A STUDY OF THE MOMENTS OF THE BETA DISTRIBUTION

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UNDERTAKING

This project was carried out by **Okonofua Godwin Junior** with matriculation number **PSC1707772** of the department of Statistics, hereby attest to writing this project by myself and where the views of others were expressed they were duly acknowledged.

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CERTIFICATION

This is to certify that this project research title “The Study of the Beta Distribution and its moments” was carried by **Okonofua Godwin Junior** with matriculation number PSC1707772 as part of the requirement for the award of Bachelor of Science (B.Sc) degree in Statistics from the Department of Statistics, Faculty of Physical Sciences, University of Benin, Benin City.

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DEDICATION

This project work is dedicated to God Almighty who in his promise kept me safe and strong all these years and also to my parents who gave me support and their love through my study I say thank you for all you have done.

ACKNOWLEDGEMENT

My infinity gratitude and thanks goes to God Almighty who kept me safe and strong all through years of study and who is still with me.

I will like acknowledge my project supervisor Prof N. Ekhousehi, for his guidance and advice throughout the process of this project I want to say I was highly honour to be one of your student I am indeed grateful.

I also want to appreciate my other lecturers for their fatherly love, they went a long way in shaping my characters and thinking I say thank you. to my friend's wisdom, Rolland, Francis and every other person, I say thank you. lastly to my brother man Andrew and my Course mate I appreciate you all.

I am very grateful to my Parents Mr. and Mrs. Okonofua for the love, care and support they have shown all through my life and my siblings for believing I me you all are most appreciated.

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ABSTRACT

This project work covers the study of the beta distribution which include its moments, the central moments, coefficient of variation, coefficient of skewness and coefficient of kurtosis. An estimation of the rth moment of the beta distribution can be symmetry (asymmetry), negatively skewed (left skewed) or positive skewed (right skewed) at different level of its fixed parameters.

CHAPTER ONE

1.0 INTRODUCTION

In probability theory of statistics, the beta distribution is a family of continuous probability defined on the interval [0,1] with two positive shape parameter, denoted *a* & [3, which appear as exponents of the random variable and control the shape of distribution, the beta distribution! can be used to represent proportion or probability outcomes.

For example, the beta distribution might be use to find now likely it is that your preferred candidate for mayor will receive 70% of the vote, it has also been use to model the behavior of random variable limited to interval of finite length in a wide variety of disciplines. For example, it has been used in the variability of soil properties and heterogeneity and in the probability of HIV transmission, and also the time it takes to complete a task and the proportion of defective treatment in a shipment, how likely that obaseki will win the next Govnorship election (you might think the probability is 0,2, my friend might think its 0.15 the beta distribution gives you a way to describe this)

Beta distribution can be applied to Bayesian inference due to the act that they provide in family of conjugate prior probability distribution for binomial Bernoulli and geometric distribution for example the beta distribution can be used in Bayesian analysis to described initial knowledge concerning probability of success such as the probability that a space vehicle will successfully complete a specified mission.

The beta distribution is a suitable model for the random behaviour of percentages and proportion.

The usual formulation of the beta distribution is also known as the beta distribution of the first kind whereas beta distribution of the second kind is an alternatives name for the beta prime distribution.

1.1 BASIC DEFINITION OF TERMS

1.1.1. PROBABILITY

Probability is a measure quantifying the likelihood that event will occur.

probability is a number between 0 and 1, where roughly speaking 0 indicates impossibility and 1 indicate certainly. The higher the probability of an event, the more likely it is that the event will occur.

1.1.2. RANDOM VARIABLE

A real value function X define on the space of Q an experiment is called a random variable. A random variable is a function which assign a real number to the point in the sample space of an experiment. Thus a random variable is a function whose domain of definition is the set of all outcome and whose range is a set of real number .it is random in the sense that the result of each outcome is dependent in chance, a random variable can either be discrete or continuous.

i. Discrete random variable

A discrete random variable is a variable which take on a finite or countable finite number determined by the outcome of random phenomenon, a discrete random variable is one in which the set or possible outcome for a random variable is countable.

The probability of each value of a discrete random variable is between Oand land all the sum of all probabilities is equal to 1.

ii. Continuous random variable

A continuous random variable is a random variable where the data can take infinitely many values, a continuous random variable X take all value in a given interval of numbers.

1.1.3 EXPECTED VALUE OF RANDOM VARIABLE X

Let x be a random variable, the expected value of the random variable X is denoted by E(X). the expected value of a discrete probability distribution is defined as:

$$E(x) = \sum_{t=1}^{n} X_1 P(X_i)$$

where P (X_j) is the probability mass function provided the summation exist.

Also, the expected value for the continuous distribution E(X) is defined by;

$$E(x) = \int_{-\infty}^{\infty} x f(x) dx$$

where; *f(x)* is the probability density function provided the integral.

1.1.4 PROBABILITY DENSITY FUNCTION PDF

A probability density function also known as the density of a continuous random variable is a statistical expression that define a probability distribution (the likelihood of an outcome) that take a continuous range of value. The probability density function of a continuous random variable is given as *f(x)*

PROPERTIES OF THE PDF

i. $f(x) \geq 0 \forall x$

ii. $\int_{-\infty}^{\infty} f(x) dx = 1$

A probability density function also known as the density of a continuous random variable is a statistical expression that define probability distribution (the likelihood of an outcome) that take a continuous range of value. The probability density function of the continuous random variable X is given as

1.2 PROBABILITY MASS FUNCTION

A probability mass function (PMF) is a function that gives the probability that a discrete random variable is exactly equal to some value. The probability mass function of a discrete random variable X is given as;

$$P(x) = P(X = x)$$

PROPERTIES OF THE PMF

i. $P(x) \geq \forall X$ that it must be positive

ii. $\sum_{t=1}^{n} P(X_i) = 1$

1.2.1. Moment generating function

The moment generating function is the expectation of a function of the random variable. Let X be a random variable with the distribution function *F(x)* now suppose $E(e^{tx})$ is finite fort in the open interval containing zero, then the moment generating function for a discrete random variable is defined by;

And the moment generating function for continuous random variable defined by the integral

1.2.2. VARIANCE OF A RANDOM VARIABLE

The variance of a random variable X is denoted by o^2 but sometimes can be written as Var(x). variance of a random variable can be defined as the expected value of the square difference the random variable and the mean L.

The variance X of any probability distribution function of discrete types is given by

$$Var(x) = \sum_{t=1}^{n} (X_i - \mu)^2 P(X_i)$$ provided the summation exist

Also the variance of any probability distribute on function of continuous random variable x is given by;

$$Var(x) = \int_{-\infty}^{\infty} (x - \mu)^2 F(x) dx$$

Theoretically can be express as;

$$Var(X) = E(X^2) - (E(X))^2$$ Provided the integral of£(X^2)m exist

1.2.3. SKEWNESS

Skewness is an asymmetry statistical distribution in which the curve appears distorted or skewed either to the left (The skewness value is negative i.e. the left tail is longer than the right tail or to the right the skewedness value is positive the right tail is longer than the left tail), the coefficient of skewness is given by:

$$Skweness = \frac{E(x-\mu)^3}{\sigma^3}$$

where μ is the mean and σ is the standard deviation.

1.2.4. KURTOSIS

Kurtosis is a statistical measure that defines how heavily the tails of a distribution differ from the tails of a normal distribution, and also measure how peaked a graph is, or how high the graph is around the mean. A positive value implies little data the tails while a negative value implies too much value or data at the tail

Kurtosis is expressed as;

$$Kurtosis = \frac{E((x-\mu))^4}{(\sigma^2)^2}$$

$$= \frac{E(x-\mu)^4}{(\sigma^4)}$$

where μ is the mean and a^2 is the variance

$$\sigma^4 = E(X^2) - (E(X^2))^2$$

1.3 COEFFICIENT OF VARIATION

The probability theory and statistics, the coefficient of variation also known as relative standard deviation is a measure of relative variability.it is the ratio of the standard deviation (σ)to mean

The higher the coefficient of variation the greater level of dispersion around the mean, the lower the value of the coefficient of variation the more, precise the estimate.

Coefficient of variation is defined as;

$CV = \frac{\sigma}{\mu}$ where σ is the standard deviation

μ is the population mean

1.4 AIMS AND OBJECTIVES

The aims of this project work is to study the beta distribution and the moments, the objectives of this study are as follows.

i. To obtain the mathematical expression of the r^{th} moment of the beta distribution

ii. To obtain the mathematical expression of the K^{th} central moment of the beta distribution

iii. To study the behaviour of the moment for some fixed value of it parameter.

1.5 SCOPE OF THE STUDY

This project covers the estimation of the beta distribution and to show the behaviour of the rth moment for some fixed parameter value and it interpretation.

CHAPTER TWO

2.1 INTRODUCTION

In this chapter, we shall discuss on the review of the beta distribution and survival of its generalization. The purpose of this will be to give a historical account of one of the most useful probability distribution which is the beta distribution.

2.2 REVIEW OF LITERATURE ON THE BETA DISTRIBUTION AND ITS GENERALIZATIONS

Historically, the work on beta distribution started as early as 1676 when Sir Isaac Newton wrote a letter to Henry Oldenburg (Dutka, 1981). In this letter dated 24th October 1676, Newton evaluated $\int_0^x ydx$ as a series. The particular case of the result was applicable to the evaluation of the beta distribution.

Gupta and Nadarajah (2004) wrote and published a book titled "Beta Distribution and its applications" and in the book they enumerated the properties of beta distribution and related Mathematical notions. Which demonstrates the applications in the fields of economics, quality control, soil sciences and biomedicine. It further discusses the centrally of beta distribution in Bayesian inference, the beta-binomial model and applications of the beta

Binomial distribution.

Libby and Novick (1982) proposed a three parameter generalized beta distribution for utility fitting. McDonald (1984) generalized beta distribution based on transformation as an income distribution and unifying the various research activities in closely related fields.

McDonald and Xu (1995) introduced a five parameter beta distribution which nest the generalized beta and gamma distributions and include more than thirty distributions as limiting or special cases.

McDonald and Richards (1987 a,b) presented a four generalized beta distribution which includes as special cases three and two parameter beta, generalized gamma, Weibull, power function pareto, lognormal, half normal, uniform and others.

Karl Pearson (1906) established that the beta distribution is couched as a solution of a differential equation. The beta distribution essentially identical to the Pearson's type 1 distribution except for arbitrary shifting and re-scaling (the beta and Pearson type 1 distribution can always be equalized by proper choice of parameter).

Armero and Bayarri (1994) suggested Guass hypergeometric function distribution in connection with marginal prior/posterior distribution. They obtained the generalized beta distribution by dividing the classical beta distribution by certain algebraic function.

Eugene et.al (2002) defined the beta-normal distribution and studied some of its properties. Since then, several author have been studying particular cases of this class of distribution. An advantage of beta-normal distribution is that the beta normal distribution can be unimodal or bimodal.

Hamza and Vallois (2016) characterize the kummer distribution of type two (the generalized beta distribution) as a solution of an equation involving gamma (respectively beta) distributions, which also give some almost sure realizations of krimmers distribution and generalized beta one using the conditioning method and the the rejection method as an application.

Tousi 2019 used the notion of generalized power function in the space of real symmetric matrices which is used to introduced the extended matrix variate beta function, with this define a different versions of extended matrix -variate beta distribution and also establish some fundamental properties of these distribution which show that using a linear transformation on the extended matrix variate beta

distribution and also used fixed iterative method provide a maximum posterior (MAP) estimator for the unknown covariance matrix of a multivariate normal distribution base on the class of the extended matrix -variate beta prior distribution.

According to Johnson et al (1995) the beta distribution are versatile and are able to model many types of uncertainties since it can be uni-modal, uni-antimodal, increasing and decreasing or constant depending on the value of their two parameters.

Barelkis and Maciulis 2017 proved that any beta distribution can be simulated by means of a sequence of distribution define through multiplicative function related to the generalized divisor function.

Chenchao etal 2012 studied asymptotic distribution of the beta distribution by means of decomposing the beta distribution properly and the classical analysis is truncation method which is proved that the asymptotic distribution of the beta distribution is normal distribution when the two parameter a and b of the beta distribution are all positives members.

Higbee et al 2019 have make the asymmetric log-laplace all and the generalized beta distribution of the second kind (GB2), in many applications which they demonstrate that all is a limiting case of the GB2 and examine their ability to model stock returns.

CHAPTER THREE

3.0 METHODOLOGY

In this chapter, we shall introduce one of the family continuous probability distributions, the beta distribution defined on the interval [0,1] parameterized by two positive shape parameters, denoted by oc and p, we shall also introduce the r^{th} moments measure of Skewness, Kurtosis, mode, variance and also the shape of beta distribution and we also establish the relationship between beta function and Gamma function.

3.1 THE CONTINOUS PROBABILITY DISTRIBUTION FUNCTION OF BETA DISTRIBUTION

Beta distribution are based on gamma function. A brief description of these two function is therefore in order

Beta function

The beta function is a special function denoted by (3 (a, b) and defined d as

$$\beta(a,b) = \int_0^1 x^{a-1}(1-x)^{b-1}\,dx, a > 0, b > 0$$

Gamma function

The gamma function is a special function denoted by T and defined as

$$\Gamma(a) = \int_0^{\infty} t^{a-1} e^{-t} dt$$

Relationship between the function and of gamma function.

3.2 CONSTRUCTION OF THE CLASSICAL BETA DISTRIBUTION

From the Beta function

$$\beta(a,b) = \int_0^1 x^{a-1}(1-x)^{\beta-1} dx, a > 0, b > 0$$

Normalizing the beta function gives;

$$\int_0^1 \frac{x^{a-1}(1-x)^{b-1} dx}{\beta(a,b)} = 1$$

As a result, the probability density function of the beta distribution with shape parameters a and b is expressed as;

$$F(x : a,b) = \frac{x^{a-1}(1-x)^{b-1}}{\beta(a,b)} . 0 < x < 1 . a > 0, > b > 0$$

The cumulative distribution of equation 3.1 is given by;

$$F(x) = \frac{1}{\beta(a,b)} \int_0^x y^{a-1}(1-y)^{b-1} dy$$

Where the parameter a and b are positive real quantities and the variable x satisfies 0<x<l. The quantity *p {a, b)* is the beta function. Equation (3.1) is also known as the standard beta or classical beta distribution.

3.3 PROPERTIES OF CLASSICAL BETA DISTRIBUTION R™ ORDER MOMENTS

Rth Moment

The r^{th} moment of a random variable x about the origin provided it exist is expressed as;

$$E\left(X^r\right) = \mu_r = \int_{-\infty}^{\infty} X^r f(x)dx, r = 1,2,3,4$$

To obtain the r^{th} moment of the beta distribution will evaluate;

$$m_r = E\left(X^r\right) = \int_0^1 X^r f\left(x : a, b\right) dx$$

$$= \int_0^1 X^r \frac{X^{a-1}\left(1-X\right)^{b-1}}{\beta X\left(a,b\right)} dx$$

$$= \frac{1}{\beta\left(a,b\right)} = \int_0^1 \frac{X^{(a+r)-1}\left(1-x\right)^{b-1}}{\beta\left(a+r,b\right)} . \beta\left(a+r,b\right) dx$$

$$= \frac{\beta\left(a+r,b\right)}{\beta\left(a,b\right)}$$

$$= \frac{\Gamma\left(a+r\right)\Gamma\left(b\right)}{\Gamma\left(a+b+r\right)} \frac{\Gamma\left(a+b\right)}{\Gamma\left(a\right)\Gamma\left(b\right)}$$

$$= \frac{\Gamma\left(a+r\right)\Gamma\left(a+b\right)}{\Gamma\left(a+b+r\right)\Gamma\left(a\right)}$$

3.4 THE FIRST FOUR MOMENTS

The mean is found by substituting r=l in equation (3.3)

$$m_1 = \mu = E\left(X\right) = \frac{\Gamma\left(a+1\right)\Gamma\left(a+b\right)}{\Gamma\left(a+b+r\right)\Gamma\left(a\right)}$$

$$= \frac{a\Gamma\left(a\right)\Gamma\left(a+b\right)}{\Gamma\left(a+b+1\right)\Gamma\left(a\right)}$$

$$= \frac{a\Gamma\left(a\right)\Gamma\left(a+b\right)}{\left(a+b\right)\Gamma\left(a+b\right)\Gamma\left(a\right)}$$

$$= \frac{a\Gamma\left(a\right)\Gamma\left(a+b\right)}{\left(a+b\right)\Gamma\left(a+b\right)\Gamma\left(a\right)}$$

$$= \frac{a}{a+b}$$

Substituting r=2 in equation (3.3)

$$m_2 = E\left(X^2\right) = \frac{\Gamma(a+b)\Gamma(a+2)}{\Gamma(a+b+2)\Gamma(a)}$$

$$= \frac{\Gamma(a+b)(a+1)\Gamma(a+1)}{(a+b+1)\Gamma(a+b+1)\Gamma(a)}$$

$$= \frac{(a+1)}{(a+b+1)(a+b)}$$

$$= \frac{a(a+1)}{(a+b)\Gamma(a+b+1)}$$

$$m_2 = \frac{(a+1)}{(a+b+1)}$$

Substituting r=3 in equation (3.3)

$$m_3 = E\left(X^3\right) = \frac{\Gamma(a+b)\Gamma(a+3)}{\Gamma(a+b+3)\Gamma(a)}$$

$$= \frac{\Gamma(a+b)(a+2)\Gamma(a+1)a\Gamma(a)}{(a+b)\Gamma(a+b+2)(a+b+1)\Gamma(a+b)\Gamma(a)}$$

$$= \frac{(a+2)(a+1)a}{(a+b)(a+b+2)(a+b+1)}$$

$$m_3 = \frac{m_2(a+2)}{(a+b+2)}$$

Substituting r=4 in equation (3.3)

$$m_4 = E\left(X^4\right) = \frac{\Gamma(a+b)\Gamma(a+4)}{\Gamma(a+b+4)\Gamma(a)}$$

$$= \frac{\Gamma(a+b)(a+3)(a+2)\Gamma(a+1)a\Gamma(a)}{(a+b)\Gamma(a+b+1)(a+b+2)(a+b+3)\Gamma(a+b)\Gamma(a)}$$

$$= \frac{(a+1)(a+2)(a+3)a}{(a+b)(a+b+1)(a+b+2)(a+b+3)}$$

$$m_4 = \frac{m_3(a+3)}{(a+b+3)}$$

MODE

Mode is obtained by solving

$$\frac{d}{dx}F(x,a,b) = 0$$

$$\frac{d}{dx}\left(\frac{y^{a-1}(1-x)^{b-1}}{\beta(a,b)}\right)$$

$$= \frac{(a-1)X^{a-2}(1-x)^{b-1} + (b-1)(-1)(1-x)^{b-2}x^{a-1}}{\beta(a,b)}$$

$$(a-1)x^{a-2}(1-x)^{b-1} = (b-1)(1-x)^{b-2}x^{a-1}$$

$$(a-1)X^{a-1}X^{-1}(1-X)^{b-1} = (b-1)(1-x)^{b-1}(1-x)^{-1}X^{a-1}$$

$$(a-1)X^{-1} = (b-1)(1-x)^{-1}$$

$$= \frac{a-1}{b-1} = \frac{x}{1-x}, a > 1, b > 1$$

$$(1-x)(a-1) = x(b-1)$$

$$(a-1-xa+x) = xb-x$$

$$\Rightarrow x(-a+1-b+1) = 1-a$$

$$x = \frac{(a-1)}{a+b+2}. x = 0, if\ a = 1\ and\ x = 1\ if\ b = 1$$

3.5. THE KTH CENTRAL MOMENT OF THE BETA DISTRIBUTION

The kth central moment of a random variable X is defined by;

$$M_k = E(X-\mu)^k$$
$$= \sum_{k=0}^{n} \binom{n}{k} (1-\mu)^k \mu^{n-k},\ k = 2,3,4$$

VARIANCE

Since variance can be expressed as;

$$Var(X) = E(X^2) - [E(X)]^2$$
$$= m^2 - m_1^2$$

We can now get the variance as;

$$Var(X) = \sigma^2 = \frac{a(a+1)}{(a+b)(a+b+1)} - \left[\frac{a}{a+b}\right]^2$$
$$\frac{a(a+1)(a+b) - a^2(a+b)(a+b+1)}{(a+b)(a+b+1)(a+b)^2}$$
$$\frac{a(a+1)(a+b) - a^2(a+b)(a+b+1)}{(a+b+1)(a+b)^2}$$
$$\frac{a^3 + a^2b + a^2 + ab - a^3 - a^2b + a^2}{(a+b+1)(a+b)^2}$$
$$\frac{ab}{(a+b+1)(a+b)^2}$$

SKEWNESS

This is the third central moment, which can be expressed as;

$M_3 = E(X^2) - [E(X)]^3$

$E(X^3) - 3E(X^2)E(X) + 2[E(X)]^3$

$= m_3 - 3m_2 m_1 + 2m_1^3$

It can be further simplified as follows:

$$M_3 = \frac{a(a+1)(a+2)}{(a+b)(a+b+1)(a+b+2)} - 3\frac{a^2(a+1)}{(a+b)^2(a+b+1)} + 2\left(\frac{a}{a+b}\right)^3$$

$$= \frac{a(a+1)(a+2) - a^2(a+1)(a+b+2)}{(a+b)^2(a+b+1)(a+b+2)} + \frac{2a^3}{(a+b)^3}$$

$$= \frac{(a^3+3a^2+2a) - 3a^4 - 3a^3b - 9a^3 - 3a^2b - 6a^2}{(a+b)^2(a+b+1)(a+b+2)} + \frac{2a^3}{(a+b)^3}$$

$$= \frac{a^4 + 3a^3 + a^3b + 3a^2b + 2ab - 3a^4 - 3a^3b - 9a^3 - 3a^2b - 6a^2}{(a+b)^2(a+b+1)(a+b+2)} + \frac{2a^3}{(a+b)^3}$$

$$= \frac{-2a^4 - 6a^3 - 4a^3 - 2a^2b + 2ab}{(a+b)^2(a+b+1)(a+b+2)} + \frac{2a^3}{(a+b)^3}$$

$$= \frac{(a+b)(-2a^4 - 6a^3 - 4a^2 - 2a^3b + 2ab) + (2a^4 + 2a^3b + 2a^2)(a+b+2)}{(a+b)^3(a+b+1)(a+b+2)}$$

$$= \frac{-2a^5 - 6a^4 - 2a^4b - 4a^3 + 2a^2b - 2a^4b - 6a^3b - 2a^3b - 4a^2b + 2ab^2 + 2a^2 + 2a^4b}{(a+b)^3(a+b+1)(a+b+2)}$$

$$= \frac{\begin{array}{c} -2a^5 - 6a^4 - 4a^3 - 2a^4b + 2a^2b + (-2a^4b - 6a^3b - 4a^2b - 2b^2a^3 + 2b^2a) \\ +2a^5 + 2a^4b + 4a^4 + 2ab + 2a^3b^2 + 4a^3b + 2a^4 + 2a^3b + 3a^3 \end{array}}{(a+b)^3(a+b+1)(a+b+2)}$$

$$= \frac{-2a^2b - 4a^2b + 2b^2a}{(a+b)^3(a+b+1)(a+b+2)}$$

$$= \frac{-2a^2b + 2b^2a}{(a+b)^3(a+b+1)(a+b+2)}$$

$$= \frac{-2a^2a + 2b^2a}{(a+b)^3(a+b+1)(a+b+2)}$$

$$= \frac{-2a^2b(b-a)}{(a+b)^3(a+b+1)(a+b+2)}$$

Hence skewness given as $\frac{M_3}{\sigma^3}$

$$E\left(\frac{X-E(X)}{\sigma}\right)^3$$

$$\frac{1}{\sigma^3}\left(\frac{2ab(b-a)}{(a+b)^3(a+b+1)(a+b+2)}\right)$$

$$=\frac{1}{\left(\sqrt{\frac{ab}{(a+b+1)(a+b)^2}}\right)^2}\left(\frac{2ab(b-a)}{(a+b)^3(a+b+1)(a+b+2)}\right)$$

$$=\frac{1}{\sqrt{\frac{ab}{(a+b+1)(a+b)^2}}\;\frac{ab}{(a+b+1)(a+b)^2}}\left(\frac{2ab(b-a)}{(a+b)^3(a+b+1)(a+b+2)}\right)$$

$$=\frac{\sqrt{(a+b+1)(a+b)^2}}{\sqrt{ab}}\;\frac{(a+b+1)(a+b)^2}{ab}\left(\frac{2ab(b-a)}{(a+b)^3(a+b+1)(a+b+2)}\right)$$

$$=\frac{\sqrt{(a+b+1)(a+b)^2}}{\sqrt{ab}}\times\frac{(a+b)^2(2(b-a))}{(a+b)^3(a+b+2)}$$

$$=\frac{\sqrt{(a+b+1)(a+b)^2}}{\sqrt{ab}}\times\frac{(a+b)^2\,2(b-a)}{(a+b)(a+b)^2(a+b+2)}$$

$$=\frac{\sqrt{(a+b+1)}\,2(b-a)}{\sqrt{ab}\;(a+b+2)}$$

$$=\sqrt{\frac{a+b+1}{ab}}\left[\frac{2(b-a)}{(a+b+2)}\right]$$

CHAPTER FOUR

In this chapter we consider the moment of the beta distribution for some fixed value of the parameters.

Recall that the moment is defined by:

$$\mu_1 = \frac{a}{a+b}$$

$$\mu_2 = m_1 = \frac{(a+1)}{(a+b+1)}$$

$$\mu_3 = m_2 = \frac{(a+2)}{(a+b+2)}, \ \mu_4 = m_3 = \frac{(a+3)}{(a+b+3)}$$

$$Variance = \frac{ab}{(a+b+1)(a+b)^2}$$

$$Skewness = \sqrt{\frac{a+b+1}{ab}}\left[\frac{2(b-a)}{(a+b+2)}\right]$$

$$Kurtosis = \frac{(a+b+1)}{ab}\left(\frac{3\left[a^2(b+2)-2ab+b^2(a+2)\right]}{(a+b+2)(a+b+3)}\right)$$

Coefficient of variance $= \dfrac{\sigma}{\mu}$

Table 4.1: Moment of the Beta distribution for some fixed value of the parameter.

Moments	$\sigma = 0.6$ $\beta = 0.2$	$\sigma = 0.6$ $\beta = 0.4$	$\sigma = 0.6$ $\beta = 0.6$	$\sigma = 0.6$ $\beta = 0.8$
μ_1	0.7500	0.600	0.5000	0.4286
μ_2	0.6667	0.4800	0.3636	0.2857
μ_3	0.6191	0.416	0.2954	0.2185
μ_4	0.5865	0.3744	1.2532	0.1788
Variance	0.1042	0.1200	0.1136	0.1020
Coefficient of variation	0.4304	0.5774	0.6741	0.7452
Skewness	-1.1066	-0.5443	0	0.2631
Kurtosis	1.3195	1.025	1.9152	2.1905

In Table 1, we observe that for a fixed value of the left shape parameter a=0.6 and as the right shape β increase β = 0.2, 0.4, 0.6, 0.8, the value of the r^{th} moments and variance decreases, the coefficient of variation, measures of skewness increases, while the measure of kurtosis decreases for $a > \beta$ and increases for $a < \beta$.

Table 4.2: Moment of the Beta distribution for some fixed value of the parameter.

Moments	$\sigma = 0.3$ $\beta = 0.5$	$\sigma = 0.5$ $\beta = 0.5$	$\sigma = 0.6$ $\beta = 0.5$	$\sigma = 0.7$ $\beta = 0.5$
μ_1	0.375	0.5	0.5455	0.5833
μ_2	0.2708	0.375	0.4156	0.3803
μ_3	0.2224	0.3125	0.3486	0.4507
μ_4	0.1931	0.2734	0.3061	0.3350
Variance	0.1302	0.125	0.1181	0.11045
Coefficient of variation	-0.9622	0.7071	0.6299	0.5698
Skewness	0.0989	0	-0.17069	-0.3134
Kurtosis	2.6330	1.9441	1.7721	1.3163

In Table 2, also we observe a fixed value of the right shape parameter β = 0.5 and as the left shape parameter increase a = 0.3, 0.5, 0.6, 0.7, the value of the r^{th} moments decreases, the coefficient of variation ,measures increases, while the measure of skewness and kurtosis decreases.

Table 4.3: Moment of the Beta distribution for some fixed value of the parameter.

Moments	$\sigma = 0.8$ $\beta = 0.2$	$\sigma = 0.8$ $\beta = 0.4$	$\sigma = 0.8$ $\beta = 0.6$	$\sigma = 0.8$ $\beta = 0.8$
μ_1	0.8000	0.6667	0.5714	0.5000
μ_2	0.7200	0.5455	0.4286	0.3462
μ_3	0.6720	0.4773	0.3529	0.2692
μ_4	0.6384	0.4318	0.3048	0.2221
Variance	0.0800	0.1010	0.1020	0.0962
Coefficient of variation	0.3536	0.4767	0.5590	0.6202
Skewness	-14142	-0.655	-0.2631	0
Kurtosis	1.2315	1.451	1.6592	1.8531

In Table 4.3, the moment of the beta distribution for a fixed value of the left shape parameter $a = 0.8$ and different value of the right shape parameter β = 0.2, 0.4, 0.6, 0.8, which increase the value of the r^{th} moment decrease, the variance increase fora > β and decreases for $a < \beta$, the coefficient of variation and measure of skewness increases, while the measure of kurtosis increase for $a > \beta$ decreases for $a < \beta$

CHAPTER FIVE

5.0 INTRODUCTION

This chapter present the summary and the conclusion of the entire study carried out on the beta distribution.

5.1 SUMMARY

This project work has been able to show the study of beta distribution in five chapters. In chapter one, we see that the beta distribution is define on the interval [0, 1]. On the other hand, we also review some work in chapter 2.

In chapter 3, we introduce the methodology in mathematical expression of the beta distribution also drive an expression for the rth moment.

The mean and variance are the first two statistical moments which provides information on the locations and variability of a set of distribution. The third moment defines the skewness of a distribution. It is a measure of the symmetry of the shape of a distribution. If the skewness of a distribution is zero, then the distribution is said to be symmetry. The distribution is positively skewed, if there is a long tail in the positive, negative and negatively skewed if there is a long tail in the negative direction.

Finally, the fourth moment is used to define the kurtosis of a distribution which measures the peakness fatness of a distribution

5.2 CONCLUSION

We reviewed that the beta distribution is one of a kind probability distribution on probabilities which typically models an ancestry of probabilities, beta curve distribution is considered to be a versatile resourceful way to described outcome, since the beta distribution represent a probability it domain is bounded between 0 to 1 for example how likely Obaseki will win the governorship election different probability value might arise from different individual but the beta distribution finds a beta way in explain this work better.

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