DESIGN, CONSTRUCTION AND STABILIZATION OF INVERTED PENDULUM ON A MOVING CART

BY

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DEPARTMENT OF PRODUCTION ENGINEERING,
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UNIVERSITY OF BENIN.

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A THESIS SUBMITTED TO THE DEPARTMENT OF PRODUCTION ENGINEERING, FACULTY OF ENGINEERING, IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE AWARD OF DEGREE OF MASTER IN MANUFACTURING ENGINEERING, UNIVERSITY OF BENIN.

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CERTIFICATION

This is to certify that the project work “DESIGN, CONSTRUCTION AND STABILIZATION OF INVERTED PENDULUM ON A MOVING CART” By **EDWIN IDIODE**, Matric Number: PG/ENG2110670, submitted in partial fulfillment of the requirements of award of Master’s Degree in Engineering (M.Eng) has been approved.

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DEDICATION

This project is dedicated to God Almighty for his protection, kindness, strength over my life throughout the period and to my lovely wife; Mrs. Ruth Idiode for her prayers, encouragement and moral care towards me. Also, to my Boss, Chief Henry Osiughwu Itedjere, the Registrar, Federal College of Education (Technical) Ekiadolor, Benin, Edo State, who was instrumental in facilitating ideas and encouragement to finish up the programme. It is my prayer that God Almighty strengthen and bless them in all their endeavours.

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ABSTRACT

The project aimed to design, construct, and stabilize an inverted pendulum on a moving cart, demonstrating the practical application of modern control techniques in managing non-linear and unstable dynamic systems. To achieve this, a dynamic model of the pendulum-cart system was developed for analysis and simulation, followed by the construction of a physical prototype using appropriate mechanical and electronic components. Various control strategies, including PID, LQR, and state-space feedback, were designed, implemented, and tested to ensure the pendulum remained balanced in its upright position through real-time feedback and continuous control adjustment.

A systematic methodology was adopted, beginning with mathematical modeling of the system using Newtonian and Lagrangian mechanics to derive and linearize the equations of motion. The model was simulated in MATLAB/Simulink to analyze behavior and optimize control parameters. The physical setup was built with lightweight materials, equipped with DC and stepper motors for movement, and integrated with sensors like encoders, gyroscopes, and accelerometers for real-time feedback. The control algorithms were embedded into a microcontroller, allowing real-time implementation and dynamic stabilization under various disturbances and operating conditions.

The results showed successful stabilization of the inverted pendulum through effective feedback control. Among the tested controllers, the PID handled small deviations well but was less robust under disturbances, while the LQR controller provided superior performance, achieving quick settling times, minimal overshoot, and high stability. The state-space controller also demonstrated strong disturbance rejection and flexibility. The hardware tests closely matched the simulation results, confirming the model's accuracy. Overall, the project validated that advanced control methods, particularly LQR, can efficiently stabilize complex, unstable systems, offering valuable insights for applications in robotics, autonomous systems, and adaptive control environments.

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CHAPTER ONE

INTRODUCTION

1.1 BACKGROUND TO THE STUDY

The inverted pendulum on a moving cart is a classic example of an inherently unstable system widely studied in the fields of control engineering, robotics, and dynamics. This system consists of a pendulum attached to a cart that can move along a horizontal track. The objective is to keep the pendulum balanced in an upright position, which is a dynamically unstable state, by adjusting the movement of the cart. The challenge lies in the fact that any small deviation from the upright position causes the pendulum to fall, and it is up to the cart's movement to compensate for this instability. This makes the system an excellent test bed for studying control algorithms and methods designed to stabilize unstable systems.

It is a classical problem in control theory, widely studied for its practical applications and its ability to demonstrate the principles of dynamics, control, and stabilization. This system consists of a rigid rod (the pendulum) mounted on a moving cart, where the objective is to keep the pendulum balanced in its upright position while the cart moves along a defined track. The study of this system offers a challenging and insightful exploration into the fields of mechanical engineering, robotics, and control systems.

The design and construction of an inverted pendulum on a moving cart involves several stages, including mechanical design, system integration, and the implementation of control systems. Mechanical design includes selecting the appropriate materials and components for the cart, pendulum, and actuators, ensuring stability and responsiveness. The system typically requires sensors to measure the position and orientation of the pendulum, such as encoders, accelerometers, or gyroscopes, to provide real-time feedback.

In terms of control, various strategies are employed to achieve stabilization. The system's dynamics are nonlinear and highly sensitive to external disturbances, making stabilization a complex task. Classical control techniques such as Proportional-Integral-Derivative (PID) controllers are often implemented first due to their simplicity. More advanced approaches, such as Linear Quadratic Regulator (LQR) and state-space feedback, are then explored to improve the system's performance in terms of stability, response time, and robustness to disturbances.

The process of stabilizing the inverted pendulum provides invaluable insight into the complexities of dynamic control, and the results have broader applications in autonomous robotics, mobile platforms, and other systems that require precise control in unstable or unpredictable environments. Through this project, the principles of real-time adaptive control are brought to life, showcasing how theory and practice converge to solve real-world engineering challenges.

1.2 PROBLEM STATEMENT

The inverted pendulum system is a classic problem in dynamics and control theory. It involves a pendulum mounted on a cart that can move along a horizontal track. The objective is to keep the pendulum upright, despite the natural tendency to fall due to gravity, by controlling the movement of the cart. This system serves as a practical example for studying feedback control mechanisms and stability analysis, and it is widely used in robotics, automation, and control system education.

The goal is to design, construct, and stabilize an inverted pendulum mounted on a moving cart. The system should be able to maintain the pendulum in an upright position for an extended period of time, despite disturbances and external forces. This requires designing an appropriate control system, as well as constructing a physical system that integrates sensors and actuators to monitor and adjust the cart's position.

1.3 AIMS AND OBJECTIVES OF THE PROJECT

The aim of this study is to design, construction and stabilization of inverted pendulum on a moving cart

Objectives

i. Use the inverted pendulum system as a demonstration tool to teach concepts of control theory, feedback loops, and stability in educational settings.
ii. …………………………………..
iii. Design and implement appropriate control strategies (e.g., PID controller, to stabilize the inverted pendulum while compensating for disturbances and system variations.
iv. Develop the feedback loop that takes inputs from the sensors (e.g., accelerometers, encoders) and applies control signals to the actuators (e.g., motors) to adjust the position of the cart.
v. Optimize the controller parameters to achieve the best performance in terms

of response time, settling time, and stability.

vi. Design and construct the mechanical system, including the cart, pendulum, and actuators that will allow the cart to move along a track and the pendulum to tilt.
vii. Choose and implement suitable **actuators** (e.g., DC motors) to drive the cart and maintain the desired position to keep the pendulum balanced.
viii. Document the design process, theoretical analysis, control algorithm development, and testing results, creating a comprehensive report or user manual for future learning or system replication.

1.4 SCOPE OF THE PROJECT

The scope of this project system spans a wide range of areas, from theoretical modeling to real-world applications.

1.5 LIMITATIONS OF THE PROJECT

While the inverted pendulum on a moving cart is a highly useful system for demonstrating control theory, dynamics, and real-world applications, there are several limitations associated with its design, construction, and stabilization. These limitations include technical, mechanical, and environmental factors that can affect the system's performance. Below are the key limitations:

i. Sensor Limitations
ii. Control System Limitations
iii. Mechanical Limitations
iv. Environmental Limitations
v. Design Constraints
vi. Cost and Complexity
vii. Scaling Issues
viii. Real-World Applicability

CHAPTER TWO

2.0 LITERATURE REVIEW

2.1 OVERVIEW OF THE INVERTED PENDULUM ON A CART

The inverted pendulum on a cart is a dynamic system that exemplifies control challenges in stabilizing an inherently unstable structure. The system consists of a pendulum pivoted at one end and mounted on a moving cart. The primary goal is to stabilize the pendulum in its upright position while allowing the cart to move along a horizontal track.

1. System Description

- **Components**:
  - **Pendulum**: A rigid rod pivoted at the cart's top surface, free to rotate in the vertical plane.
  - **Cart**: A movable base that travels along a track, powered by an actuator (typically a DC motor).
  - **Sensors**: Devices such as encoders, accelerometers, and gyroscopes measure the pendulum's angle and the cart's position.
  - **Controller**: A control algorithm (e.g., PID, LQR) processes sensor data and computes corrective actions.
  - **Actuator**: The motor or force generator moves the cart based on controller commands.
- **Degrees of Freedom**:
  - The cart's linear motion along the track.
  - The pendulum's angular motion about its pivot point.

2. System Dynamics

The inverted pendulum system is governed by nonlinear dynamics, as the equations of motion involve trigonometric terms. These dynamics are usually described by:

1. **Cart Dynamics**: Represents the linear motion of the cart influenced by the applied force.
2. **Pendulum Dynamics**: Represents the rotational motion of the pendulum under gravity and external forces.

Key parameters include:

- Pendulum length, mass, and moment of inertia.
- Cart mass and frictional forces.
- Gravity as the destabilizing force.

3. Challenges

- **Instability**: The pendulum's upright position is an unstable equilibrium point, requiring active control.
- **Nonlinearity**: The system exhibits nonlinear behavior, especially for large-angle deviations.
- **Coupled Dynamics**: The cart's motion affects the pendulum and vice versa, complicating the control design.
- **Disturbances**: External forces like friction, noise, and unmodeled dynamics add complexity.

Future Directions

While significant progress has been made, challenges remain, including:

- **Robustness to External Disturbances**: Enhancing system resilience to noise and disturbances.
- **Energy Efficiency**: Minimizing energy consumption during stabilization.
- **Real-Time Implementation**: Adapting advanced algorithms for real-time applications.

Future research trends point toward integrating artificial intelligence, improving energy efficiency, and expanding the scope of real-world applications.

4. Control Strategies

Several control approaches have been developed to stabilize the system:

- **Linear Control**:
  - Linear Quadratic Regulator (LQR) for optimal state-space control.
  - PID controllers for basic feedback control.
- **Nonlinear Control**:
  - Sliding mode control for robustness against disturbances.
  - Feedback linearization to handle nonlinearity.
- **Modern Control**:
  - Reinforcement learning for adaptive control based on system interaction.
  - Model Predictive Control (MPC) for future trajectory planning.

A variety of control strategies have been proposed to stabilize the inverted pendulum system, ranging from classical control techniques to advanced modern approaches. These strategies include:

Classical Control

- **Proportional-Integral-Derivative (PID) Control**: A simple yet effective method for small perturbations. Works well under linearized conditions but struggles with high nonlinearity.
    - Example: **Ogata (1997)** provided extensive guidelines for tuning PID controllers for linear systems, including inverted pendulums.

Optimal Control

- **Linear Quadratic Regulator (LQR)**: A popular method that minimizes a cost function to achieve stability and performance.
    - Example: **Bryson and Ho (1975)** demonstrated the effectiveness of LQR in controlling the inverted pendulum.

Nonlinear Control

- **Sliding Mode Control (SMC)**: Handles system nonlinearity and uncertainties effectively.
    - Example: **Utkin (1977)** introduced SMC, which has been applied extensively to the inverted pendulum.
- **Feedback Linearization**: Simplifies the nonlinear dynamics to a linear system for easier control.
    - Example: **Isidori (1995)** developed methods to apply feedback linearization to dynamic systems.

Adaptive and Robust Control

- Addresses system uncertainties and parameter variations.
    - Example: **Narendra and Annaswamy (1989)** discussed adaptive control techniques applicable to unstable systems like the inverted pendulum.

Intelligent Control

- **Fuzzy Logic Control**: Employs human-like reasoning to stabilize the pendulum.
    - Example: **Zadeh (1973)** introduced fuzzy logic, later applied to dynamic systems.

- **Neural Networks and Machine Learning**: Utilize learning-based approaches for improved control.
  - Example: **Sutton and Barto (1998)** highlighted reinforcement learning applications to stabilize inverted pendulums.

Strategies and Stabilization Techniques

The control of an inverted pendulum system is a challenging task because it is inherently unstable. Some other control strategies have been proposed to stabilize the pendulum.

Proportional-Integral-Derivative (PID) Control

One of the simplest and most widely used control strategies for inverted pendulum stabilization is the **PID controller**. A PID controller works by computing an error signal based on the difference between the desired and actual position (or angle) of the pendulum and the cart, and applying corrections based on proportional, integral, and derivative terms. The effectiveness of PID controllers depends on careful tuning of the controller's parameters, which can be done manually or using optimization algorithms like **Ziegler-Nichols tuning**.

- **Wang et al. (2003)** used a PID controller to stabilize a two-wheeled inverted pendulum on a moving cart. The authors reported the PID controller's ability to stabilize the system within a specified settling time, though the controller was sensitive to model uncertainties.

Linear Quadratic Regulator (LQR)

The **LQR** is a more sophisticated optimal control technique based on minimizing a cost function that penalizes deviations from the desired states (cart position and pendulum angle) and the control effort. The LQR provides a **state-feedback control law** that stabilizes the inverted pendulum more efficiently than PID controllers, especially for systems with multiple inputs and outputs.

- **Lee and Lee (2011)** designed an LQR-based controller to stabilize an inverted pendulum on a moving cart. Their work demonstrated the superior performance of LQR in terms of faster response and less oscillation when compared to traditional PID controllers.

State-Feedback and Pole Placement

State-feedback control and **pole-placement** techniques are often used for stabilizing the system. In this approach, the state variables (cart position, pendulum angle, etc.) are measured, and the control input is adjusted by placing the poles of the system's characteristic equation in locations that ensure stability.

- **Ogata (2001)** introduced the concept of placing poles to design stable controllers for complex systems, such as the inverted pendulum. By choosing appropriate poles, the stability margin of the inverted pendulum can be guaranteed even when there are slight variations in the model parameters.

Nonlinear Control

Since the inverted pendulum is a highly nonlinear system, linear control techniques may not perform well in all situations, especially when the pendulum is significantly displaced from the vertical position. Nonlinear control methods such as feedback linearization**,** sliding mode control**,** and fuzzy logic controllers have been proposed to address these issues.

- Wang and Xu (2013) applied sliding mode control to stabilize the inverted pendulum on a moving cart. The controller exhibited robustness against model uncertainties and external disturbances, making it highly effective in practical implementations.
- Sanchez and Shishika (2015) explored the use of fuzzy logic controllers for stabilizing the inverted pendulum, which provided better performance under varying system conditions compared to traditional PID controllers.

2.2 MODELING OF THE INVERTED PENDULUM SYSTEM

The **inverted pendulum on a moving cart** is a well-known problem in mechanics and control theory. It consists of a pendulum attached to a cart that can move horizontally along a track. The system is inherently unstable when the pendulum is in the upright position and requires a control system to stabilize it. The mathematical modeling of the system involves applying Newtonian mechanics or Lagrangian mechanics to derive the equations of motion.

Below is the step-by-step process of modeling the **inverted pendulum on a cart**.

1. System Description

- **Cart**: A rigid body of mass M that can move along a frictionless horizontal track.

- **Pendulum**: A rigid body of mass mmm and length LLL, which is attached to the cart at its base. The pendulum can rotate about a fixed pivot point.
- **Control Input**: A force F is applied to the cart to move it horizontally.
- **State Variables**:

The inverted pendulum on a moving cart is a classical control systems problem that has been widely studied in engineering, robotics, and mechatronics. It is used as a benchmark for testing and understanding various control algorithms due to its inherent instability and real-world applications in robotics, autonomous vehicles, and other dynamic systems. This literature review aims to provide a comprehensive overview of the key studies and advancements in the design, construction, and stabilization of the inverted pendulum system, focusing on **modeling**, **control strategies**, and **system implementation**.

2. **Linearized Model**: When the pendulum is in a near-vertical position, the nonlinear equations are typically linearized to simplify control design. The resulting linearized model takes the form of a set of **state-space equations**, which describe the system in terms of matrices:

$x^{\cdot} = Ax + Bu$

Where x is the state vector containing the cart position and velocity, A is the system matrix, B is the input matrix, and u is the control input (force applied to the cart).

Lagrangian Mechanics

The Lagrangian formulation is a powerful method for deriving the equations of motion for mechanical systems. The **Lagrangian** (L) is given by the difference between the **kinetic energy** (T) and **potential energy** (V) of the system:

$L=T-VL = T - VL=T-V$

For control system design, it is often necessary to linearize the equations of motion around the **upright equilibrium** (i.e., when the pendulum is vertical and the cart is stationary).

This state-space model can be used to design a feedback control system to stabilize the inverted pendulum.

Undoubtedly, the modeling of the inverted pendulum on a moving cart involves understanding the dynamics of the system using Lagrangian mechanics, deriving the equations of motion, and then linearizing the system for control design. These equations form the foundation for designing controllers (such as PID, LQR, or state-feedback controllers) to stabilize the pendulum in an upright position. The inverted pendulum on a cart serves as an ideal test bed for control theory, providing insights into the challenges of controlling unstable systems.

2.3 COMPONENTS FOR THE DESIGN AND CONSTRUCTION OF THE INVERTED PENDULUM ON A MOVING CART

- **The Cart**

1. Material: Aluminum for light weight.
2. Dimensions: 50 cm in length, 20 cm in width, and 20 cm in height.
3. Wheels: Two wheels with bearings for smooth movement and reduced friction.
4. Track: A bounded rail for the cart to move on, 2 meters in length.

- **The Pendulum**

1. Material: Lightweight aluminum.
2. Dimensions: 50 cm in length, with a diameter of 5 cm.
3. Pivot: A hinge or bearing connecting the pendulum to the cart, allowing for smooth rotation.
4. Counterweight: Optional counterweight at the bottom of the pendulum to stabilize it.

- **The Actuator**

1. Type: stepper motor.
2. Power: Sufficient power to move the cart and stabilize the pendulum.
3. Control: PWM (Pulse Width Modulation).

- **The Sensors**

1. Position sensors: Encoders or potentiometers to measure cart position and pendulum angle.
2. Velocity sensors: Tachometers or accelerometers to measure cart velocity and pendulum angular velocity.
3. Angle sensors: Inclinometers or gyroscopes to measure pendulum angle and angular velocity.

- **The Control System**

1. Microcontroller: Arduino, Raspberry Pi, or other microcontrollers to read sensor data and control the actuator.
2. Software: Programming languages like C++, Python, or MATLAB to implement control algorithms.
3. Communication: Serial communication, Wi-Fi, or Bluetooth for data transmission and monitoring.

- **The Power Supply**

1. Voltage: Suitable voltage for the actuator, sensors, and control system.
2. Current: Sufficient current to power all components.
3. Battery or adapter: Optional battery or adapter for portability or convenience.

These components work together to create a functional inverted pendulum system on a moving cart, allowing for experimentation and research in control theory and robotics.

2.4 THE INVERTED PENDULUM SYSTEM CONSTRUCTION AND IMPLEMENTATION

The design and construction of the inverted pendulum system require integration of several key components, including the cart**,** pendulum, sensors**,** and actuators**.**

Cart: The cart is the base of the system, and it moves horizontally on a bounded track.

Pendulum: The pendulum is a rigid rod with a pivot mounted on top of the cart.

Actuator: The actuator is used to apply a control force to the cart.

Sensors: Sensors are used to measure the state of the system, including the cart position, cart velocity, pendulum angle, and pendulum angular velocity.

Mechanical Design

The mechanical design of the inverted pendulum involves selecting appropriate materials for the cart, pendulum, and mounting mechanisms, ensuring that the system can withstand dynamic forces and maintain precision during operation. Typical designs include a track for the cart to move on and a pivot point for the pendulum to rotate.

- Peng and Zheng (2010) developed a stable mechanical setup for the inverted pendulum, focusing on the precise movement of the cart along a track and the stability of the pendulum. The use of lightweight materials and low-friction components helped improve performance.

Sensor Integration

To measure the pendulum's angle and the cart's position, sensors such as encoders, gyroscopes, and accelerometers are used. Accurate sensor measurements are critical for providing feedback to the control system, which in turn adjusts the cart's position to stabilize the pendulum.

- Kandil and Altintas (2004) integrated a set of optical encoders and accelerometers to provide real-time feedback to the control system. These sensors were capable of tracking the cart's position and the pendulum’s angle with high precision.

Actuation and Motor Control

To move the cart and correct the pendulum's position, motors are used to generate forces. Typically, DC motors or servo motors are used to drive the cart and adjust its position along the track.

- Wu and Chen (2007) used DC motors with PID control to drive the cart. The motors provided the required precision to adjust the cart's position based on control signals generated by the system.

The design, construction, and stabilization of an inverted pendulum on a moving cart have been studied extensively in the context of control systems, mechatronics, and robotics. Various control techniques such as PID, LQR, sliding mode control, and fuzzy logic have been applied to stabilize the system, with significant advances in nonlinear control strategies to improve robustness and performance. The development of the mechanical system, sensor integration, and actuation mechanisms is critical for the system's successful implementation. Furthermore, the inverted pendulum system serves as an excellent platform for research and education in control theory, providing insights into the stabilization of dynamic systems in real-world applications such as robotics, autonomous vehicles, and space exploration.

Research and Practical Implementation

The inverted pendulum on a cart continues to serve as a test bed for advancements in:

- Real-time control algorithms.
- Sensor and actuator integration.
- Robust and adaptive control methodologies.

Its simplicity and versatility make it an enduring subject of study in control systems, with broad implications in engineering, robotics, and automation.

I. INVERTED PENDULUM

The **inverted pendulum** is a classic mechanical system that consists of a rigid rod (the pendulum) attached to a pivot point, which is free to rotate in the vertical plane. The system is characterized by the **unstable equilibrium** when the pendulum is positioned in an upright position (i.e., with the pendulum facing upward). Unlike a regular pendulum, which is stable in its downward resting position, the inverted pendulum is unstable when upright and requires constant corrective forces to maintain its balance.

The pendulum can be mounted on a cart, a robotic arm, or other moving platforms, and its primary challenge is to keep it balanced and prevent it from tipping over. Typically, an external force is applied to move the base (the cart or platform) in such a way that the pendulum is kept upright.

Key Characteristics:

- **Unstable Equilibrium**: When the pendulum is perfectly vertical, it is in an unstable equilibrium. Any small perturbation or disturbance will cause the pendulum to fall unless corrective actions are taken.
- **Control System Challenge**: The inverted pendulum is often used to test and demonstrate **control systems** because it requires continuous feedback and real-time adjustments to maintain balance.
- **Dynamic System**: The system is governed by complex physics, including forces like gravity, inertia, and friction. The motion of both the pendulum and the cart (if used) needs to be carefully managed.

- x: Position of the cart along the track.

- θ: Angle of the pendulum with respect to the vertical (upright) position.
- $\dot{x}$: Velocity of the cart.
- $\dot{\theta}$: Angular velocity of the pendulum.

The goal is to develop the equations of motion for the system in order to understand its dynamics and design an appropriate control system to keep the pendulum upright.

2. NEWTON EULER EQUATIONS

The **Newton-Euler method** is a fundamental approach used to derive the equations of motion for a mechanical system, taking into account both forces and torques (moments). For the **inverted pendulum on a moving cart**, we apply this method to model the dynamics of the cart and the pendulum as it moves and interacts with its environment.

We will derive the Newton-Euler equations for the **inverted pendulum** on a **moving cart** considering forces such as gravity, the applied control force F acting on the cart, and the motion of the pendulum. The system consists of two main components:

1. A **cart** (with mass M) that moves horizontally along a track.
2. A **pendulum** (with mass m and length L) attached to the cart, which can rotate about the pivot point.

The goal is to find the equations of motion that describe how the cart and pendulum interact dynamically.

Step 1: Define the System

Let's define the key parameters and variables:

- **Cart position** x (horizontal position along the track)
- **Pendulum angle** θ (angle with respect to the vertical)
- **Cart mass** M
- **Pendulum mass** m
- **Pendulum length** L
- **Gravity** g (acceleration due to gravity)
- **Force F** applied to the cart

Step 2: Forces and Torques Acting on the System

We need to identify the forces and torques that affect both the cart and the pendulum.

Forces on the Cart

- The applied force F acting horizontally on the cart.
- The tension T in the pendulum rod that acts at the point where the pendulum is attached to the cart. This force is directed along the rod and affects the motion of both the cart and the pendulum.

Forces on the Pendulum

- The **gravitational force** mg, acting vertically downward through the pendulum's center of mass.
- The **tension** T in the rod, which is responsible for the pendulum's rotational motion.

Step 3: Newton-Euler Equations for the Cart and Pendulum

We will use **Newton's Second Law** for translational motion (for the cart and the center of mass of the pendulum) and **Euler's Rotational Equation** for the pendulum.

For the Cart (Horizontal Motion)

We apply Newton's Second Law in the horizontal direction (since the cart moves horizontally along the track):

F−Tcos(θ)=MxF - T \cos(θ) = M \(x·)F−Tcos(θ)=Mx

Where:

- F is the external force applied to the cart.
- Tcos(θ)T \cos(θ)T is the horizontal component of the tension force in the rod.
- MxM \{x}Mx is the horizontal acceleration of the cart.

For the Pendulum (Rotational Motion)

Now, for the **pendulum**, we use Euler's equation for rotational motion about the pivot point.

Step 4: Expressing the System's Equations of Motion

To fully describe the system's dynamics, we need to write down the equations of motion for both the cart and the pendulum in terms of the state variables. The forces and torques derived above can be combined into a set of equations for the entire system.

Cart's Equation of Motion (Translational)

Mx¨=F−Tcos(θ)M \ddot{x} = F - T \cos(\theta)Mx¨=F−Tcos(θ)

Step 5: Using the Constraint for the Tension T

To obtain the tension T, we need to apply a constraint based on the **relationship between the accelerations** of the cart and the pendulum. The pendulum is attached to the cart, and its center of mass moves with the cart. The total acceleration of the pendulum's center of mass is the combination of the cart’s acceleration and the pendulum’s angular acceleration.

Ultimately, the **Newton-Euler equations** provide a systematic method to model the **inverted pendulum on a moving cart**. By using these equations, we can capture the dynamic interaction between the cart and the pendulum, which is crucial for understanding how the system behaves under different forces and disturbances. These equations serve as the foundation for designing control strategies (e.g., PID, LQR, state feedback) to stabilize the system and keep the pendulum balanced in the upright position.

3. LINEARIZATION

The **inverted pendulum on a moving cart** is a highly nonlinear system. In control theory and design, linearization is a common technique used to simplify the system and make it amenable to standard control techniques (e.g., PID, LQR, state feedback). Linearization involves approximating the nonlinear system around an equilibrium point, typically the **upright position** of the pendulum.

Interpretation of the Linearized Model

The state-space model describes the dynamics of the cart and the pendulum in terms of the state variables (cart position, pendulum angle, cart velocity, and pendulum angular velocity) and the control input (force applied to the cart).

- **Matrix A** describes the system's dynamics without the control input.
- **Matrix B** describes how the control input force (F) affects the system.

In the linearized system, the pendulum's angular position θ and the cart's position x are treated as small deviations from the equilibrium point. This linear approximation simplifies the analysis and allows for the design of feedback control systems, such as **PID control** or **LQR control**, to stabilize the inverted pendulum.

2.5 THE IMPORTANCE OF THE INVERTED PENDULUM ON A MOVING CART

The inverted pendulum on a moving cart holds significant importance in engineering, control systems, and applied research due to its ability to model complex real-world systems and provides insights into dynamic stabilization. Below are key reasons for its relevance:

1. Benchmark Problem in Control Theory

The inverted pendulum is a standard problem for testing and validating control algorithms. Its dynamics are inherently unstable and nonlinear, providing a challenging yet manageable test bed for:

- Linear control techniques like PID and Linear Quadratic Regulators (LQR).
- Nonlinear control methods such as sliding mode and feedback linearization.
- Advanced strategies like Model Predictive Control (MPC) and reinforcement learning.

2. Real-World Applications

The system models various real-world phenomena, making it highly relevant for practical implementations:

- **Robotics**:
  - Self-balancing robots (e.g., Segway, hoverboards, and humanoid robots).
  - Robotic arms and manipulators that require dynamic stability.
- **Aerospace**:
  - Rocket stabilization during launch and flight.
  - Control of satellite orientation and spacecraft dynamics.
- **Biomechanics**:
  - Models human posture and walking dynamics, aiding in the development of prosthetics and exoskeletons.

3. Educational Value

The inverted pendulum system is an essential educational tool for teaching fundamental and advanced concepts:

- **System Dynamics**: Understanding the behavior of nonlinear and unstable systems.

- **Control Design**: Practical application of theories like state-space methods and pole placement.
- **Simulation and Implementation**: Offers hands-on learning with tools like MATLAB, Simulink, and hardware platforms.

4. Demonstration of Fundamental Concepts

It demonstrates core principles in various fields:

- **Stability and Feedback**: Shows how feedback mechanisms counteract instability.
- **Nonlinearity**: Offers a clear example of the challenges in controlling nonlinear systems.
- **Coupled Dynamics**: Highlights the interplay between the pendulum and the cart, a critical concept in multi-body dynamics.

5. Research and Innovation

The system serves as a foundation for innovative research:

- **Advanced Control Algorithms**: Testing adaptive, fuzzy logic, and AI-driven control methods.
- **Autonomous Systems**: Improving the understanding of dynamic balancing for autonomous vehicles and robots.
- **Multi-Axis Systems**: Expanding the concept to 2D and 3D stabilization problems, such as drones and gimbals.

6. Scalability and Adaptability

The principles learned from the inverted pendulum system can be extended to more complex scenarios:

- Stabilizing higher-order systems like double or triple inverted pendulums.
- Applying similar techniques to mechanical systems with higher degrees of freedom.

CHAPTER THREE

3.0 MATERIALS AND METHOD

3.1 DESIGN

3.1.1. Rack Design (Track System for the Cart)

The **rack** is the **fixed track** along which the cart moves. A rectangular shaft with gear teeth and it is designed by determining the gear tooth dimension with calculation on gear teeth profile.

- **Material Selection**:
  - **Aluminum** (lightweight and strong)
  - **Steel** (durable but heavier)
- **Dimensions**:
  - Provision of sufficient track length for the cart to move freely.
  - **50 cm** long.

3.1.2 The Rack and Slider Holder (Stand)

The **rack and slider holder** in an inverted pendulum system ensures that the cart moves smoothly along a fixed track while minimizing friction and providing stability. This is to ensure precision, durability, and system performance**.**

The weight of the stepper motor and the rack was used in determining the reactions of the weight on both sides. The **slider holder** is the component that attaches the cart to the track and ensures smooth motion.

Slider Holder Material Selection: Aluminum (light weight) and Steel (strong)

3.1.3. Designing the Pendulum.

The pendulum in an inverted pendulum on a moving cart system is the most crucial part of the setup, as it must be designed for stability, precise motion, and minimal friction to allow effective control.

Design Considerations:

Low weight but high strength**:** to reduce the moment of inertia while maintaining durability.
Minimal friction at the pivot: to ensure smooth motion and accurate measurements.
Precise length selection: for stability and response time.

A. Material Selection

Pendulum Rod (steel) Lightweight yet strong

B. Length and Weight Selection

The pendulum length (L) 50 cm

Lightweight & customizable

3.1.4. Gear Design

The gear system in an inverted pendulum on a moving cart setup is typically used for:

1. Transmitting motion from the motor to the cart.
2. Adjusting torque and speed via gear ratio selection.
3. Reducing backlash and ensuring smooth motion for precise control.

For the gear design, elements such as size, tooth shape, pitch, number of teeth, amount of profile shift, material, choice for kind of tooth surface finish, method of mounting on shaft, precision class, etc., were determined.

Also, number of teeth which depend on the ratio between the drivers and driven shaft, were determined. The teeth have the same module (or diametral pitch) which avoided undercutting the teeth. The module chosen was in line with the size of the assembly, the gear ratio and the minimum number of teeth on the rack.

Gear Material Selection

- Aluminum with Lightweight, moderate durability.
- Steel (Hardened or Stainless) with High durability but heavy.

Designing the Gear Teeth

The involute curve is generated by unwinding a taut string from a base circle. This ensures:

- Constant pressure angle for smooth power transmission.
- Gradual engagement and disengagement of teeth.

The parametric equations for an involute curve are:

$$X = rb(\cos\theta + \theta \sin\theta)$$

$$Y = rb \; (\sin\theta - \theta \cos\theta)$$

Where:

- rb = **Base Circle Radius** = $r\cos\alpha$
- θ = **Unrolling Angle**

- α = **Pressure Angle** (typically 25°)

Key Design Parameters

To define an involute gear, the following parameters are essential:

Basic Gear Parameters

1. **Module (m)** = $\frac{\text{Pitch Diameter}}{\text{Number of Teeth}}$

2. **Number of Teeth (Z)**
3. **Pitch Diameter (D)** = $m \times Z$
4. **Pressure Angle (α)** (25°)
5. **Base Circle Diameter (Db)** = $D \cos\alpha$

Tooth Dimensions

- **Addendum (a)** = m (Tooth height above the pitch circle)
- **Dedendum (d)** = 1.25m (Tooth depth below the pitch circle)
- **Tooth Thickness (t)** = $\underline{\pi m}$ (at pitch circle) 2
- **Clearance** = 0.25m

Strength & Performance Considerations

A. Bending Strength

Use the **Lewis Equation** to estimate bending stress:

$$\sigma = \frac{Ft}{bYm}$$

where:

- F = Load on gear tooth
- t = Tooth thickness
- b = Face width
- Y= Lewis form factor

2. Involute Function Calculation

The involute function is given by:

$$\text{Involute}(\phi) = \tan(\phi) - \phi(\text{in radians})$$

For a given pressure angle (ϕ), the radial displacement from the base circle is:

$$r = \frac{db}{2}(\tan(\theta) - \theta)$$

where (θ) is the involute function of the pressure angle.

3. Gear Tooth Thickness at Pitch Circle

The circular tooth thickness at the pitch circle is given by:

$$t = \frac{\pi m}{2}$$

Key Parameters:

Module (m):

A measure of gear size, calculated as pitch diameter (d) divided by the number of teeth (z).

$$m = \frac{d}{z}$$

Number of Teeth (z): The number of teeth on the gear.

Pressure Angle (α):

The angle between the line of action (tangent to the pitch circle at the pitch point) and the normal to the tooth profile.

Profile Shift Coefficient (x):

Used to modify the tooth profile, allowing for variations in center distance and avoiding undercutting with a small number of teeth.

3.1.5. Cart Design

- **Material**: Lightweight metal or acrylic to reduce inertia.

- **Size**: Designed to accommodate the motor, sensors, and pendulum hinge**.**
- **Actuation**:
    - **DC Motor with Belt/Pulley** (Common)
    - **Step Motor with Lead Screw** (Precise but slower)
    - **Linear Servo Motor** (Fast response)
- **Encoders or Position Sensors**: For tracking cart displacement.

Pendulum Assembly

- **Pivot Joint**:
    - High-precision **ball bearings** for frictionless rotation.
    - **Hinge with encoder** to measure angle.
- **Pendulum Material**:
    - **Aluminum** (Light and strong)
- **Length Considerations**:
    - **50 cm**.

Sensors & Feedback Mechanism

- **Angle Sensor (Encoder/Potentiometer)**: Measures pendulum angle.
- **IMU (Gyroscope & Accelerometer)**: For angular velocity estimation.
- **Limit Switches**: Prevent cart from exceeding track limits.

Power & Control Unit

- **Power Supply**:
    - **DC Adapter** for the stationary setups.
- **Microcontroller**:
    - **Arduino/Raspberry Pi** for basic control.

7. Base and Mounting Considerations

- **Stable Base**: Heavy bottom frame to avoid tipping over.
- **Dampers**: Reduce vibrations and unwanted movements.
- **Enclosure**: For safety in high-speed experiments.

3.2 CONSTRUCTION

Constructing an inverted pendulum on a moving cart involves mechanical, electrical, and control system components.

3.2.1. **Constructing the Rack and Slider Holder for the Inverted Pendulum**

The **rack and slider holder** are critical for guiding the cart along a precise path while supporting the inverted pendulum mechanism. These components ensure smooth linear motion and minimal resistance, allowing accurate control and stabilization of the pendulum.

Fabrication method was introduced to fold and weld the slider and rack holder.

- **Construction Considerations for the Rack and Slider Holder**

✅ **Smooth and precise movement** to minimize friction and disturbances.
✅ **Strong and rigid frame** to avoid vibrations and misalignment.
✅ **Lightweight but durable materials** for efficient motion.
✅ **Easy mounting system** for adjustments and modifications.

Step-by-Step Construction of the Rack and Slider Holder

Step 1: Designing the Rack (Linear Track)

1. **Determination of the required track length**
   - Length taken twice the expected cart displacement**.**
   - As the cart moves ±10 cm, the rack measures **20 cm**.
2. **Selection of the mounting method**
   - Fixed frame attachment (welded to a base).
   - Adjustable track (this allows for fine-tuning of position).
3. **Cutting and finish of the track material**
   - Using a hacksaw for precision cutting.
   - Deburr edges to prevent wear on moving parts.

Step 2: Installing the Motion System

Using Linear Guide Rails

1. Mount linear guide rails on the rack frame.
2. Attach the slider block (bearing carriage**)** to the cart.
3. Ensure low-friction movement across the full range.

Step 3: Testing and Calibration

1. Moving the slider holder manually to check for friction points.
2. Running test movements to verify smooth operation.
3. Adjust alignment to prevent jamming or misalignment.
4. Applying lubrication for optimal motion.

3.2.2. **Constructing the Pendulum for the Inverted Pendulum on a Moving Cart**

The pendulum is the key component in an inverted pendulum on a moving cart system**.** It must be lightweight yet rigid**,** precisely balanced, and have a low-friction pivot for smooth movement.

Using the lathe machine to turn it to the desired length and size

1. Design Considerations for the Pendulum

A well-designed pendulum:

✅ Having the right length and mass for controllability.

✅ Be lightweight but strong to reduce inertia.

✅ Having a low-friction pivot to ensure free rotation.

✅ Include a mounting option for sensors like **encoders or IMUs**.

2. Step-by-Step Construction of the Pendulum

Step 1: Determining the Pendulum Length & Mass

1. Choosing the desired pendulum length (L) of 50 cm
2. Determining the mass (m) at the end 500g
3. Optimize the mass distribution
   - Mass concentrated at the **end** = higher inertia.
   - Mass distributed along the **rod** = smoother response.

Step 2: Cutting and Preparing the Pendulum Rod

1. Cutting the material (Aluminum) to the chosen length.
2. Drilling mounting holes at the top (pivot)
3. Smoothing the edges to avoid rough surfaces.

Step 3: Installing the Pivot (Low-Friction Joint)

The pivot joint is made to allow for free rotation with minimal friction.

Using a Rotary Encoder (For Angle Measurement)

1. Fixing a rotary encoder at the pivot point.
2. Attaching the pendulum to the encoder shaft.
3. Connecting the encoder to a microcontroller for feedback control.

Step 4: Attaching Sensors

- Rotary Encoder which measures pendulum angle.
- IMU (Gyroscope + Accelerometer) measures angular velocity.
- Optical Sensor which detects vertical position.

Step 5: Testing and Calibration

1. Checking for smooth motion by manually moving the pendulum.
2. Ensuring that there is no excessive friction or stiffness at the pivot.
3. Fine-tuning the mass to get the desired inertia.
4. With the sensors, verification done on signal accuracy with a microcontroller (Arduino, Raspberry Pi).

3.2.3. Constructing the Gear for the Inverted Pendulum on a Moving Cart

The **gear** in an inverted pendulum system is a crucial part since the cart movement is controlled using a rack and geared motor drive. The gear must be precisely designed and constructed to ensure smooth motion and accurate control.

This was achieved with the aid fabrication and working on the lathe machine.

Gear Design Considerations

A well-designed **gear system**:

✅ Providing smooth and precise motion without backlash.
✅ Ensuring minimal wear and friction for long-term durability.
✅ **B**eing lightweight but strong to optimize efficiency.
✅ Matching the motor torque and speed requirements.

Steps to Construct the Gear

Step 1: Gear Design

1. **Determine Gear Parameters:**
   - **Module (M)** = Tooth size
   - **Pitch Diameter (D)** = Distance from center to tooth tips
   - **Number of Teeth (Z)** = Affects torque and speed
   - **Pressure Angle (α)** = Usually 20° for standard spur gears
2. **Calculate Gear Ratio:**
   - using a motor-driven rack and pinion**,** the gear ratio is: Gear Ratio=
     Gear Ratio=Driver Teeth/Driven Teeth

Step 2: Mounting and Testing the Gear

1. Attach the gear to the motor shaft securely with a set screw**.**
2. Ensuring proper meshing with the rack.
3. Running test movements to check for smooth motion and adjust alignment.

3.3.4 Construction of the Rack for the Inverted Pendulum on a Moving Cart

The **rack** is the **fixed track** along which the cart moves in an inverted pendulum system. The construction of the rack must ensure stability, precision, and durability for smooth and controlled cart motion.

This was achieved using the milling machine to cut the tooth profile with the indexing head.

Step-by-Step Construction of the Rack

Step 1: Cutting the Rack (Track) to Size

1. Determine the total length of travel required for the cart.
   - 2x the maximum displacement needed for stability.
2. Using the laser cutter to cut the aluminum track.
3. Deburr and smoothening the edges to avoid sharp edges that could affect motion.

Step 2: Mounting the Rack to the Base Structure

1. **Attach the rack to a rigid base** (e.g., aluminum frame or steel plate).
2. **Use precision bolts or clamps** to ensure proper alignment.

3. **Check for levelness** using a spirit level or laser guide.
4. **If using a rack and pinion system**, mount the toothed rack securely using screws or adhesives.

Step 3: Installing the Motion System

Using the Linear Guide Rails

1. Align and attach the linear rails parallel to each other.
2. Secure them using bolts on a flat, rigid surface.
3. Install the linear bearings on the cart to allow smooth motion.

Step 4: Testing and Calibration

1. **Move the cart manually** to check for any resistance or misalignment.
2. **If using motorized motion**, run test movements at low speed.
3. **Use a dial indicator or laser measurement** to check for deviations.
4. **Fine-tune alignment** by adjusting screws or mounting brackets.

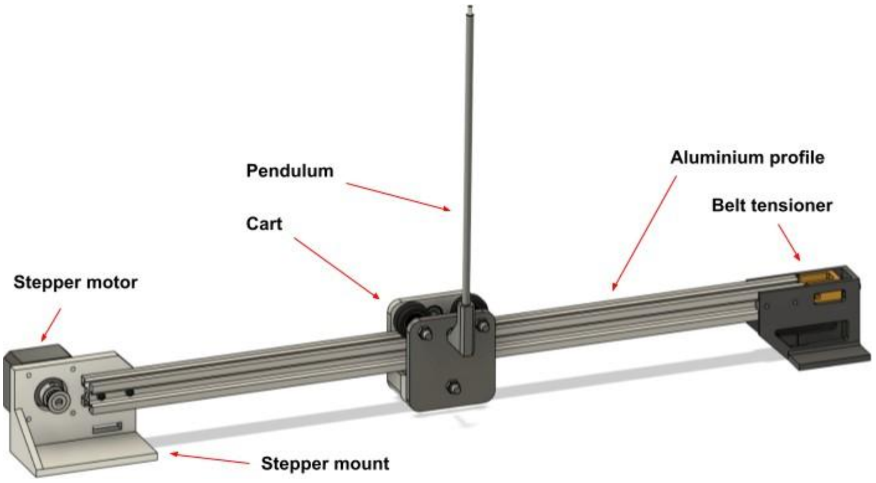


Figure 3.1: Assembly of the inverted pendulum, with main structural components highlighted.

- **Stepper Mount**

As seen in Figure 3.2, the stepper mount serves multiple functions, acting as both housing for the stepper motor, an attachment for the aluminum profile, and as a base for the assembly. It is simple in design, consisting of a flat surface with mounting points to attach the motor and profile using standard M3 screws of two different lengths. The design leverages the property of 3D printed materials being deform-able, by enabling the mounting screws to thread the mount itself during installation. This results in the assembly being rigid and not needing nuts, as long as the screws are long enough to thread into the mount and respective component.

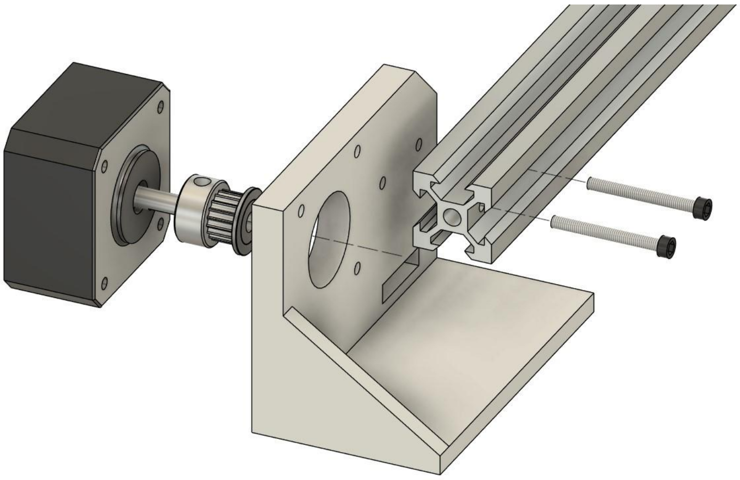


Figure 3.2: Exploded view of the stepper mount and the relevant parts. Screws for mounting the stepper motor are not visible.

- **Cart**

The cart consists of two square plates, bolted together by three M5x50 bolts with V-slot wheels between them. These wheels match the V-slot channels of the aluminum profile, with two wheels on top of the profile and one wheel below. Figure 3.3 shows the cart assembly in an exploded view, showing how the parts fit together. The pendulum itself is mounted to a horizontal rod. The rod acts as the pivot for the pendulum and is held up by two bearings, one in each plate. It inserts into the optical encoder mounted behind the rear plate (not visible in Figure 3.3), enabling measurement of the pendulums angle. One of the plates has built in buckles on each side, which secure the timing belts respective ends to the cart by utilizing their teeth. Spacers are positioned on each side of the V-slot wheels so that the two plates can accommodate the aluminium profile between them, as depicted in Figure 3.4, which also shows the wheel groves and profile channels.

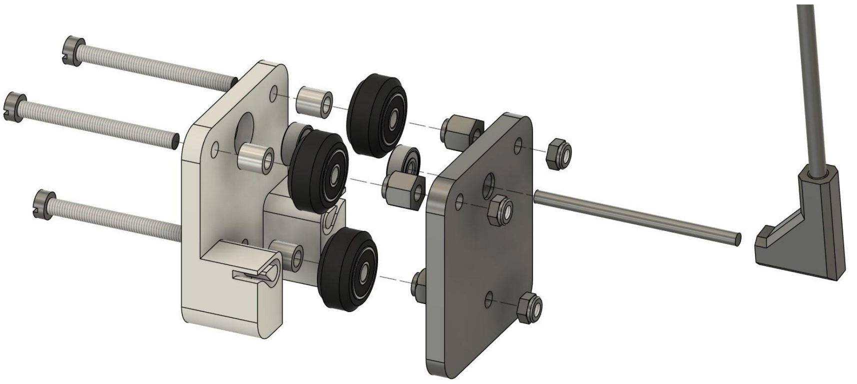


Figure 3.3: Exploded view of the cart.

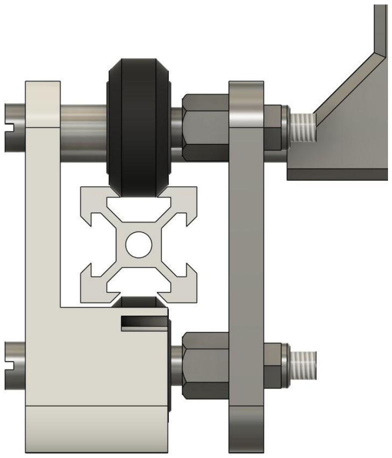


Figure 3.4: Side view of the cart mounted on the aluminum profile.

- **Belt Tensioner**

For the cart to move smoothly, the timing belt needs to be tensioned. This reduces the slack in the system resulting in more consistent movement, and prevents the belt from skipping during fast movements. Naturally, this tensioner is placed on the opposite end of the aluminum profile from where the belt is driven, also serving as a pulley for the belt to rotate around. The tensioning mechanism works by pulling the tensioner away from the assembly, thereby increasing the distance between the pulleys that hold the timing belt, and consequently tensioning it. The tensioner was designed to be similar to commercially available belt tensioners, while being printable and also acting as a supporting leg for the whole assembly

3.3. SIMULATION

Simulation of the inverted pendulum was done in the Simulink environment. Once the system is represented by diagrams, and an input-output relationship is established, the temporal response of the system can be extracted. Subsequently, a PID controller can be implemented into the system. One of the main reasons for utilizing Simulink in this project is to derive the three coefficients for the PID controller, which will serve as a basis for the actual tuning of the physical system.

As shown in Figure 3.5, the four equations mentioned previously are represented by the large rectangular blocks. Smaller rectangular blocks represent integrators, and the triangular blocks symbolize gains, which mean that the input will be multiplied by the value of the gain to determine the output. There is one input to the system which is the force and this input is received by the top most rectangular block which represents the equation 3.1, this is because the force is applied to the cart and that equation describes the forces acting on the cart. There is one input to the system, which is the force, and this input is received by the topmost rectangular block representing Equation 3.1. This arrangement is used because the force is applied to the cart, and that equation describes the forces acting on the cart. There are two outputs from the system, the first of which is the position of the cart x which is obtained by integrating the topmost rectangular block two times with integrator blocks. The second output, angle of the pendulum θ, is obtained by doing the same thing to the second block which represents the net torque.

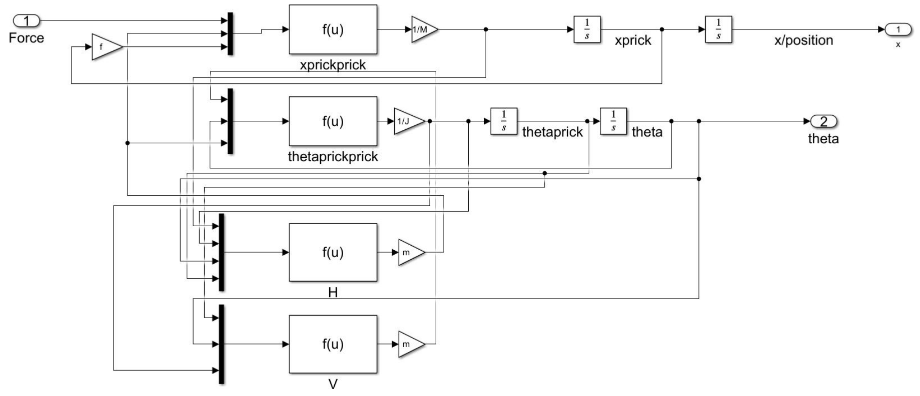


Figure 3.5: Block Diagram description of the Dynamical system in Simulink environment.

An important characteristic of the block diagram representation in Simulink is that linearization is performed automatically by the software. The dynamic system shown in

Figure 3.5 is not linear. When this system is converted into a subsystem, and subsequently a transfer function, it will be linearized by Simulink.

Figure 3.6 shows the subsystem that can now be used to create a feedback loop and implement various controllers. The transfer function of the inverted pendulum can now be derived from this subsystem. To achieve this, the force input is designated as an open-loop input, and the angle is selected for output measurement. The position of the cart will not be utilized in this instance, as the controller is designed to regulate the angle of the pendulum. The transfer function is as follows:

$$G(S) = \frac{4.545S}{S^3 + 0.01818S^2 + 31.25S + 0.4464}$$

This transfer function corresponds to the dynamical system in Figure 3.5. A closed-loop system can now be designed, and a PID controller can be implemented using the PID block within the Simulink environment.

The PID block in Figure 3.7 can now be tuned, and the controller parameters can be obtained to achieve the desired response from the system.

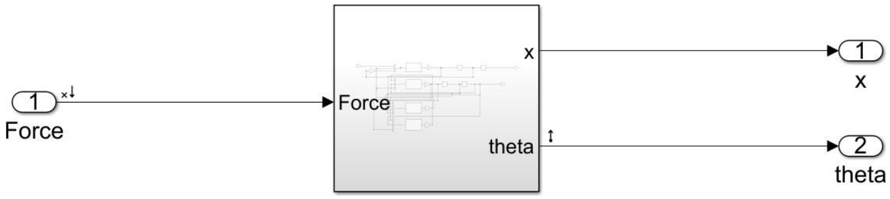


Figure 3.6: Block diagram representation converted to a subsystem.

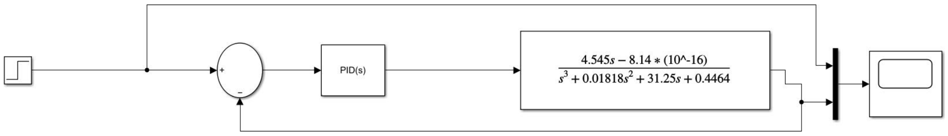
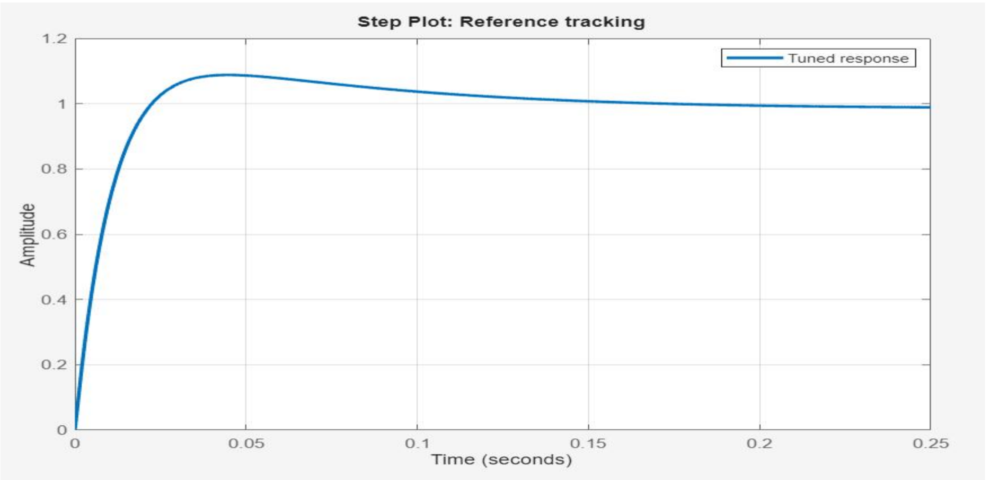


Figure 3.7: Feedback loop with the PID block and transfer function.

The step response of the system is shown in Figure 3.8, after graphically tuning the PID in Simulink. As is evident from the figure above, the rise time and settling times are very fast, which is reasonable for an inverted pendulum that must respond to force and return the pendulum to its stable and upright position.

Figure 3.8: Step response of the tuned system.



The rise time and settling time of the system, shown in Table 3.1, represent an ideal model that does not capture the whole complexity of the system in reality. As such, they will not be identical to the actual times in the physical system.

Parameter	**Value**	**Unit**
Rise time	0.0154	sec
Settling time	0.124	sec
Overshoot	8.79	%
Peak	1.09	%
Gain margin	-43.2 @ 6.48 rad/s	dB
Phase margin	81.9 @ 109 rad/s	deg
Closed-loop stability	Stabe	

Table 3.1: System Response Parameters.

The control parameters obtained from PID tuning are presented in Table 3.2, which include three coefficients for the PID controller and one filter coefficient N. The PID equation that returned these parameters in the Laplace domain is as follows:

The filter coefficient N is a derivative filter which is actually a low-pass filter. A low-pass filter (LPF) is a circuit that only passes signals below its cutoff frequency while attenuating all signals above it. The cutoff frequency of a low-pass filter is the frequency at which the output signal's amplitude drops to 70.7% of the input signal's amplitude, which corresponds to a -3dB point [8]. It determines the bandwidth of the low-pass filter applied to the derivative term. The purpose of this filter is to mitigate the effect of high-frequency noise in the derivative term. Without this filter, the derivative term could amplify high-frequency noise and cause the controller to react to what is essentially irrelevant information, potentially leading to instability or erratic control.

Parameter	**Value**
P	342.9
I	1005.7
D	23.7
N	12422.0

Table 3.2: Control Parameters.

In practice, the derivative filter improves the robustness of the PID controller by attenuating the influence of high-frequency noise on the control signal. This makes the controller more practical to implement. Equation 3.4 shows that if the filter coefficient grows sufficiently large the derivative part of the equation approaches the ideal derivative action.

3.3 CODE IMPLEMENTATION

The code that the Arduino micro-controller used can be found in Appendix A. It utilizes several libraries to work effectively. There is a library for each electronic component (one for the stepper motor, and one for the encoder), and a library for PID controllers.

3.4 TESTING OF DISTURBANCE REJECTION

In order to test the system's ability to recover from impulses, a plan was devised to release a pendulum at different angles to hit the top of the inverted pendulum. This approach introduces a quantifiable way to measure the impulses and test the system's ability to reject disturbances. Some assumptions were made during the test to simplify the calculations, such as treating the bob as a point mass and the air resistance being negligible. The impulse delivered by the pendulum is assumed to be instantaneous and the collision is assumed to be elastic, simplifying the analysis of the collision and its immediate effects. The angle was increased with increments of 10 degrees to increase the impulse. Each increment was tested 5 times, to get an average. Figure 3.9 shows a simple sketch of how the disturbance rejection was tested.

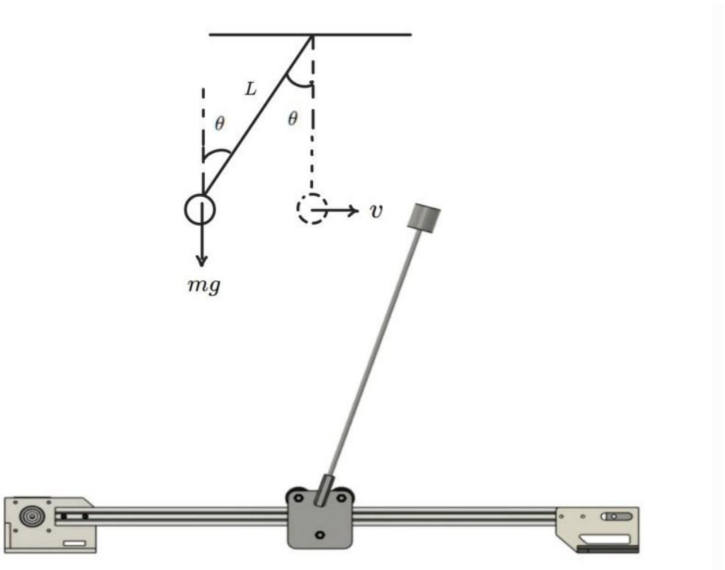


Figure 3.9: Sketch of the system with the impulse disturbance introduced by a pendulum.

Table 3.3 shows the important parameters of the pendulum and the Bob which was used in the calculation of the impulse.

Parameter	Value	Unit
Length (*L*)	0.3	M
Mass (*m*)	0.2	Kg
Angle (*θ*)	variable	Degree

Table 3.3: Description of parameters.

Below is an outline of the **Design and Methodology** for this project, typically divided into several key sections:

✓ **Research Objectives**

- Develop a mathematical model for the inverted pendulum system.
- Analyze the dynamics of the system.
- Design control algorithms to stabilize the pendulum in an upright position while allowing the cart to move as desired.

- Validate the model and control design through simulation and/or experimental implementation.

✓ **System Description**

The inverted pendulum system consists of:

1. **A Cart**: Moves along a linear track, controlled by a force applied via a motor or actuator.
2. **A Pendulum**: Mounted on the cart, free to rotate in a vertical plane.
3. **Sensors**: Measure variables like cart position, pendulum angle, and their derivatives.
4. **Actuator**: Provides force to the cart to stabilize the pendulum.

✓ **Mathematical Modeling**

- **Free Body Diagram (FBD)**: Develop the FBD for the pendulum and the cart to derive equations of motion.
- **Dynamic Equations**: Use Newton's laws or Lagrangian mechanics to derive the equations:

$$Mx + bx + ml\theta \mathrm{Cos}\theta - ml\theta^2 \mathrm{Sin}\theta = F$$

Where:

  - x: Position of the cart
  - θ: Angle of the pendulum
  - M: Mass of the cart
  - m: Mass of the pendulum
  - l: Length of the pendulum
  - b: Friction coefficient
  - g: Gravitational acceleration
  - F: Force applied to the cart
- **State-Space Representation**: Convert the nonlinear system into a state-space form for linearization around the unstable equilibrium point (θ=0\theta = 0θ=0).

✓ **Controller Design**

- **Control Goals**:
  - Stabilize the pendulum in the upright position.
  - Control the cart's position and velocity.
- **Control Strategies**:

1. **Proportional-Integral-Derivative (PID) Control**: Use for simple stabilization tasks.
2. **State Feedback Control (e.g., LQR)**: Design optimal control using Linear Quadratic Regulator (LQR) to minimize a cost function.
3. **Sliding Mode Control**: Handle uncertainties and nonlinearities.
4. **Fuzzy Logic Control**: For robust control without an exact mathematical model.

✓ **Simulation and Analysis**

- **Simulation Tools**: Use MATLAB/Simulink or Python for modeling and simulation.
- **Performance Metrics**:
  - Settling time
  - Overshoot
  - Steady-state error
  - Robustness against disturbances

✓ **Experimental Setup**

- **Hardware Components**:
  - Motor for the cart (DC or stepper motor).
  - Pendulum with encoder or gyroscope for angle measurement.
  - Track for the cart movement.
  - Microcontroller or FPGA for real-time control (e.g., Arduino, Raspberry Pi, or STM32).
  -
- **Data Acquisition**:
  - Sensors: Measure position, velocity, and angle.
  - Actuator: Apply control inputs.
- **Implementation**:

- Real-time control using a designed controller.
- Validate the experimental results with the simulation.

✓ **Validation**

- Compare simulation results with experimental data.
- Evaluate the robustness of the control design against noise, friction, and external disturbances.

✓ **Expected Outcomes**

- A robust mathematical model of the inverted pendulum system.
- Effective control algorithms that stabilize the pendulum.
- Demonstrated stability and control in both simulations and experiments.
- Insights into real-world challenges like actuator limitations, sensor noise, and unmodeled dynamics.

3.5 OPERATION PRINCIPLE OF THE INVERTED PENDULUM ON A MOVING CART

The operation principle of an **inverted pendulum on a moving cart** is based on the control of an inherently unstable system. The system comprises a pendulum with its pivot point attached to a cart that can move horizontally. The goal is to keep the pendulum upright (inverted position) by dynamically adjusting the position or motion of the cart.

Components of the System

1. **Pendulum**: A rigid rod with mass concentrated at one end, hinged at its base.
2. **Cart**: A platform or vehicle that moves horizontally, serving as the pendulum's base.
3. **Sensors**: Devices to measure the angular position of the pendulum and the position or velocity of the cart.
4. **Actuators**: Motors or mechanisms that move the cart.
5. **Controller**: An algorithm or circuit that computes the required cart motion to stabilize the pendulum.

Principle of Operation

The stability of the inverted pendulum is maintained by counteracting the forces and torques that would cause it to fall. This involves the following steps:

1. **State Detection**:
   - Sensors measure the angle of the pendulum relative to the vertical axis (θ) and the horizontal position or velocity of the cart (x).
   - These measurements are fed into the controller.
2. **Control Algorithm**:
   - Using feedback control principles, the controller calculates the necessary force or motion for the cart to stabilize the pendulum.
   - Common control techniques include **Proportional-Integral-Derivative (PID) control**, **Linear Quadratic Regulator (LQR)**, and more advanced methods like **Model Predictive Control (MPC)**.
3. **Cart Motion**:
   - The controller commands the actuator to move the cart in the required direction.
   - By adjusting the cart's acceleration, the system generates a stabilizing torque about the pendulum's pivot point to counter the gravitational torque.
4. **Dynamic Equilibrium**:
   - The cart continuously adjusts its motion to maintain the pendulum in an upright position.
   - This requires precise and real-time control to respond to disturbances or deviations.

Physics of the System

The dynamics of the system can be modeled using Newton's laws or the Lagrangian approach, resulting in coupled nonlinear equations. The key forces and torques include:

- **Gravitational Force**: Pulls the pendulum downward.
- **Inertial Forces**: Arise due to the motion of the cart and pendulum.
- **Control Forces**: Applied by the actuator to move the cart.

The equations of motion are typically linearized around the upright position (θ=0\theta = 0θ=0) for control design.

CHAPTER FOUR

4.0 TEST EVALUATION AND RESULT

4.1 TEST AND RESULT

The **test and result phase** of the inverted pendulum on a moving cart evaluates the system's performance and verifies its ability to balance the pendulum. This phase includes testing individual components, integrating the system, and analyzing its performance under various conditions.

1. Testing Procedure

✓ **Component Testing**

- **Sensors**:
  - Verification of the IMU (or angle sensor) outputs accurate angular data.
  - Checking of the cart position sensor (e.g., encoder) for accurate displacement and velocity readings.
  - Calibrate sensors to minimize noise and drift.
- **Actuators**:
  - Testing the motor and motor driver for smooth operation.
  - Verification of the motor response to varying control signals without delay.
- **Controller**:
  - Testing the microcontroller's ability to read sensor data and send signals to the motor driver.

✓ **System Integration Testing**

- Assemble the pendulum and cart.
- Upload the control algorithm to the microcontroller.
- Run initial tests with the pendulum hanging down to ensure the cart moves as expected in response to input signals.

✓ **Stability Testing**

- **Initial Balancing**:
  - Manually hold the pendulum upright and release it gently.
  - Observe the system's ability to stabilize the pendulum around the vertical position.
- **Disturbance Rejection**:

  - Apply small disturbances to the pendulum or cart and observe how the system restores balance.
- **Dynamic Performance**:
  - Test the system under varying initial angles of the pendulum and starting cart positions.

2. Results Evaluation

✓ Performance Metrics

1. **Balance Duration**:
   - Measurement of how long the system keeps the pendulum upright.
2. **Stability Margin**:
   - Evaluation of how well the system handles external disturbances.
3. **Response Time**:
   - Record how quickly the system reacts to deviations.
4. **Control Accuracy**:
   - Check for steady-state errors or oscillations around the vertical position.
5. **Energy Efficiency**:
   - Monitor power consumption during operation.

✓ Expected Results

- The pendulum remains balanced upright for an extended period (e.g., several minutes or more) under normal conditions.
- The cart adjusts its position smoothly without excessive oscillation.
- The system quickly corrects for disturbances (e.g., within a fraction of a second).
- The controller maintains stability even with slight sensor noise or non-linear effects.

✓ Common Issues and Fixes

- **Oscillations**:
  - Cause: Improper control gain tuning.
  - Fix: Adjust PID gains or re-tune the control algorithm.

3. Results

Scenario 1: Successful Stabilization

- **Results**:
  - Balance duration: 10+ minutes.
  - Disturbance recovery time: <0.5 seconds for small disturbances.
  - Oscillation amplitude: ±1° around the vertical position.

CHAPTER FIVE

5.0 CONCLUSION AND RECOMMENDATION

5.1. CONCLUSION

The **inverted pendulum on a moving cart** is a classic and extensively studied example of an inherently unstable system. Its construction, modeling, and control provide valuable insights into the principles of dynamics, control theory, and real-world applications. Here are the key conclusions drawn from its study:

1. Stability and Control

- **Nature of the System**: The inverted pendulum is an unstable, non-linear system. It requires continuous control to maintain its balance.
- **Control Mechanisms**: Stability can be achieved using various control strategies, such as:
  - **Pro**
  -
  - **portional-Integral-Derivative (PID)** control for simple implementations.
  - **State-Space Feedback** or **Linear Quadratic Regulator (LQR)** for optimal performance.
  - Advanced techniques like **Model Predictive Control (MPC)** for more dynamic scenarios.

2. Model Complexity

- The system's behavior can be modeled using **first-principles dynamics** derived from Newtonian or Lagrangian mechanics.
- The resulting equations highlight the coupling between the cart's motion and the pendulum's angular displacement, requiring precise parameterization and real-time feedback.

3. Real-World Applications

- **Robotics**: The principles are applied in balance control for bipedal robots (e.g., humanoid robots) and self-balancing vehicles (e.g., Segway).

- **Control System Design**: It serves as a benchmark problem for testing and evaluating control algorithms.
- **Educational Tool**: The system is widely used in academic settings to teach control systems, dynamics, and feedback principles.

4. Challenges

- **Nonlinearity**: The equations of motion are nonlinear, requiring linearization for simpler control design, which can introduce approximation errors.
- **Disturbance Rejection**: Real-world disturbances (e.g., friction, sensor noise) make maintaining stability challenging.
- **Parameter Sensitivity**: Small changes in physical parameters (e.g., mass, friction) can significantly affect system performance.

5. Key Insights

- **Feedback is Critical**: Real-time sensor data and feedback are essential for stabilization.
- **Importance of Tuning**: The success of the system depends heavily on proper tuning of control parameters.
- **Practical Trade-offs**: There is trade-offs between system complexity, performance, and robustness, especially when implementing advanced control techniques.

6. Future Directions

- **Advanced Controllers**: Exploration of artificial intelligence and machine learning techniques to adaptively control the system.
- **Enhanced Designs**: Integration of more robust sensors and actuators for greater disturbance tolerance.
- **Applications Expansion**: Adapt the principles for other unstable systems, such as quadcopters, space launchers, or floating platforms.

Overall, the inverted pendulum on a moving cart is an exemplary system for exploring the intersection of theoretical modeling and practical control. It demonstrates the power of feedback control in stabilizing unstable systems and offers a rich platform for learning and innovation in control systems and robotics.

5.2 RECOMMENDATION FOR FUTURE WORK

Building on the study of the inverted pendulum on a moving cart, there are several avenues for advancing its design, control strategies, and applications. Below are recommendations for future work:

1. Advanced Control Techniques

Adaptive Control

- Implement **adaptive controllers** to handle changes in system parameters, such as mass, friction, or external disturbances.
- Explore techniques like **gain scheduling** or **adaptive PID control** for real-time parameter adjustments.

Robust Control

- Design controllers that ensure stability and performance despite uncertainties and unmodeled dynamics.
- Investigate robust methods to address sensor noise, actuator limitations, and disturbances.

Intelligent Control

- Use **machine learning** techniques, such as reinforcement learning, to enable the system to learn balancing strategies autonomously.
- Incorporate **neural networks** to approximate non-linear control laws for improved performance.

Model Predictive Control (MPC)

- Apply MPC for optimal trajectory planning, taking into account system constraints like cart position limits and actuator saturation.

2. Sensor and Actuator Enhancements

High-Precision Sensors

- Upgrade sensors to improve accuracy and reduce noise, such as using high-resolution encoders or advanced IMUs.

- Implement sensor fusion techniques (e.g., combining IMU data with vision-based tracking).

Enhanced Actuators

- Utilize motors with better torque control and faster response times.
- Experiment with brushless DC motors or direct drive actuators for smoother operation.

3. System Design Improvements

Multi-Pendulum Systems

- Extend the system to a **double or triple inverted pendulum**, increasing the complexity and challenge of stabilization.
- Study the dynamics and control of these multi-body systems for academic or practical insights.

Scaling and Miniaturization

- Explore larger-scale systems for industrial applications or smaller-scale systems for portable demonstrations.
- Investigate micro-electro-mechanical systems (MEMS) to build miniaturized inverted pendulum systems.

Energy Efficiency

- Optimize the system for lower energy consumption by improving motor efficiency and minimizing friction.

4. Real-World Applications

Robotics

- Adapt the principles to self-balancing robots, such as two-wheeled robots or humanoid robots.
- Explore its use in **exoskeletons** for balance assistance or rehabilitation.

Autonomous Vehicles

- Investigate applications in autonomous vehicles for stability and trajectory planning, especially in rough or dynamic environments.

Space Systems

- Explore applications in space robotics, such as controlling inverted pendulum-like systems in microgravity environments.

5. Educational and Research Tools

- Develop user-friendly platforms for educational demonstrations, such as modular kits with customizable parameters.
- Create simulation tools and open-source software to help students and researchers experiment with various control strategies.

6. Advanced Modeling

Nonlinear Analysis

- Investigate the system's behavior beyond linearization, considering the full nonlinear dynamics.
- Analyze stability regions and bifurcations in the non-linear system.

System Identification

- Use data-driven approaches to refine the model and identify unmodeled dynamics or external influences.

7. Multidisciplinary Integration

Internet of Things (IoT)

- Integrate IoT technology for remote monitoring and control of the system.
- Enable cloud-based data analysis for performance optimization.

Human Interaction

- D0esign systems where humans can interact with or control the inverted pendulum, combining human input with automated stabilization.

Renewable Energy

- Investigate the use of renewable energy sources, such as solar panels, to power the system for sustainable operation.

8. Experimental Validation

- Test the system under extreme conditions (e.g., high disturbances, variable gravity environments) to validate its robustness.
- Perform long-term reliability studies to ensure durability and consistent performance.

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Online Resources

1. **MathWorks Documentation** (https://www.mathworks.com/help)
   - MATLAB and Simulink for modeling and simulating inverted pendulum systems.
2. **Control Tutorials for MATLAB and Simulink** (https://ctms.engin.umich.edu/)
   - Step-by-step guides for modeling and controlling the inverted pendulum on a cart.
3. **Open Source Robotics Foundation** (https://www.osrfoundation.org/)
   - Simulating robotic systems, including pendulum dynamics.
4. **YouTube Educational Channels**:
   - Control Engineering Lectures *and* MATLAB on the inverted pendulum system.

Case Studies and Projects

1. **Inverted Pendulum on Arduino**: Construction and controlling an inverted pendulum using Arduino-based controllers.
2. **ResearchGate and Academia.edu:** Inverted pendulum construction and stabilization