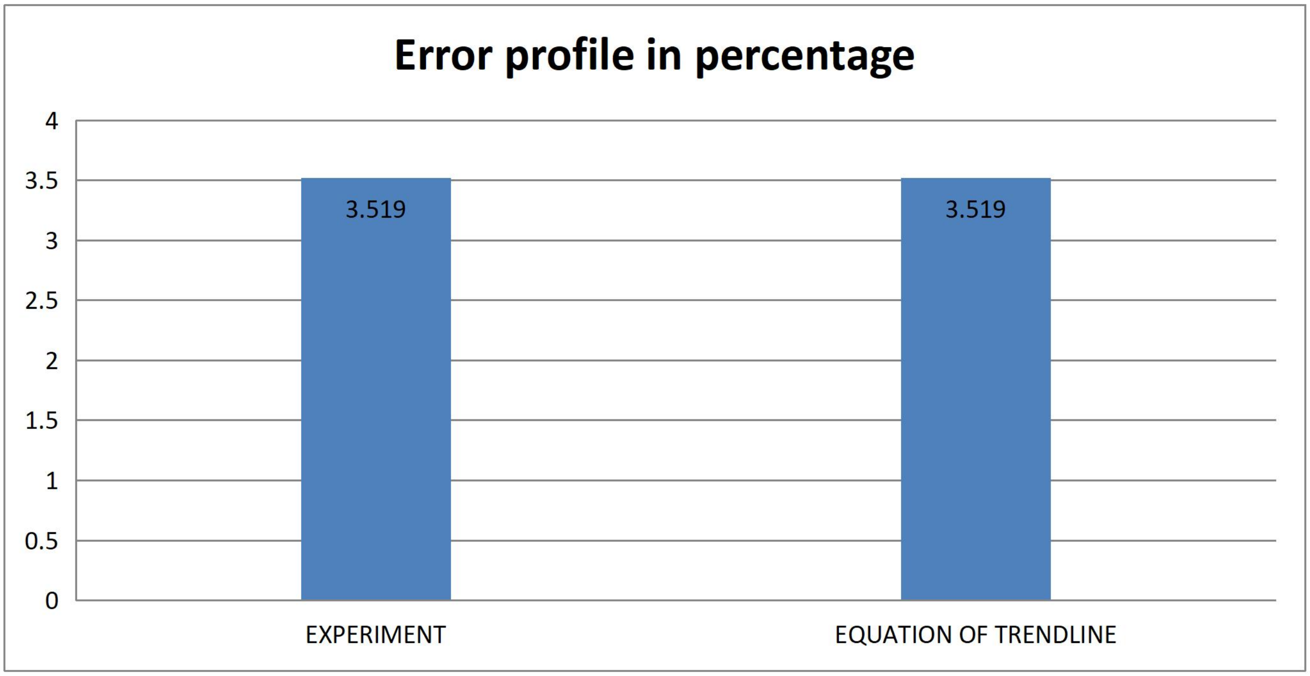


Figure 84 : Error incurred by the prediction methods for a surfactant polymer flooding.

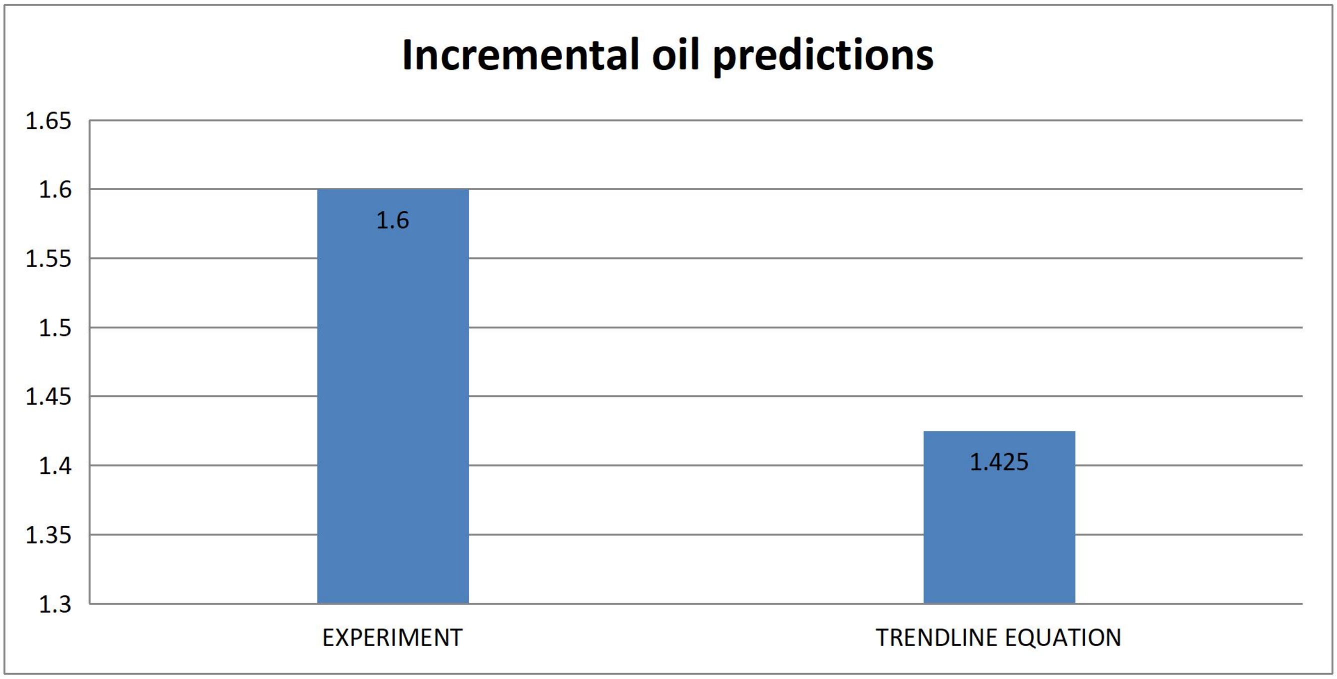


Figure 85: Incremental oil prediction for a surfactant polymer flooding.

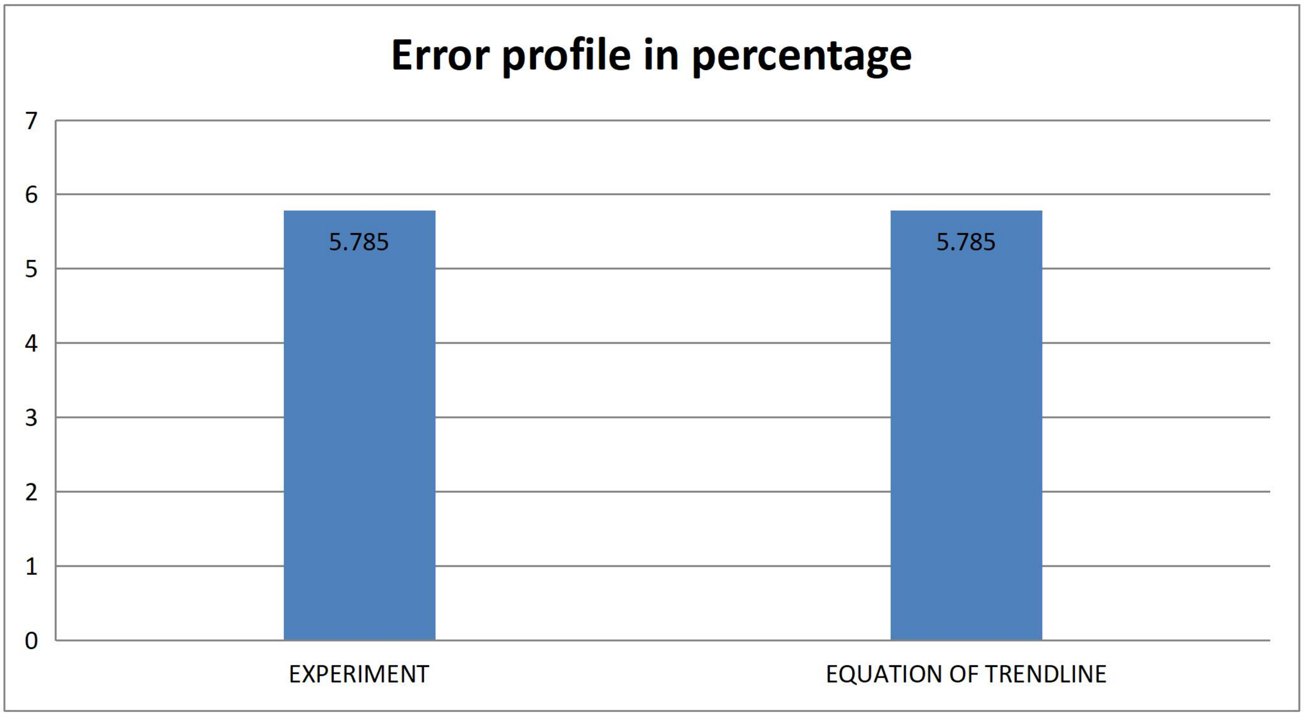


Figure 86 : Error incurred by the prediction methods for a surfactant polymer flooding.

CHAPTER FIVE

CONCLUSIONS AND RECCOMMENDATIONS

CONCLUSIONS

“From the analysis of the” result “the following conclusion can be drawn:”

- “The amount of incremental oil predicted by the” Simpson rule “and” trapezoidal “rule” are close to that measured in the laboratory.
- “The error due to trapezoidal can be attributed to its assumption of” the “linear relationship between data in approximation given data points, which is hardly obtainable in real field data.”
- “The error due to” Simpson’s “can be attributed to its” assumptions “of” the parabolic “relationship between data in approximation given data points, which is hardly obtainable in real field data.”
- “The introduction of Finite difference effectively helps account for the error terms in both” trapezoidal “rule and Simpson’s rule respectively and consequently” enhanced “their accuracies” by “the” quadrature.
- “Simpson’s rule” represents “a viable Numerical tool and gives a reliable prediction of incremental oil regardless of the number” of odd “data points considered”
- “The difference between the integral under the incremental oil recovery curve and the integral under the corresponding curve of the projected decline represents the incremental oil.”
- “The” Simpson’s error S_n and the Trapezoidal error T_n are attributed to the number of step sizes of the abscissa (flow time) for a rate time profile.
- Using a higher degree polynomial, as adopted by the “Simpson rule to” approximate “data points effectively and therefore when integrated gives better incremental oil” because this gives a robust definition of each data point, giving a more accurate result for the respective integral evaluation than its counterpart.

- The accuracy of the final result of the incremental oil is independent of the number of numerical methods used. Therefore thorough attention and concern are needed to the positive and negative deviation of the numerical methods result and/or estimation of the incremental oil for a particular EOR process with reference to one another and the experiment result.
- The incremental oils when Hele-shaw system is used are close to that of the real reservoir than the cylindrical core (physical model). A potential pit fall “of the 1D nature linear core flood arises during water” flooding; water “channels” will “be blocked” because of “limited pathways, hence producing the additional oil before” the “chemical consequently” break “through” unlike the Hele-shaw system with more water pathways present, there is possibility that this blockage mechanism will not be significant and only linear pathways are encounter in cylindrical core model unlike hele-shaw system that possess non linear flow pathways.

RECCOMENDATIONS

In other to make the numerical method more accurate, the following recommendation was made:

- Further studies should be done by “increasing the number of points” to analyze “the” volume “of the” incremental oil from the plot/graph of volumetric flow rate against time.
- Other Numerical methods such as the cubic spine, integral and variation method, rectangular method and finite difference could be “used in” the “study for the” volume “of the” incremental oil “governing the whole system. This will accommodate” more accurate results easily and quickly.
- The number of step/discrete sizes of the abscissa (time) for rate time curve should be reduce as possible.

- The mathematical models showing “the relationship between the” variables (“dependent and independent) is” ordinary differential equation with only one dependent variable, continuous function with lower and upper boundary. The volume of the additional oil can be calculated by taking direct anti-derivative of the integrand.
- The mathematical models showing the relationship between the variables (independent and dependent) is Partial and/or ordinary differential equation. It can be linearise and Transformed by Numerical Methods such as Finite difference, direct differential linearisation and laplace transformation, Fourier transformation, Bessel functions, z transformation respectively. Could “be used in the” quick study “for” the “cumulative oil recovery” “by taking the summation of the cumulative oil” volume at each step as a discrete function V=$\sum_{i=1}^{i=n}(qt)_i$…………………………………………………………………………equation 56

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NOMENCLATURE

T_n	Trapezoidal Error
S_n	Simpson Error
EOR	Enhanced Oil Recovery
I	Area under the graph
IOR	Improved Oil Recovery
RF	Recovery Factor
x	“Independent variable”
x_i	“Node”
$f_{i,i+1}andf_{i,i-1}$	“Interpolant or cubic”
$f_{i,i+1}'$	“First derivatives”
$f_{i,i+1}''$	“Second derivatives”
$\Delta\mathrm{f}_i$	“First forward difference”
N_p	“Cumulative production” cc
q	Volumetric “Recovery” flow “rate” cc/min
y_i	Knot
k_i	“Coefficient of the second derivatives”
$i,,J$	unit “vectors” or “position” vectors
h	Discrete step size in minutes
d^ny/dx^n	nth order differentials