APPLICATION OF TRANSPORTATION MODEL TO SOLVE TANKERS ROUTINE PROBLEM

BY

ODUAH NWACHUKWU JOSHUA
PSC2105453

DEPARTMENT OF MATHEMATICS
FACULTY OF PHYSICAL SCIENCES
UNIVERSITY OF BENIN, BENIN CITY, NIGERIA

NOVEMBER, 2025

APPLICATION OF TRANSPORTATION MODEL TO SOLVE TANKERS ROUTINE PROBLEM

BY

ODUAH NWACHUKWU JOSHUA
PSC2105453

A RESEARCH PROJECT SUBMITTED TO THE DEPARTMENT OF MATHEMATICS, UNIVERSITY OF BENIN, BENIN CITY
IN PARTIAL FULFILMENT TO THE REQUIREMENT FOR THE AWARD OF B.SC HONOURS IN MATHEMATICS (INDUSTRIAL MATHEMATICS)

NOVEMBER, 2025

UNDERTAKING

This project work was carried out by me ODUAH NWACHUKWU JOSHUA with the matriculation number PSC2105453.

I have not plagiarized any existing work. All published work used in this project work have been cited and referenced appropriately

ODUAH NWACHUKWU JOSHUA
Date

CERTIFICATION

This is to certify that this project work was carried out by ODUAH NWACHUKWU JOSHUA, under the supervision of PROF C.I Nkeki, in the partial fulfillment of the requirements for the award of the degree of Bachelor of science in Industrial mathematics in the department of mathematics, faculty of physical science, university of Benin, Benin city.

PROF. C. I. NKEKI (Project Supervisor)	Date

PROF DANIEL OKUONGHAE (Head of Department)	Date

DEDICATION

I dedicate this project to the owner of my life, the lover of my life, the one and only through GOD, JESUS CHIRST who was there for me throughout my journey in University and always be there for me and I also dedicate this project to my lovely parents MR&MRS ODUAH for being there lovely support and care, prayers and encouragement bestowed on me

ACKNOWLEDEGEMENT

I sincerely want to appreciate GOD Almighty for seeing me through during my stay of study in the university

I wish to express my gratitude to the following persons, support me and make my educator and project a success which cannot be over emphasized

First and foremost, I wish to express my sincerely appreciation to my project supervisor Prof C.I Nkeki, for his attention, guidance suggestion and most of all for the knowledge passed across me. Despite your busy schedules you still find the time o assist me in making my project work a reality and successful. Thank you very much sir, God bless you sir.

My sincere gratitude and appreciation to my loving parent MR & MRS Oduah, for love, support, care, assistance, and encouragement throughout my study in the university, may the almighty God continue to bless

I specially want to appreciate my pastors; firstly the General Overseer of Mountain of Fire and Miracle Ministry DR D.K Olukoya, The Regional Overseer Pastor Chike Willams, The Blessed Deliverance Ministers and Pastors in mountain of Fire and Miracle Ministry, and also sincerely want to appreciate Pastor Clement Moses and the rest Pastors and Leaders in my church, and fellowship, that have always be there for me in journey in the university may the Almighty God continue to bless you richly.

Also specially want to appreciate my loving siblings (my big sister Oduah Grace, my brother Oduah Godwin), for their support, advice, prayers and care may the Lord God bless you

Finally, I would like to thank my colleagues, friends course mates which i exchange ideas with such as mercy, Blessed, Solomon and all my course mates and lecturers in mathematics department may the almighty God continue to bless you all in Jesus name

TABLE OF CONTENTS

ABSTRACT

This study applies the transportation model as an optimization tool to address tanker routes problem in petroleum distribution. The research focuses on minimizing transportation cost reducing fuel consumption, and improving operational efficiency in the delivery of petroleum products from supply depots to various demand centers using linear programming techniques, the transportation problem was formulated to determine the optimal routes and cost- effective allocation of tankers. Data were obtained for selected depots and filling stations in Benin city Edo state and analyzed using the Northwest Corner Rule (NCR), Least Cost Method (LCM), Vogel Approximation Method (VAM), Russell Approximation Method (RAM) and Stepping-Stone Optimal Solution.

The result revealed that the optimal total transportation cost obtained through the stepping stone was ₦49,700,000 per tanker, representing a significant reduction from the initial feasible solution derived through other methods ₦55,400,000. The findings demonstrate that the transportation model effectively minimizes operational cost, enhance fleet utilization and ensures timely delivery or petroleum products. The study concluded that adopting linear programming and transportation models provides a reliable decision -support tool for logistics managers in the petroleum industry, ensuring improved cost-efficiency and sustainability in tanker operation.

CHAPTER ONE

INTRODUCTION

Transportation models are a type of linear programming problem. Transportation problems involve selection of minimum cost routes in which goods and services are shipped from a supply center (origin) to the demand centre (destination).

The origin of transportation model sets been to 1941 when F.L Hitchcock presented a study entitled the distribution of a product from several sources to Numerous localities. The presentation (F.L Hitchcock)is said to be the first important contribution to the solution of transportation problems. In 1947,J.C Koopmans presented a study, not related to Hitchcock called optimum utilization of the transportation methods, which involves a number of shipping sources and a number of destination within a given time period, each destination certain requirement with a given cost of shipping from the source has a certain capacity and each destination certain requirement with a given cost of shipping from to the destination.

Various definitions highlighted the characteristics of the transportation problem as given by several authors. Hiller and Liabarmu (1980) describe the transportation problem as probably the most important special type of linear programming problem (Lpp), according to them, the general transportation problem in practice is concerned with distribution of any commodity from any group of supply centers called sources, to any group of receiving centers called distributions, in such a way as to minimize total distribution costs. A basic assumption is that the cost of distributing unit from a source say I, to a destination say j, given e_{ij} is directly proportional to the numbered distributed.

Ehadurn (1996) defined the transportation problem as a decision problem concerned with finding the minimum cost combination of routes linking origins of goods or services to their destination.

In the recent, we aim at discussing transportation models in general and apply them to tanker's-routing problems via linear programming techniques.

The problem of Tankers routing was first investigated in Dantzig and Fulkerson (1954). Their model aims at finding the least number of tanker to satisfy a fixed schedule in the delivery of petroleum products.

This research aims at extending a tanker's-routing model which uses a discrete integer programming approach to determine efficient and effective distribution of petroleum products.

1.1 STATEMENT OF THE PROBLEMS.

Many logistics companies face high operational costs due to inefficient routing of tankers, challenge such as; utilized tankers, longer travel times, and fuel wastage are common without segmentic model, routing decisions are often suboptimal. The need arises for a data -driven model that ensure cost_effective and timely delivery of goods using tanker vehicles.

1.2 SCOPE OF THE STUDY

The study focuses on applying the classical transportation model to their logistics of tankers distributing liquid goods while the model is scaleble, this project assume known supply and demand point transportation cost and fleet availability.

1.3 OBJECTIVE AND AIM OF THE STUDY

. To develop a transportation model suitable for routing tankers.

. To analyze the existing tanker routing system and identify an unnecessary costs.

. To apply Linear programming and transportation modeling techniques in designing an optimized routing strategy for tanker operation.

. To stimulate real-world scenarios

. To provide decision - support tools for logistics managers in the tanker industry using mathematical optimization.

. To minimize the total transportation and operation cost.

. To ensure timely delivery of good to the required destinations.

. To increase fleet utilization and reduce empty miles

. To demonstrate the effectiveness of optimization model in logistics planning.

1.4 AIMS OF THE STUDY

The aim of the study is to develop and apply transportation model that optimizes the routing of the tankers, with the primary goal of minimizing transportation costs and maximizing operational efficiency.

1.5 SIGNIFICANCE OF THE STUDY

The significance of the study lies in it's potential to address real world logistical and economic challenge in tanker transportation. Tanker which transport bulk liquid like fuel oil and chemicals, often face routing inefficiencies, due to poor planning, fluctuating demands, traffic conditions and operational constraints.

The study aims to demonstrate how transportation models, particularly linear programming and optimization techniques ,can be effectively applied to reduce transportation costs and increase efficiency.

CHAPTER TWO

LITERATURE REVIEW

2.1 INTRODUCTION TO TRANSPORTATION MODEL

Transportation models are a subset of linear programming models designed to minimize the cost of transportation goods and services to several destinations while satisfying supply and demand constraints. Transportation models have long been used to solve logistics challenges involving the optimal allocation and routing of tankers to minimize operational costs and enhance efficiency

2.2 THEORETICAL FOUNDATION OF TRANSPORTATION MODEL

Transportation models are based on various theoretical foundations from Operation Research (OR), mathematics, and economics. Some key theoretical foundations included:

1. Linear programming (LP); Linear programming is a method used to optimize linear constraints. Many transportation problems, such as transportation cost minimization, can be modeled as LP problems.
2. Network Flow Theory: This theory deals with the flow of goods, services, or information through a network.it provides a framework for modelling and

solving transportation problems such as finding the shortest path or maximum flow in a network.

3. Graph Theory: Graph theory is used to represent transportation networks as graphs, when nodes represent locations and edges represent routes. The Theory provides a basis for solving routing and scheduling problems.
4. Dynamic Programming: Dynamic Programming is a method to solve complex problems by breaking them down into smaller sub- problems. It is often used in transportation models to solve problems such as inventory routing and dynamic routing.

5.Integer Programming: Integer Programming is a method used to solve optimization problems where some or all of the variables are restricted to integer values. Many transportation problems, such as vehicle routing and scheduling, can be modeled as integer programming problems

6.Stochastic Process: Stochastic Processes are used to model uncertainty in transportation systems, such as random demand or travel times.

7. Game theory: Game theory is used to model competitive situations in transportation, such as competition between carriers or models of transportation.

2.3 KEY CONCEPTS:

1. Optimization: The process of finding the best solution of the set of feasible solutions
2. Feasibility: The ability of a solution to satisfy all constraints
3. Optimality: The property of a solution that is the best among all feasible solutions
4. Scalability: The ability of a model to handle large problem instances

2.4 APPLICATIONS:

1.Route Optimization: Finding the shortest or most efficient route for vehicles or goods

2.Scheduling: scheduling vehicles, drivers, or other resources to minimize costs or maximize efficiency

3. Capacity planning: Determining the optimal capacity of transportation infrastructure or vehicles

4. Inventory Management: Managing inventory levels and distribution to minimize costs and maximize customer satisfaction.

These theoretical foundations and concepts provide a basis for developing and applying transportation models to solve complex problems in various industries.

2.5 REPRESENTING THE TRANSPORTATION PROBLEM AS A LINEAR PROGRAMMING PROBLEM

With the special transportation condition that the total amount of units variable at the various sources must be equal to the total amount demanded at all the destinations, the mathematical description or model of transportation is ;

$$C_{11}X_{11} + \cdots + C_{1n}x_{1n} + C_{21}X_{21} + \cdots + C_{2n}X_{2n} + C_{m1}X_{m1} + \cdots + C_{mn}X_{mn} \qquad (1)$$

Subject to

$$\left.\begin{array}{lllll} X_{11} & +X_{12} & +\cdots & +X_{1n} & = S_1 \\ X_{21} & +X_{22} & +\cdots & +X_{2n} & = S_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ X_{m1} & +X_{m2} & +\cdots & +X_{mn} & = S_m \end{array}\right\} \text{(Demand) (2)}$$

$$\left.\begin{array}{lllll} X_{11} & +X_{21} & +\cdots & +X_{m1} & = D_1 \\ X_{12} & +X_{22} & +\cdots & +X_{m2} & = D_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ X_{1n} & +X_{2n} & +\cdots & +X_{mn} & = D_n \end{array}\right\} \text{(Supply) (3)}$$

$$X_{11},\ X_{12},\ \ldots,\ X_{1n},\ X_{21},\ X_{22},\ \ldots,\ X_{m1},\ X_{m2},\ \ldots, X_{mn} \geq 0 \quad (4)$$

In the standard interpretation of the model m constraints, (given by (2)) State that the amount shipped from sources, $_i$, to all the n destination, must equal the amount available at the source must equal the amount available at the source, that is s_i, the remaining n constraints (given by (3)) assert that the amounts shipped to a destination, $_j$, from all the m sources must, in total, be equal to amount required at the destination. The available supplies at the various sources and the demand requirement are fixed in reference to a stated time interval or planning horizon

The cost of shipping each unit from source $_i$ to demand point $_j$, is c_{ij}. The non-negative quantity, x_{ij} represents the amount of goods shipped from source I to destination $_j$. It should be observed that the quantity is integer - Valued. This is obtained provided the amounts a available at the various sources and these demanded at the various destinations are all positive integers. Hiller and Lieberman (1980) observe that because of the special structure of the model, if such a model has any feasible solution, it always will have an optimal solution with just integer values.

It should be noted that if the unit cost of prior an item differs from source then differs from solution to sources then this cost is included in the determination of c_{ij}.

The transportation condition asserts that

$$\sum_{i=1}^{m} S_i = \sum_{j=1}^{n} Dj \qquad (5)$$

This condition requires that the system be in balance. Below is a typical transportation table.

Sources Destination

	1	2	3	…	N	Supply
1	C_{12} X_{11}	C_{12} X_{12}	C_{13} X_{13}	…	C_{1n} X_{1n}	S_1
2	C_{21} X_{21}	C_{22} X_{22}	C_{23} X_{23}	…	C_{2n} X_{2n}	S_2
3	C_{31} X_{31}	C_{32} X_{32}	C_{33} X_{33}	…	C_{3n} X_{3n}	S_3
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
M	C_{m1} X_{m1}	C_{m2} X_{m2}	C_{m3} X_{m3}	…	C_{mn} X_{mn}	S_m
Demand	D_1	D_2	D_3	…	D_n	

In apply the model, Wegner (1989) and Agbedude(1996),stated that implied in the model b(I) to (4) assumption that only a single type of commodity is being shipped, that is the commodities are homogeneous. Wagner went on to explain that this because, in meeting the demands requirements, the models does not distinguish among the sources of supply.

2.6 THE TANKER ROUTING PROBLEM

The tanker Routing problem (TRP) refers to the optimal scheduling and routing of tanker vehicles that transport with liquid (e.g. , petroleum, chemicals or water) access a net of supply and demands nodes. The problem is a type of Vehicle Routing problem (VRP), a man complex generalization of the transportation model. Some of the complexion include:

1. Multi- Commodity Flow (e.g: Different types of fuels).
2. Routing constraints due to road conditions, safety standards and time on schedules.
3. Fleet limitations and maintenance schedule
4. Dynamic traffic and delivery demands
5. Environmental and cost concerns (fuel, Labour, carbon emissions)

2.7 OPTIMIZATION APPROACHES FOR TANKERS -ROUTING

Some of the recent studies apply optimization techniques to improve tankers routing efficiency are;

1.Linear Programming (LP) and Integer Linear programming (ILP):

LP and ILP techniques allow for more complex constraints like binary decision (e.g, tanker goes or doesn't go to a nodes), capacity limitation, and the windows and also their techniques used to model basic routing and cost minimization (Laporte 1992)

2. Metaheuristic Algorithms; Metaheuristic algorithms such as Genetic Optimization (GA),Ant colony Optimization (ACO), and Particle Swan Optimization(PSO), provide new - optimal solutions for complex Nphard routing problems.

3. Mixed - Integer programming (MIP): Mixed - integer programming, offers flexible modelling of constraints and objectives.

2.8 REAL- WORLD APPLICATION ON TRANSPORTATION MODELS IN PETROLEUM AND CHEMICAL INDUSTRIAL.

Numerous Research studies, highlights the application of transportation model in the petroleum and chemical industries, where tankers are used;

1. Hitchcock (1941)

   Work: The distribution of a product from several sources to numerous localities.

   Application: Laid the foundation for the classical transportation model. Generally it directly applies to fuel distribution from refineries(supply).

2. Koopmans (1947)

   Work: Optimum utilization of the transportation system

   Application: provided an economic interpretation of the transportation model, used later in petroleum products from multiple depots to high-demand urban center at minimum cost

3. Dantzig and Ramser (1959)

   work: The truck dispatching problem

   Application: First to propose the Vehicle Routing Problem (VRP),a direct model for tanker routing in oil and chemical distribution, minimizing cost and distance while meeting demand at multiple filling stations

4. Brown, Grace and Rones(1987)

   Work: scheduling ocean transportation of crude oil (operation Research,35(1) 147_159).

Application: Develop optimization models for crude oil shipping using large tankers minimizing chartering and operating costs while meeting refinery demand

5. Huang and Li(2008)

   Work : A transportation model for hazardous chemical logistics.

   Application: Applied multi- objective transportation model to chemical tanker routing, considering not only cost but also safety and environment risks.

6. Shafice and Berglund (2013)

   Work: Optimized distribution planning petroleum products

   Application: Applied a mixed - integer linear programming model to optimize the distribution of petroleum products by road tankers, reducing transportation cost while improving service reliability.

7. Al- Khayyal and Hwang(2007)

   . Work: Inventory routing problem in the petroleum industry.

   . Application: Studied integrated inventory and routing models for oil tanker distribution , focusing on balancing depot inventory with customer demand though optimal tankers dispatching.

8. Hemmati (2016)

   . Work: Benching mark supply chain optimization in the oil industry.

. Application: Developed transportation optimization method for fleet management of oil tankers , reducing both operating costs and delivery delays in petroleum logistics

2.9 CHALLENGES AND FUTURE TRENDS ON TANKER ROUTING

2.9.1 CHALLENGES IN TANKER'S ROUTING PROBLEMS

1. Complex Network Design

   . Tankers must serve multiple depots ,ports or customers with varying demand.

   . The network often spans long distances, involving both land (truck tankers) and sea(oil/ chemical tanker)

2. Uncertainty in Demand and Supply

   . Demand for petroleum products fluctuates (seasonal, daily, emergencies).

   . Refineries may face production delays, impacting routing schedule.

3. Dynamic Routing

   . Unexpected events (road congestion, accidents, port delays, bad weather) disrupt pre- planned routes.

   . Tankers may need real-time rerouting.

4. High Operational cost

   . Fuel cost is a large portion of total expenses

   . Empty return trips lead to inefficiency

5. Safety and Environmental concerns.

   . Transportation of hazardous liquids requires

   . Oil spills, accidents, and emissions and risk and cost

6. Fleet Heterogeneity

   . Different tankers have varying capacities, compartments and restrictions (flammable vs non flammable).

7. Regulatory and Infrastructural Limitations

   . Road weight limits, restricted operating how in cities, and report traffic

   . International maritime laws for chemical tankers.

8. Computational complexity

   . Tanker routing is a variant of the Vehicle Routing Problem (VRP), which is NP- hard.

   . Large - scale instances require advanced and heuristics

2.9.2 FUTURE TRENDS IN TANKER'S -ROUTING PROBLEM

1. Intelligence of Artificial Intelligence (AI) and Machine Learning.

   . Productive analytic for demand forecasting

. Al- driven routing system that adapts in the real-world.

2. IOT and Telematics in Tanker Fleet Management

   .GPS, sensors, and Telematics for monitoring routes, speed

   .Real -time tanker tracking with automated alerts for delays or hazards

3. Autonomous Tankers and Smart Logistics.

4. Block Chain For Transparency and Security.

   . Used for tracking petroleum transactions, constraints and logistics data reduce fraud and ensure compliance.

5. Muti-Model Interaction

   . Coordinating tanker trucks, ships, and pipelines in one optimized network

6. Cloud -Based Decision Support System

   . Cloud computing enabling large- scale, real-time

   . Collaboration platforms for oil companies, distributors and regulators

CHAPTER THREE

METHODOLOGY

3.1 TANKERS -ROUTING PROBLEM

The Tanker's Routing Problem (TRP) is a specialized optimization problem in transportation and logistics that focuses on finding the most cost- effective and efficient routes for tankers (such as oil, fuel, chemical, or water tankers) to deliver commodities from supply depots to demand locations. It is a variant of the transportation and vehicle routing problem, where the aim is to minimize total operational costs while ensuring timely delivery and compliance with safety capacity and regularity.

The routing of petroleum tankers is formed as a transportation problem, where;

- Supply Nodes; represent depots, refineries, or terminals where petroleum products are loaded.
- Demands Nodes; represent filling stations, industries or bulk buyers requiring supply.
- Costs; represent fuel consumptions, distance traveled, tools or time converted into monetary terms.

- Decision Variable: Represents the allocation of petroleum products from supply to demand Nodes via tanker.

3.2 OBJECTIVE OF TRP (TANKER ROUTING PROBLEM)

The primary objective of Tanker Routing Problem (TRP) include;

1. Minimizing Transportation Cost
2. Maximizing Efficiency
3. Meeting demand requirement at all destinations without shortages or excesses
4. Ensuring safety and regulatory compliance; especially for hazardous Cages like petroleum or chemical.

3.4 DATA COLLECTION TOOLS AND TECHNIQUES

This study uses primary data collection gather observations of a refinery that has a supply center(source) and a demand center(filling station)

3.4.1 DATA COLLECTION TOOLS

1. Questionnaire and Interview - For collecting real - world information from logistics managers, tanker operators, and oil distribution companies.
2. Observation and Field Survey-To note real travel times, fuel consumption ay road conditions
3. Company Records and Database- Historical data on cost, routes, fuel usage

and demands at depots/stations

3.4.2 TECHNIQUES USED

1. Transportation Model Techniques:
    - North - West corner Method: to obtain initial feasible solutions
    - Least -Cost Method (LCM)- To minimize cost from the start.
    - Vogels Approximation Method (VAM)- To generate better initial feasible solutions.
    - The Russell Approximation Method (RAM)
2. Routing and Optimization techniques:
    - Linear programming (LP)-For cost minimization subject to supply and demand constraints.
    - Integer programming (IP)-For discrete tankers assignment.
    - Heuristics and Metaheuristic-(Genetic Algorithms, Tabu search, Ant colony Optimization)-For complex real- life routing optimization when exact methods are difficult.

3.5 TRANSPORTATION MODEL

Transportation models have long been used to solve logistics challenges involving the optimal allocation and routing of tankers to minimize operational costs and enhance efficiency

3.6 DESCRIPTION OF THE MODEL

Let $\mathrm{i} = 1,2,3,\dots,\mathrm{m}$ be the number of loading depots of the petroleum products, at which tankers are loaded for deliveries to discharge centres, $\mathrm{j} = 1,2,3,\dots,\mathrm{n}$. A time at which a tanker is to be loaded at i depot for making a delivery to a j destination is fixed or known. In this model, we shall consider the case where all tankers are identical (so that they are interchangeable) and where deliveries are quantified in units of tanker capacity so that a tanker must make a full delivery. In other words, a tanker cannot make a part delivery. Having made a delivery, a tanker can travel to any depot if it can get there in time to pick up another cargo for some selected destinations.

3.7 MODEL ASSUMPTIONS

Assumptions associated with the model formulation and operations are as follows:

1. The product/ commodity being transported must be identical such that customers can accept them from any source.

2. The number of tankers to be supplied and demanded is known otherwise a non-existing depot which takes excess supply of the product at no cost is added.
3. The cost of shipment through each route is known.
4. The tankers are available at the beginning of the period wherever they are needed irrespective of where they end up at the end of the period Methodology

The methodology applied includes

1. Russell's Approximation Method (RAM)
2. Linear Programming Problem (LPP)
3. Vogel’s Approximation Method (VAM)
4. Transportation Algorithm/Northwest Corner Rule
5. Optimal solution method

The objective of the study is to formulate a transportation model which determines the minimum cost of shipping petroleum products from depots (Refinery center) in their demand centers (filling stations).

3.8 MATHEMATICAL NOTATIONS/VARIABLES

The variables used in this research are defined as follows:

$\mathrm{S_i}$ = number of supply centre for $\mathrm{i} = 1,2,\ldots,\mathrm{m}$

D_j = number of demand centre for $j = 1,2,\dots,n$

X_{ij} = number of tankers transported from supply centre i

C_{ij} = transportation cost per tanker.

3.9 TRANSPORTATION MODEL FORMULATION

The mathematical expressions of the transportation model described above can be stated thus:

$$\mathrm{MinH} = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}X_{ij} \tag{1}$$

$$\sum_{j=1}^{n} X_{ij} \leq S_i \quad \text{for} \quad j = 1,2,\dots,m(\text{supply constraint}) \tag{2}$$

$$\sum_{i=1}^{m} X_{ij} \geq D_j \quad \text{for} \quad j = 1,2,\dots,m(\text{demand constraint}) \tag{3}$$

$$X_{ij} \geq 0(\text{Nonnegative constraints}). \tag{4}$$

In such cases, we employ at most $(m + n - 1)$ routes of feasible transportation schedule. Where the quantity supplied and the quantity demanded are the same then, we have a balanced transportation problem. This can mathematically be expressed as:

$$\sum_{i=1}^{m} S_i = \sum_{j=1}^{n} D_j. \tag{5}$$

If

$$\sum_{i=1}^{m} S_i \geq \sum_{j=1}^{n} D_j. \tag{6}$$

equation (6) implies that supply is either equal to or greater than demand.

If supply and the demand are not the same in quantity then, the transportation problem TP is unbalanced. This can be expressed mathematically as:

$$\sum_{i=1}^{m} S_i \neq \sum_{j=1}^{n} D_j. \tag{7}$$

Theorem 1 (Existence of Feasible Solution). $\sum_{i=1}^{m} S_i = \sum_{j=1}^{n} D_j$.A necessary and sufficient condition required for a transportation problem to have a feasible solution, is that the quantity supplied must be equal to quantity demanded. That is:

Proof. (a) Necessary condition. Suppose, the solution that is feasible if TP exists, then we shall have

$$\sum_{i=1}^{m} \sum_{j=1}^{n} X_{ij} = \sum_{i=1}^{m} S_i. \tag{8}$$

And

$$\sum_{j=1}^{n} \sum_{i=1}^{m} X_{ij} = \sum_{j=1}^{n} D_j. \tag{9}$$

$\sum_{i=1}^{m} S_i = \sum_{j=1}^{n} D_j$.Equation (8) represents the quantity supplied while equation (9) is the demanded quantity. Since equation (8) and equation (9) are the same it means

(b) Sufficient condition. Assuming the quantity supplied is equal to demanded quantity, then

$\sum_{i=1}^{m} S_i = \sum_{j=1}^{n} D_j = h$ (10)

Suppose k $(k \neq 0)$ is a real number such that

then the value of $k_i X_{ij} = k_i D_j \neq 0 \text{for alliandj},$ is given by

$$\sum_{j=1}^{n} X_{ij} = \sum_{j=1}^{n} k_i D_j = k_i \sum_{j=1}^{n} D_j = hk_i or k_i = \frac{1}{h} \sum_{j=1}^{n} X_{ij} = \frac{S_i}{h}$$

Thus

$X_{ij} = k_i D_j = \frac{S_i D_j}{h}$ for all i and j. (11)

Since $S_i > 0$ and $D_j > 0$ for every i and j, then $S_i \frac{D_j}{h} \geq 0$ and hence a solution which is feasible exists, that is $x_{ij} > 0$. □

Theorem 2 (Basic Feasible Solution). In any basic feasible Solution, we have $m + n - 1$ basic variables and $m + n - 1$ independent constraint, where m rows is the supply constraint and n columns is the demand constraint equations.

Proof. In all mathematical formulations of transportation problems including the ones discussed in Sharma (2006) and Ekoko (2011), it is observed that, if mrows which represent supply constraints exists, then, we will have a total of $m + n$ constraints. But due to Theorem 1 which states that the quantity supplied and the quantity demanded must be equal out of $m + n$ constraint equations, one of the equations must be unused and consequently, removed. Hence, we have $m + n - 1$ equations which are linearly independent. We can prove this if we add all the equations of the m rows and deducting from the sum of the first $n - 1$ column equations, consequently obtaining the last column equation. That is,

$$\sum_{i=1}^{m}\sum_{j=1}^{n} X_{ij} - \sum_{j=1}^{n-1}\sum_{i=1}^{m} X_{ij} = \sum_{i=1}^{m} S_i - \sum_{j=1}^{n-1} D_j$$

$$\sum_{i=1}^{m}\left[\sum_{j=1}^{n} X_{ij} - \sum_{j=1}^{n}\sum_{i=1}^{m} X_{ij} - \sum_{i=1}^{m} X_{in}\right] = \sum_{i=1}^{m} S_i - \left[\sum_{j=1}^{n} D_j - D_n\right]$$

since

$\sum_{i=1}^{m} S_i = \sum_{j=1}^{n} D_j$. □. (12)

3.11Unbalanced Transportation Problem

A necessary condition required for the existence of a feasible solution, is that the quantity supplied and quantity demanded must be equal as earlier stated in equation (6). That is,

$$\sum_{i=1}^{m} S_i = \sum_{j=1}^{n} D_j.$$

But that is not always true, since there could be cases in real life situations where supplied quantity does not match the quantity required. Two cases can be derived from an unbalanced TP.

Case 1: If the supply is higher than demand, the constraints of the transportation problem can be mathematically stated as,

$\sum_{j=1}^{n} X_{ij} \leq S_i, i = 1,2, \dots, m.$ (13)

$\sum_{i=1}^{m} X_{ij} = D_j, j = 1,2, \dots, n.$ (14)

$X_{ij} \geq 0 \text{ for everyi, j.}$ (15)

If we add $h_{i,n+1}$ as slack variable for $(i = 1,2, \dots, m)$ in the first m equations, we obtain

$$\sum_{j=1}^{n} X_{ij} + h_{i,n+1} = S_i$$

$$\sum_{i=1}^{m} \left[\sum_{j=1}^{n} X_{ij} + h_{i,n+1} \right] = \sum_{i=1}^{m} S_i$$

$\sum_{i=1}^{m} \sum_{j=1}^{n} X_{ij} + \sum_{i=1}^{m} h_{i,n+1} = \sum_{i=1}^{m} S_i - \sum_{j=1}^{nsf} D_j$ available excess supply. (16)

If we proceed further to denote the available excess supply by D_{n+1}, then, the modified version of the TP can be written as

$mH = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} X_{ij} + \sum_{i=1}^{m} C_{i,n+1} h_{i,n+1}.$ (17)

Subject to

$\sum_{j=1}^{n} X_{ij} + h_{i,n+1} = S_i, i = 1,2,\dots,m,$ (18)

$\sum_{i=1}^{m} X_{ij} = D_j, j = 1,2,\dots,n+1,$ (19)

$X_{ij} \geq 0$ for everyi, j. (20)

Note that $C_{i,n+1} = 0(i = 1,2,\dots,m)$ and

$$\sum_{i=1}^{m} S_i = \sum_{j=1}^{n} D_j + D_{n+1} \text{ or } D_{n+1} = \sum_{i=1}^{m} S_i - \sum_{j=1}^{n} D_j.$$

The mathematical analysis presented in equations (17)–(20) implies that, if the quantity supplied is higher than the quantity required (i.e. $\sum S_i > \sum D_j$), then a dummy column (demand centre) which absorbs the surplus supply is added to the TP table. Then transportation cost per unit for the cells in this column is equated to zero. (See Ekoko (2011) and Oguneyo and Panya (2024)).

Case 2: If demand is higher than the quantity supplied, the equation of constraints of the TP will appear as

$\sum_{j=1}^{n} X_{ij} = S_i, i = 1,2,\dots,m.$ (21)

$\sum_{i=1}^{m} X_{ij} \leq D_j, j = 1,2,\dots,n.$ (22)

$$X_{ij} \geq 0 \text{ for everyi, j.}$$

If we add slack variables $h_{m+1,j}$ for $j = 1,2,\dots,n$ in the last n constraints, we obtain

$\sum_{j=1}^{n} X_{ij} = S_i, i = 1,2, \dots, m.$ (23)

$\sum_{i=1}^{m} X_{ij} + h_{m+1,j} = D_j, j = 1,2, \dots, n.$ (24)

$\sum_{j=1}^{n} h_{m+1,j} = \sum_{j=1}^{n} D_j - \sum_{i=1}^{m} S_i$ surplus demand. (25)

If we denote the surplus demand by h_{m+1}, then we can rewrite the modified transportation problem as:

$H' = \sum_{i=1}^{m} \sum_{j=1}^{n} \left(C_{ij} X_{ij} + C_{m+1,j} h_{m+1,j}\right)$ (26)

Subject to

$\sum_{j=1}^{n} X_{ij} = S_i, i = 1,2, \dots, m + 1$ (27)

$\sum_{i=1}^{m} X_{ij} + h_{m+1,j} = D_j, j = 1,2, \dots, n$ (28)

$$X_{ij} \geq 0 \text{ for everyi, j.}$$

Note that $C_{m+1,j} = 0$ for all j and

$$\sum_{i=1}^{m} S_i + h_{m+1} = \sum_{j=1}^{n} D_j \text{ or } \sum_{j=1}^{n} D_j = \sum_{i=1}^{m} S_i + h_{m+1}.$$

Equations (26)–(28) imply that if total quantity demanded is higher than total quantity supplied (i.e. $\sum S_i > \sum D_j$), then, we need to add a dummy row (supply centre) to the TP to absorb the surplus quantity demanded. In this case, the transportation cost for the dummy row is equated to zero.

3.10 FORMULATION OF TP AS LPP

If the size of the transportation problem is very large such that applying the transportation simplex technique becomes computationally inefficient, then the TP can be formulated as an LPP. This is made possible by taking advantage of the duality relationships of its network. The general mathematical formulation of the transportation problem as earlier stated in Section 3.0 is:

$\mathrm{mZ} = \sum_{\mathrm{i}=1}^{\mathrm{m}} \sum_{\mathrm{j}=1}^{\mathrm{n}} \mathrm{C}_{\mathrm{ij}} \mathrm{X}_{\mathrm{ij}}.$ (29)

Subject to

$\sum_{\mathrm{j}=1}^{\mathrm{n}} \mathrm{X}_{\mathrm{ij}} = \mathrm{S}_{\mathrm{i}}, \mathrm{i} = 1,2, \dots, \mathrm{m}$ (S constraints).(30)

$\sum_{\mathrm{i}=1}^{\mathrm{m}} \mathrm{X}_{\mathrm{ij}} = \mathrm{D}_{\mathrm{j}}, \mathrm{j} = 1,2, \dots, \mathrm{n}$ (D constraints).(31)

$\mathrm{X}_{\mathrm{ij}} \geq 0$ for everyi, j (Nonnegative condition). (32)

Note that all S_{i} and D_{j} are positive numbers satisfying the equation

$\sum_{\mathrm{i}=1}^{\mathrm{m}} \mathrm{S}_{\mathrm{i}} = \sum_{\mathrm{j}=1}^{\mathrm{m}} \mathrm{D}_{\mathrm{j}}$ (Quantity supplied = Quantity demanded) (33)

The technology table for the TP is shown in Table 1 below for the case of a 3 rows × 4 columns ($\mathrm{m} = 3$ and $\mathrm{n} = 4$) TP.

Since the total quantity supplied and the total quantity demanded in equations (30) and (31) are equal, the TP is a dummy cell because if any $\mathrm{m} + \mathrm{n} - 1$ constraints in equations (30) and (31) are met, then the remaining constraint will also be met. The Table 1 is divided into an upper and a lower section.

Technology table for transportation problem

U_{11}	U_{12}	U_{13}	U_{14}	U_{21}	U_{22}	U_{23}	U_{24}	U_{31}	U_{32}	U_{34}	U_{35}	
1	1	1	1									B_1
				1	1	1	1					B_2
								1	1	1	1	B_3
1				1				1				M_1
	1				1				1			M_2
		1				1				1		M_3
			1				1				1	M_4
Q_{11}	Q_{12}	Q_{13}	Q_{14}	Q_{21}	Q_{22}	Q_{23}	Q_{24}	Q_{31}	Q_{32}	Q_{33}	Q_{34}	Minimize

Fig 1: Technology table for transportation problem

The dual linear programming model corresponding to Table 1 is displayed in Table 2. Letters P and R are used to represent the dual variables corresponding to the supply and demand partition in Table 1. The duality of the TP in Table 1 can be expressed as:

Duality of the TP

R1	**R2**	**R3**	**P1**	**P2**	**P3**	**P4**	
1			1				$\leq Q_{11}$
1				1			$\leq Q_{12}$
1					1		$\leq Q_{13}$
1						1	$\leq Q_{14}$
	1		1				$\leq Q_{21}$
	1			1			$\leq Q_{22}$
	1				1		$\leq Q_{23}$
	1					1	$\leq Q_{24}$
		1	1				$\leq Q_{31}$
		1		1			$\leq Q_{32}$
		1			1		$\leq Q_{33}$
		1				1	$\leq Q_{34}$
B1	B2	B3	M1	M2	M3	M4	Minimize

By applying the procedure described above, we have

Supply and Demand Cost Structure

	1	**2**	**3**	**SUPPLY**
1	Q_{11} U_{11}	Q_{12} U_{12}	Q_{13}	B_1
2	Q_{21} U_{21}	Q_{22} U_{22}	Q_{23} U_{23}	B_2
3	Q_{31} U_{31}	Q_{32} U_{32}	Q_{33} U_{33}	B_3
Dd	M_1	M_2	M_3	

Using Table 3, the TP can be expressed as follows: Minimize

$$Z = C_{11}X_{11} + C_{12}X_{12} + \cdots + C_{33}X_{33} = \sum\sum C_{ij}X_{ij} \qquad (34)$$

Subject to:

$$\sum_{j=1}^{n} X_{ij} = S_i, \mathrm{for} i = 1, \ldots, m, \mathrm{Supply}.(35)$$

$$\sum_{i=1}^{m} X_{ij} = D_j, \mathrm{for} j = 1, \ldots, n, \mathrm{Demand}.(36)$$

$$X_{ij} \geq 0, \mathrm{for} i = 1, \ldots, m, j = 1, \ldots, n, \mathrm{non-negativity\ condition}. \qquad (37)$$

But the sum of the m equations in (35) is equal to the sum of the n equations in (36). This shows that one linear constraint is redundant, i.e., equations (35) and (36) are not linearly independent. We can therefore delete any one of the $m+n$ linear constraints in (35) and (36). If we delete, say, the constraint

$$\sum_{j=1}^{n} X_{1j} = S_1,$$

then the reduced';[transportation problem becomes:

$C = C_{11}X_{11} + C_{12}X_{12} + \cdots + C_{33}X_{33}$.(38)

Subject to:

$\sum_{j=1}^{n} X_{ij} = S_i, \text{for} i = 2, \ldots, m.$ (39)

$\sum_{i=1}^{m} X_{ij} = D_j, \text{for} j = 1, \ldots, n.$ (40)

$X_{ij} \geq 0$.(41)

CHAPTER FOUR

DATA PRESENTATION AND ANALYSIS

4.1 INTRODUCTION

This chapter present the data collected from the application of transportation model, and the analysis of the tanker routing problem

The aim is to determine the optimal routes for distributing petroleum products from depots to various filling stations at minimum total transportation cost

The data obtained from the petroleum depots and the filling stations are used to construct a transportation table and the problem is solved using optimization techniques such as the Big M method, North corner Rule (NCR), Least Cost Method (LCM),Vogel Approximation Method (VAM), Russell Approximation Method (RAM), and the Optimal Steeping Stone Methods

4.2 DATA PRESTATION

The data for this study were obtained from the distribution records of petroleum products within the selected area in Edo state

4.3 DATA DESCRIPTION

- Supply centers (sources): This represent the petroleum depots where tanker are loaded (i.e Depot 1 Depot 2 Depot 3)

- Demand centers (Destination): These represent the filling stations that receive the petroleum products (e.g stations 1, stations 2 etc)
- Transportation Cost; This refers to the cost incurred in transporting one unit per tanker from a depot to a filling station

4.4 NUMERICAL ILLUSTRATION

A refinery has three depots (supply centers) and two filling stations (Demand centers) as shown below. Transportation cost from any deport to any demand is fixed in hundreds of thousands of naira (₦100,000)per tanker. Transportation unit cost from supply center 1 to demand center 1 is ₦4/tanker and transportation unit cost from supply center1 to demand center 2 is ₦5/tanker and so on. The full data of the transport cost from each supply center to demand center is summarized below

TABLEAU OF ILLUSTRATION

Sources	STATIONS 1	STATIONS 2	Supply center(₦100,000pertanker)
DEPOTS 1	₦4	₦5	38
DEPOTS 2	₦9	₦7	22
DEPOTS 3	₦3	₦6	40
Demand Center (₦100,000pertanker)	30	70	100

MODEL FORMULATION

$$\sum_{i=1}^{2} \sum_{j=i}^{3} C_{ij}X_{ij} = \sum_{i=1}^{2}[C_{i1}X_{i1} + C_{i2}X_{i2} + C_{i3}X_{i3}]$$

$$= C_{11}X_{11} + C_{21}X_{21} + C_{12}X_{21} + C_{22}X_{22} + C_{13}X_{31} \quad + C_{23}X_{23}$$

Where C_{IJ} is the transportation unit cost of source i to demand center j

X_{ij} = The quantity of petroleum product to be transported from source i to demand j)

C_{11}=₦4(Transportation unit cost from sources 1 to demand center1)

C_{21}= ₦5 (Transportation unit cost from sources 2 to demand center 1)

C_{12} =₦9(Transportation unit cost from sources 1 to demand center2)

C_{22} =₦7 (Transportation unit cost from sources 2 to demand center2)

C_{13}= ₦3 (Transportation unit cost from sources 3 to demand center1)

C_{23}=₦ 6 (Transportation unit cost from sources 3to demand center2)

Using the Big M method to solve the primal LPP of the transportation problem

SOLUTION

minz $4X_{11}+5X_{12}+9X_{21}+7X_{22}+3X_{31}+6X_{22}$

S.t

$$X_{11}+X_{12} = 38$$

$$X_{21}+X_{22} = 22$$

$$X_{31}+X_{32} = 40$$

$$X_{11} + X_{21} + X_{31} = 30$$

$$X_{12} + X_{21} + X_{32} = 70$$

Let

$X_{11}= X_1$, $X_{12}= X_2$, $X_{21} =X_3$, $X_{22} = X_4$, $X_{31}= X_5$, $X_{32}=X_6$

And the primal LPP becomes

$\text{Min}z=4X_1+5X_2+9X_3+7X_4+3X_5+6X_6$

s.t

x_1+x_2

x_3+x_4

x_5+x_6

x_1 $+x_2$ $+x_3$

x_1…….x6≥0

x_1	x_2	x_3	x_4	x_5	x_6	$\underline{x}_7$	$\underline{x}_8$	$\underline{x}_9$	$\underline{x}_{10}$	RHS
4	5	9	7	3	6	M	M	M	M	0
-M[1	1	0	0	0	0	1	0	0	0	38]
-M[0	0	1	1	0	0	0	1	0	0	22
-M[0	0	0	0	1	1	0	0	1	0	40
-M[1	0	1	0	1	0	0	0	0	1	30
(-2M+4)	(-M+5)	(-2M+9)	(-M+7)	(-2M+3)	(-M+6)	0	0	0	0	130M

we update each of the linear constraints the artificial variables x_7 x_8 x_9 and x_{10} to obtain tableau 1 as follows

B V	x_1	x_2	x_3	x_4	x_5	x_6	$\underline{x}_7$	$\underline{x}_8$	$\underline{x}_9$	$\underline{x}_{10}$	R.H. S.
$\underline{x}_7$	1	1	0	0	0	0	1	0	0	0	38
$\underline{x}_8$	0	0	1	1	0	0	0	1	0	0	22
$\underline{x}_9$	0	0	0	0	1	1	0	0	1	0	40
$\underline{x}_{10}$	1	0	1	0	(1)	0	0	0	0	1	30 →
$-z$	(-2M+4)	(-M+5)	(-2M+9)	(-M+7)	(-2M+3)	(-M+6)	0	0	0	0	-130 M

Tableau 2

BV	x_1	x_2	x_3	x_4	x_5	x_6	$\underline{x}_7$	$\underline{x}_8$	$\underline{x}_9$	$\underline{x}_{10}$	R.H.S.
$\underline{x}_7$	1	(1)	0	0	0	0	1	0	0		38→
$\underline{x}_8$	0	0	1	1	0	0	0	1	0		22
$\underline{x}_9$	-1	0	-1	0	0	1	0	0	1		10
$\underline{x}_{10}$	1	0	1	0	1	0	0	0	0		30
$-z$	1	-M+5	6	-M+7	0	-M+6	0	0	0		-70M-90
		↑									

Tableau 3

x_2	1	1	0	0	0	0	0	0	38
$\underline{x}_8$	0	0	1	1	0	0	1	0	22
$\underline{x}_9$	-1	0	-1	0	0	(1)	0	1	10→
x_5	1	0	1	0	1	0	0	0	30
$-z$	M-4	0	6	-M+7	0	-M+6	0	0	-32M-280
						↑			

Tableau 4

BV	x_1	x_2	x_3	x_4	x_5	x_6	$\underline{x}_7$	$\underline{x}_8$	$\underline{x}_9$	$\underline{x}_{10}$	R.H.S.
x_2	1	1	0	0	0	0		0			38
$\underline{x}_8$	0	0	(1)	1	0	0		1			22→
x_6	-1	0	-1	0	0	1		0			10
x_5	1	0	1	0	1	0		0			30
$-z$	2	0	-M+12	-M+7	0	0		0			-22M-340

↑

Tableau 5

x_2	1	1	0	0	0	0		38
x_3	0	0	1	**(1)**	0	0		22→
x_6	-1	0	0	1		1		10
x_5	1	0	0	-1	1	0		30
$-z$	2	0	0	-5	0	0		-604

Tableau 6

x_2	1	1	0	0	0	0	38
x_4	0	0	1	1	0	0	22
x_6	0	0	0	0		1	10
x_5	0	0	0	0	1	0	30
$-z$	2	0	5	0	0	0	-494

The Optimal Solution is $X_1=0=X_{11}$, $X_2=38=X_{12}$, $X_3=0=X_{21}$, $X_4=22=X_{22}$, $X_5=30=X_{31}$, $X_6=10=X_{32}$

Therefore, the transportational cost Z = ₦49,400,000/tanker

4.5 INITIAL FEASIBLE SOLUTION METHODS

The thrust of the initialization step is to obtain an initialization step to obtain an initial b.f.s select the (m +n - 1) basic variable one at a time. After each selection a value is assigned to that variable which will satisfy one additional constraint and eliminate that constraints row or column from further consideration for providing allocations. Thus, after (m+ n -1) selection an entire basic solution has been constructed in such a way as to satisfy all the constraints (The constraint limits are also called the rim requirements).

The initial b. f. s methods include the following;

1. The Northwest Corner Rule (N.C.R)
2. The Least -Cost Method (or Low Cost First Method)
3. The Vogel Approximation Method (V.A.M)
4. The Russell Approximation Method (R.A.M)

4.5.1 THE NORTHWEST CORNER RULE

In this method, placement of quantities being transported from sources to destinations must start from the left- hand cell (1,1): In each allocatable cell (basic variable cells), quantity as large as possible must be allocated, in such a way that supply or demand be exhausted. If the diagonal movement to any cell, it is possible to exhaust the row requirement of that cell, then the movement is row wise. This kind of movement continues until both supply and demand of the final cell are simultaneously exhausted. This terminates the process. At this stage we find that the table is filled from the uppermost lefthanded cell down to the lowest right-hand cell, using the supply and demand units fully.

4.5.1.1 STEPS FOR SOLVING A TRANSPORTATION PROBLEM USING NORTHWEST CORNER RULE ALGORITHMS.

Step 1: Start by allocating as much as possible (without violating supply and demand requirements) to the uppermost left hand cell (1,1). That is $X_{11} = a_1$ if $a_1 <$

b_1 and row1 is eliminated from further consideration. Or is $X_{11} = a_1$ if $b_1 < a_1$ and column 1 is eliminated from further consideration.

Step 2: Move and allocate to the immediate next cell in the direction of (i) excess demand if $a_1 < b_1$, i.e select $x_{1+1,1} = \min(\bar{a}_2, \bar{b}_1)$ or (ii) excess supply if $b_1 < a_1$ i.e $x_{1+1,1} = \min(\bar{a}_1, \bar{b}_2)$ In general, if x_{ij} was the last basic variable selected then $x_{1+1,1} = \min(\bar{a}_{j+i}, \bar{b}_j)$ or $x_{i,j+1} = \min(\bar{a}_i, \bar{b}_{j+1})$. Where $\bar{a}_i$ and $\bar{b}_j$) are the updated units of supply and demand respectively at the iteration under consideration

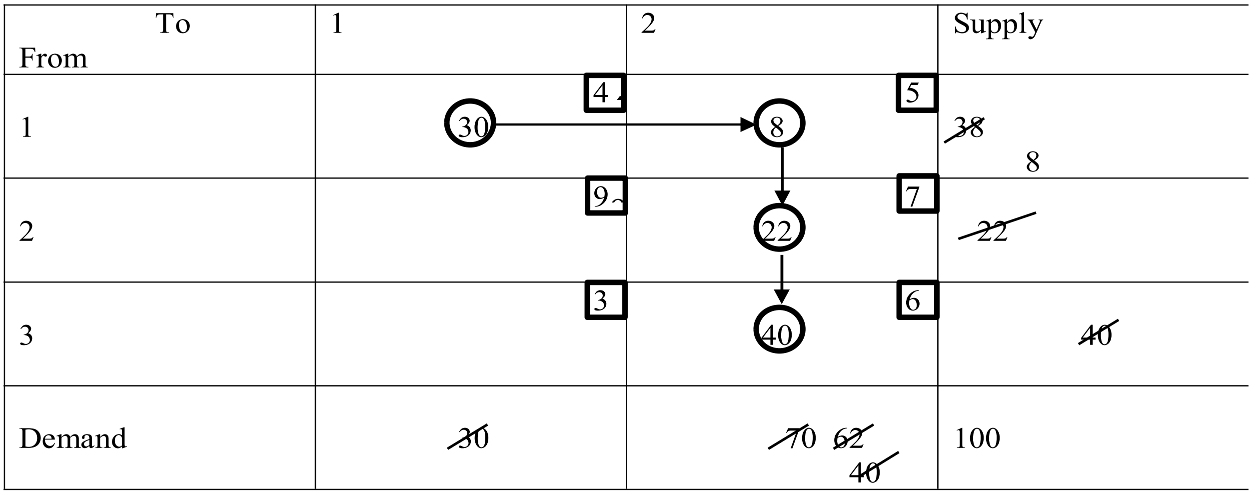
Step 3: The procedure in atep2 is continued diagonally until the supply ($\bar{a}_{m,}$) and demand ($\bar{b}_n$) of the final cell are exhausted and GOTO step 4. However, if before the lowest lefthand cell is reached, the row and column requirement of say cell (i, j) are simultaneously exhausted (i.e $\min(\bar{a}_i, \bar{b}_j)$ then proceed to the next diagonal cell.

Step 4: When all supply band demand requirement have been attained band STOP.

ILLUSTRATION;;

Sources	STATIONS 1	STATIONS 2	Supply center(₦100000)pertanker
DEPOTS1	₦4	₦5	38
DEPPOTS 2	₦9	₦7	22
DEPOTS 3	₦3	₦6	40
Demand Center(₦100,000 per tanker)	30	70	100

SOLUTION



To From	1	2	Supply
1	$\boxed{4}$ 30	$\boxed{5}$ 8	~~38~~ 8
2	$\boxed{9}$	$\boxed{7}$ 22	~~22~~
3	$\boxed{3}$	$\boxed{6}$ 40	~~40~~
Demand	~~30~~	~~70~~ ~~62~~ ~~40~~	100

The initial feasible solution to the Transportation problem using the North Corner Rule (N.C.R) is x_{11}=30, x_{12}=8, x_{13}=22, x_{14}=40

Therefore the total cost is

3000000(4)+800000(5)+2200000(7)+4000000(6)=₦55,400,000/tanker

4.5.2 THE LEAST COST METHOD

This is another method of finding initial basic feasible solution to the transportation problem. In this method, the cell with the least cost is chosen for the first allocation and the maximum possible quantity is allocated to this route. The resulting exhausted row or column is eliminated from further consideration. The cell with the lowest cost, (that is not in an exhausted row or column) is used for the second allocation and so on until all the supply and demand requirements have been satisfied.

4.5.2.1 STEPS FOR LEAST COST METHOD

Step 1: Start by the allocating as much as possible to the cell having the lowest unit cost , update the rim requirements and eliminate from further consideration the exhausted row or colomn. That is if the lowest cost cell is (i,j), then $x_{ij} = a_i$ if $a_i < b_j$ or $x,_{ij} = b_j$ if $b_j < a_i$. If at any iteration there is a tie in lowest cost, break tie by choosing the cell that will allow the largest allocation.

Step2: The next allocation is made to the cell having the lowest unit transportation cost among the remaining cells under consideration. That is $x_{ij} = \bar{a}_i$ if $\bar{a}_i < b_j$ or $x^1_{ij} = \bar{b}_j$ if $b_j < \bar{a}_i$. Where x^1_{ij} is the present lowest cost cell allocation and $\bar{a}_i$ and b_j are as explained be earlier in Northwest Corner Rule Method

Step 3: Go To step 2 until all the supply and demand requirements bare exhausted. The entire allocations cnstitute the initial feasible solution by the Least cost method and STOP.

ILLUSTRATION

Source	Depots 1	Depots 2	Supplycenter (₦100,000per tanker)
1	₦4	₦5	38
2	₦9	₦7	22
3	₦3	₦6	40
Demandcenter(filling station)	30	70	100

(₦100000per tanker)			

SOLUTION, USING LEAST COST METHOD

To From	1	2	Supply
1	4 30	5 8	~~38~~ 8
2	9	7 22	~~22~~
3	3	6 40	~~40~~
Demand	~~30~~	~~70~~ ~~62~~ ~~40~~	100

The initial feasible solution to the Transportation problem using the Least Cost Method is x_{11}=30, x_{12}=8, x_{13}=22, x_{14}=40

Therefore the total cost is

3000000(4)+800000(5)+2200000(7)+4000000(6)=₦55,400,000/tanker

4.5.3 THE VOGEL APPROXIMATION METHOD (VAM).

This method was originally developed to also produce starting solutions for the TP.VAM is a heuristic, that are near optimal.

4.5.3.1 STEPS FOR VOGEL APPROXIMATION METHOD

Step 1: Calculation the difference between the two lowest distribution costs for each row and each column; which is also known as minimum penalty cost.

Step2: select the row or column with the greatest difference and circle this value. In case of a tie, select the row or column allowing the greatest movement of units

Step3: Assign the largest possible allocation within the rim requirements to the lowest cost cell of the brow or column selected. The rational behind this bis to avoid making allocation to incur the largest penalty

Step 4: Cross out any row or column satisfied by the assignment made in step 3. Or simply mark 'X' in the said row or column in the next iteration to avoid further consideration.

Step5: Repeat step 1 through 4, expect for rows and columns that been crossed out (or marked 'X') until all assignments have been made. The final allocation is made when only one row or only one column is yet to be satisfied.

ILLUSTRATION

Source	STATIONS 1	STATIONS 2	Supply center(100,000/tanker)
DEPOTS 1	₦4	₦5	38
DEPOTS 2	₦9	₦7	22
DEPOTS 3	₦3	₦6	40
Demand center(₦100,000 pertanker)	30	70	100

SOLUTION

USING THE VOGEL APPROXIMATION METHOD

5th Iteration col diff. X

4th Iteration col diff. 1

3rd Iteration col diff. 1

2nd Iteration col diff. X 1

1st Iteration col diff. 1 1

Ij	1	2	Ss					
1	$\boxed{4}$	$\boxed{5}$ ㊳	38	1	5	5	⑤	X
2	$\boxed{9}$	$\boxed{7}$ ㉒	22	2	⑦	X		
3	$\boxed{3}$ ㉚	$\boxed{6}$ ⑩	~~40~~ 10	③	6	⑥	X	
Dd	~~30~~	~~70~~ ~~48~~ ~~38~~	100					
				1st	2nd	3rd	4th	5th

Row difference

The Vogel Approximation Method(VAM) Initial solution is;$x_{12=35}$,$x_{22=22}$,$x_{31=30}$,$x_{32=10}$ with the total transportation cost of 3800000(5)+2200000(7)+300000(3)+1000000(6)= ₦49,400,000/tanker

4.5.4 THE RUSSELL'S APPROXIMATION METHOD (RAM)

This method compared to VAM is relatively new. It was developed also to produce a near optimal solution to the TP.

4.5.4.1 STEPS FOR RUSSELL'S APPROXIMATION METHOD(RAM)

Step 1: For each source row (I) still remaining under consideration, determine its $\bar{\upsilon}_j$, which is the largest unit cost c_{ij} still remaining in that column.

Step 2: For each destination column j remaining under consideration, determine its $\tilde{v}_j$, which is the largest unit cost c_{ij} still remaining in that column.

Step 3: For each variable X_{ij} not previously selected in these rows and columns (i.e non basic variables) calculate $\Delta_{ij}=c_{ij} - \bar{\upsilon}_i - \tilde{v}_j$).

Step4: If only row/ column has its rim requirements yet to be satisfied, allocate in such a way that the remaining column/ row rim requirements are not violated and STOP. Otherwise, select the variables in step 3 that have the largest negative value of Δ_{ij} (ties should be broken using the cell with least unit cost) and allocate the largest possible units to that cell without violating the rim requirements.

ILLUSTRATION

Using Russell Approximation Method to solve the transportation problem

To From	1	2	Supply
1	$\boxed{4}$	$\boxed{5}$	38
2	$\boxed{9}$	$\boxed{7}$	22
3	$\boxed{3}$	$\boxed{6}$	40
Dd	30	70	100

1^{st} Iteration

From the tableau, row1= u_1=5, row2=u_2=9,row3=u_3=6, similarly for colomn1=v_2=9, colomn2=v_2 = 7

. $\Delta_{ij=} C_{IJ} – u_i - v_j$

$$\Delta_{11} = C_{11} – u_1 - v_1$$

$$= 4 – 5 – 9$$

$$= -10$$

$$\Delta_{12} = C_{11} . u_1 - V_2$$

=

$$= 5 - 5 - 7$$

$$= -7$$

$\Delta_{21} = C_{21} – U_2 - V_1$

$= 9 - 9 - 9$

$= -9$

$\Delta_{22} = C_{22} – U_2 - V_2$

$= 7 - 9 - 7$

$\Delta_{31} = C_{31} – U_3 - V_1$

$= 3 – 6 – 9$

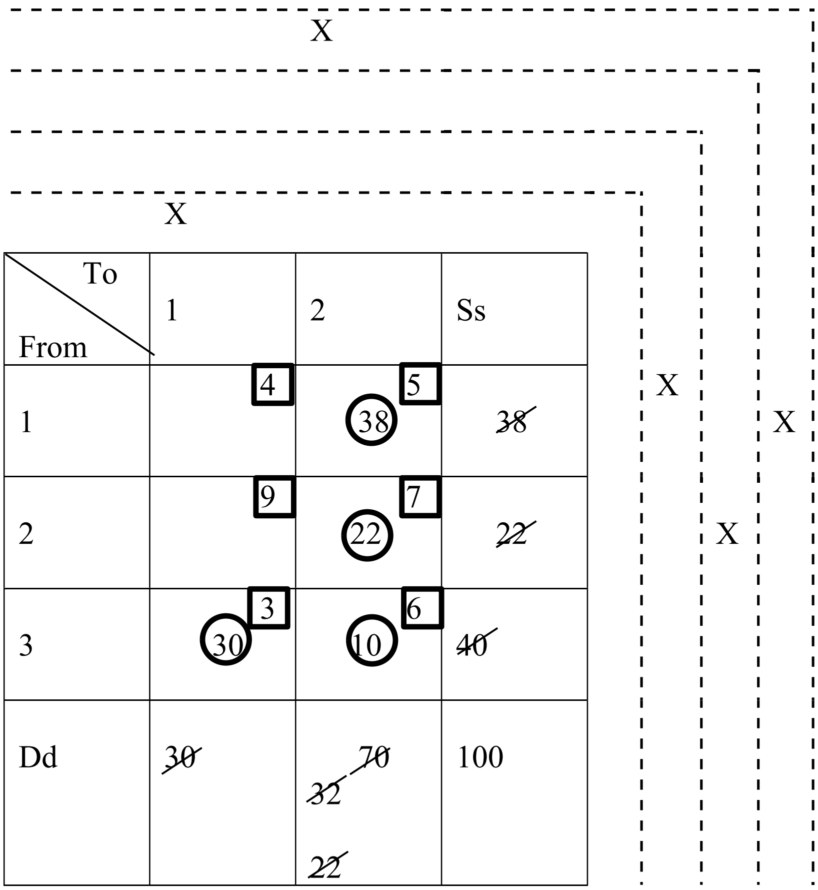
$= -12$

$\Delta_{32} = C_{32} - U_3 - V_2$

$= 6 - 6 - 7$

$= -7$

Thus, from iteration 1, the variable with the largest negative value of Δ_{ij} is x_{31}, so $X_{31} = 30$ is selected as the first basic variable (allocation) and column 1 is eliminated from further consideration



To / From	1	2	Ss
1	$\boxed{4}$	$\boxed{5}$ 38	~~38~~
2	$\boxed{9}$	$\boxed{7}$ 22	~~22~~
3	$\boxed{3}$ 30	$\boxed{6}$ 10	~~40~~
Dd	~~30~~	~~70~~ ~~32~~ ~~22~~	100

ITERATION2

$\underline{u_1}$ =5, $\underline{u_2}$ = 7, $\underline{u_3}$ =6, $\underline{v_2}$ =7

$\Delta_{12} = C_{12} - \underline{U_2} - \underline{V_2}$

$= 5 – 5 -7$

$\Delta_{22} = C_{22} - \underline{u_2} - \underline{v_2}$

$= 7 – 7 - 7$

$= 7$

$\Delta_{32} = c_{32} - \underline{u_3} - \underline{v_2}$

$= 6 - 6 - 7$

$= -7$

Similarly, $\Delta_{12} = -7$, $\Delta_{22} = -7$, $\Delta_{32} = -7$

The variable with the largest negative value of Δ_{ij} are x_{12}, x_{22}, $x_{32.}$ We break the tie here by choosing the one that has the smallest unit cost, which is x_{12}, so $x_{12} = 32$,and this row 1 is eliminated from further consideration.

3rd ITERATION

$\underline{u_2} = 7$, $\underline{u_3}=6$, $\underline{v_2}=7$

$\Delta_{22} = C_{22} - \underline{U_2} - \underline{V_2}$

$= -7$

$\Delta_{33} = C_{32} - \underline{U_3} - \underline{V_2}$

$= 6 - 6 - 7$

$=-7$

Similarly $\Delta_{22} = -7$ and $\Delta_{32} = -7$ the variable with the largest negative value of Δ_{ij} are x_{22} and x_{32}. We break the tie by choosing the one that has the smallest unit cost which is $x_{32,}$ $x_{32} = 10$, then row 3 is eliminated.

4th ITERATION

$\underline{u_2}$ =7, $\underline{v_2}$=7

Δ_{22}= C_{12} - $\underline{u_1}$ - $\underline{v_2}$

= 7 – 7 – 7

= - 7

Therefore Δ_{22} = -7, we break the tie here with the variable of the unit cost 6 and eliminating row2 and column 2, we make the final allocation of x_{22} =22 to exhaust the requirement of the row2 and column 2

Thus using the Russells Approximation Method (RAM) to solve the transportation problem.

The initial feasible solution is x_{12}=38, x_{22}=22, x_{32}=10, x_{31}=30

And the total cost Z= 3800000(5) + 2200000(7) +1000000(6) +300000(3)

= ₦ 49,400,000/tanker

4.5.5 OPTIMAL SOLUTION

They are two type of optimal solu

tion which are;

1.The stepping stone method

2.The Modifiied Distribution (MODI) Method

But will be considering the stepping stone optimal solution.

4.5.5.1 THE STEPPING-STONE ALGORITHM

Step1: Test for Degeneracy

This means testing to confirm if the number of occupied cells obtained from the given initial feasible solution method is (m +n -1)

Step2 : Evaluate All Unfilled Cells

Step3: Select Empty Cell With Largest Negative

Step4: Movement of Units into Selected Cell

ILLUSTRATION FROM THE DATA PRESENTATION TABLEAU

Source	STATIONS 1	STATIONS 2	Supply center(₦100,000pertanker)
DEPOT 1	₦4	₦5	38
DEPOT 2	₦9	₦7	22
DEDOT 3	₦3	₦6	40
Demand Center (₦100,000pertanker)	30	70	100

SOLUTION

We Start from the Northwest Corner Rule(N C R) feasible solution in Tableau 2 to determine the optimal solution using stepping-stone.

1ST ITERATION (Northwest Corner Rule Solution)

To From	1	2	Supply
1	4 30	5 8	~~38~~ 8
2	9	7 22	~~22~~
3	3	6 40	~~40~~
Demand	~~30~~	~~70~~ ~~62~~ ~~40~~	100

To From	1	2	Supply
C 1	4 30 **1.**	5 8 +	~~38~~ 8
2	9 +	7 22 -	~~22~~
3	3 +	6 - 40	~~40~~
Demand	~~30~~	~~70~~ ~~62~~ ~~40~~	100

Step 2; Evaluate all unfilled cell

$\Delta 21 = 9 - 7 + 5 - 4$

$= 7 - 4$

= 3

Δ31 = 3 - 6 + 5 - 4

= -2

Step 3: Δij(most negative) = Δ31 which will lead to a new solution with lower transportation total qcost

Step 4; New solution is shown in tableau 2

But in the 1st iteration the initial feasible solution is

X11=30, X12=8, X13=22, X14 =40

Therefore total transportation cost= 3000000(4) +800000(5) +2200000(7) +4000000(6)

= 55400000

2nd iteration

4 0 +	5 38 -
9 + -	7 22
3 30 1.	6 + 10

Step1: No Degeneracy

Step2: Evaluate all unfilled cost.

Δ_{11} = 4-5+6-3

= 2 ($\Delta_{11} \geq 0$)

Δ_{21} = 9 – 3 + 6 -7

= 5 ($\Delta_{ij} \geq 0$)

Since all $\Delta_{ij} \geq 0$, Then STOP.

Therefore the final optimal solution: X_{12} =38, X_{22} =22,X_{31} =30, X_{32} = 10

There the total transportation minimum cost in hundreds per tanker= 3800000(5) +22000000(7) + 1000000(5) + 3000000(3) = ₦49,400,000

4.5.6 INTERPRETATION, ANALYSIST OF RESULTS AND CONCLUSION

The need for efficient distribution of petroleum products from refineries (supply centers) to demand centers cannot be over emphasized considering their domestic and industrial importance in our daily activities. In this study, we have represented a transportation problem which is being formulated as LPP, and consequently applied a Tankers -Routing problem of distribution of petroleum products from the supply centers (refinery depots) to the demands centers(filling stations).

The aim of this study is to select the minimum costs that will reduces the cost of the petroleum from the refinery depots to the filling stations. Other methods of the transportation model for determination of minimum routes such as NCR, LCM, VAM, RAM, are be discussed, then we intend to use the results we got from Big M and the Optimal Steeping Stone Methods to cross check that of the initial feasible solution (NCR, LCM,VAM, and RAM)

We observed that the minimum total cost of transportation using Big M method and the Optimal Steeping Stone Methods=₦49,700,000/tanker While the minimum total cost for the intial feasible solution such as the NCR(North corner Rule) and the Least Cost Method= ₦55,400,000/tanker, The Vogel Approximation Method (VAM) and Russell Approximation Method has a minimum total cost of= ₦49,400,000/tanker.

It's also observed the optimal total cost(stepping stones solution) is found to be ₦49,700,000 compared to the initial total cost (i e North corner Rule) ₦55,400,000. This shows a. cost saving of ₦6,000 meaning that throughout the transportation model, the routing of tankers was optimized to minimize cost and improve efficiency.

CHAPTER FIVE

SUMMARY, CONCLUSION AND SUMMARY

5.1 SUMMARY OF FINDINGS

This research focuses on applying the transportation model as an Optimization tool to solve tanker routing problem which is a common challenge in logistics, petroleum distribution and supply chain Management.

The main objective was to minimize transportation cost, reduce cost consumption and improve efficiency in distributing petroleum products from depots to various demands centers

The introductory chapter of the study analyzed transportation problem as an operation research technique which is used to determine what quantity of a homogeneous product should be transported from each source to each destination in such a way that the total transportation cost is kept minimum

In chapter 2 various literature pres enting there definitions of the transportation problems as put forward by numerous authors were reviewed and the linear programming representation of the transportation problem where also given

In chapter 3, we discussed the Methodology of transportation model, the formulation of Linear programming of the transportation problem and a comparison between the methods of initial feasible solution was given such as the North corner Rule (NCR), Least Cost Method (LSM),Vogel Approximation Method (VAM), Russell Approximation Method (RAM).

In chapter 4,will use the initial feasible solution and the Optimal Steeping Stone Methods to carry out an illustration obtained from a petroleum depots and filling stations to construct a transportation table,and the problem is solved using optimization techniques.

Key findings include:

1. Optimization through the transportation model:

   The study demonstrated that by formulating the tanker routing problem as a transportation model specifying source (depots), destinations(customers), supply quantities, demand requirements, and transportation costs. It's possible to identify to the Least -Cost distribution plan.

2. Reduction in transportation cost; The optimizes routing plan shows a significant reduction in total transportation cost compared to the existing or manual routing system.

3. Efficient Resources Utilization

   The optimal plans ensures that each tanker was efficiently utilized

4. Improvement in Decision-Making

The transportation model serves as a decision -support tool for management, enabling planners to make data -driven decisions on tanker allocation, routing,and scheduling.

5.2 CONCLUSION

Based on the findings of the study,it can be concluded that:

1. The transportation model is an effective quantitative approach for solving the tanker routing problem
2. The use of Optimization techniques such as the stepping stones methods or MODI (Modified Distribution) method provides a systematic way of minimizing total cost while meeting all supply and demand constraints
3. By adopting the model, organisations (especially oil marketing or logistics component) can enhance operational efficiency, reduce cost and increases profitability.
4. The approach also minimizes unnecessary fuel consumption and vehicle weer, contributing to environmental sustainability and better fleet management

5.3 RECOMMENDATION

Based on the findings and conclusion, the following recommendations are made:

1. Implementation of optimization software:

Organization should adopt linear programming or transportation Optimization software(like LINGO, Excel Solution, or TORA) for routine tanker routing and scheduling.

2. Data Accuracy:

   Accurate data or supply, demand, distances, fuel costs, and road conditions should be regularly updated to ensure reliable moded output

3. Tracing of personnel.

   Staff involved in logistics and planning should be trained in operations research techniques to understand and apply optimization modes effectively.

4. Policy Support:

   Management should create polices that encourage the adoptions of mathematical models for planning and decision - makiy in logistics operations

5. Further Research ;

   Further studies could extend the model to include time windows, vehicle capacities and dynamic demand - turning it into a vehicle Routing Problem (VRP) for more psalm realistic application.

5.4 LIMITATIONS OF STUDY

Although the study achieved its objective,it faced limitations;

1. Limited data availability on actual fuel costs and destinations

2. Simlifed assumption of constant cost per route
3. Exclusive of unpredictable factors such as road conditions, traffic,and mechanical break down

REFERENCES

Agbadudu A.B(1998) “Elementary Operation Reasearh”. A.B Mudiaga Limited

Ekoko P.O (1998) “ Basic Operation Research In Science and Social Science”. United city Press,Benin City Edo state.

Ekoko, P.O (2011). Operation Research for Science and the Social Science. Third Edition.Minidex Publishing Company Limited, Benin city

Garuin W.W(1960) “Introduction to Linear Programming”. New york Wiley

Gupa P.K and Hira D.S., (2005). “Operation reasearch; Principle and solution”. S.Chend and Co. Ltd New Delhi

Hiller, F.S.,& Lieberman, G.J.(2010) “Introduction to Operation Research(9th edition.)”.McGraw-Hill.

Koopman T.C (1947), Optimum Utilization of Transportation System. Proc. International Conference Washington D.C. Nagurney, A. (2004).

Lee, S.M(1966) Introduction to Management Science New York wiley

Nobe, H (2023). Transportation Problem in Operations Reasearch. Available online at www . Linkedin.Com

Ogumeyo, S.A and Panya, A.L(2024). Transportation Model to Determine Minimum Cost Routes Using a Dynamic Programming Approach.

Sharma S.D(1996). Operation Research, India: MC Kinsey Company Inc.