HYDRAULIC FRACTURING: A STUDY OF THE EFFECT OF HYDRAULIC FRACTURE PARAMETERS ON PRODUCTIVITY INDEX

BY

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CERTIFICATION

This is to certify that this project work titled ‘HYDRAULIC FRACTURING: A STUDY ON THE EFFECT OF FRACTURE PARAMETER ON PRODUCTIVITY INDEX carried out

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Engineering, University of Benin, in partial fulfillment of the requirement for the award of Bachelor of Engineering, (B.Eng.) in Petroleum Engineering

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DEDICATION

This project is dedicated to Almighty God whom his infinite mercy made it possible for me to complete my program.

ACKNOWLEDGEMENT

Foremost, I thank my parents; Rev. Dr Femi Samson and Mrs Josephine Enweliku JP for their unending support and love to see me through this project work. More so, I thank my siblings; David Samson and Peter Samson for their prayers and words of encouragement throughout this process.

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Lastly, to my numerous school families, the Christian Union UNIBEN/UBTH, friends, mentors, well-wishers and acquaintances thank you so much. This success story just got started.

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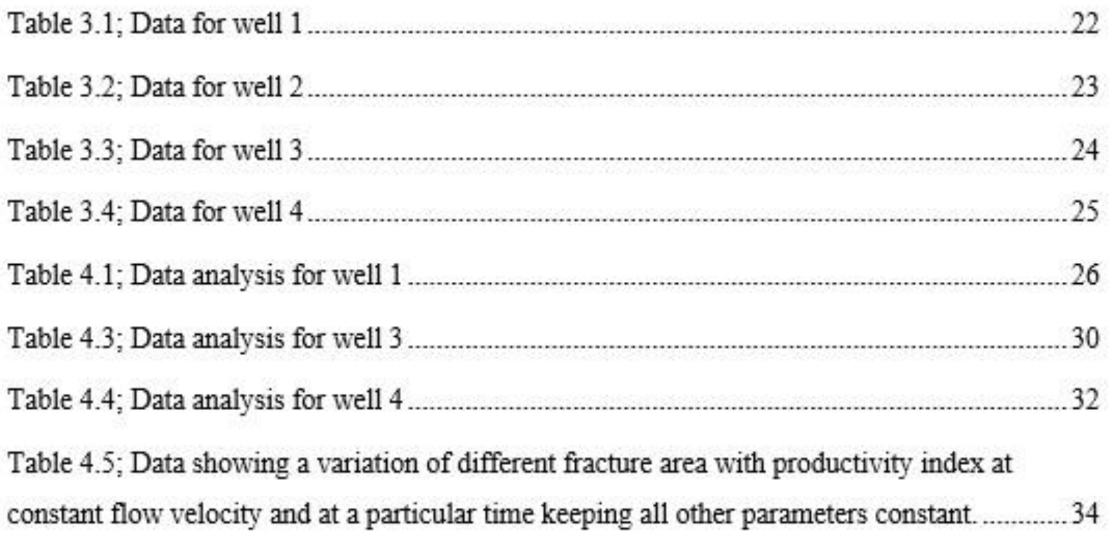

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ABSTRACT

This research presents the results of an investigation of fracture parameters and how it affects the production rate or recovery from a hydraulically fractured well. The problem that occurs without taking into consideration the important role each of these parameters play in the ultimate recover from a low permeability reservoir. In dealing with hydraulic fracturing design, it is necessary to be able to determine the area of fracture extent to be able to calculate or estimate the productivity index of the well.

The aim of the study is to show the relationship between the fracture area which is one of the important fracture parameters and the productivity index of a well. Field data was acquired, an analytical method was applied and mathematical correlations was established to determine the parameters of interest, and Microsoft excel was employed for computational purposes to ensure error elimination.

For the analysis of data, an investigation was also made to ascertain the various basic parameters and their respective values from four different wells that was fractured. Their different fracture area and productivity index were computed using the equations established and the field data acquired to attest the accuracy of the adopted correlation. Graphs were also plotted to show a clear relationship between the fracture area and productivity index.

The results from the research shows that an increase in the area of fracture extent will variably increase the production rate or recovery from the well. That is the fracture area varies directly with the well’s productivity index.

CHAPTER ONE
INTRODUCTION

Unconventional rock formations do not have enough permeability for oil and/or gas to flow into the wellbore at significant flow rates. These types of formations are also referred to as “tight” formations, (G.C Howard 1970) because the low-permeability rock is so tight that fluid cannot flow through the rock easily. The key to unlocking these types of formations is to get as much contact area as possible and to create artificial permeability. This is achieved by a technology known as hydraulic fracturing.

Hydraulic fracturing is the process of creating small fissures, or fractures, in underground geological formations to allow natural gas and oil to flow into a wellbore and up to the surface**.** **(**Card, David C., Jr 1962). This is critical to the development of unconventional oil and gas reservoirs that would otherwise be uneconomical. Unlike conventional reservoirs that will produce oil and gas solely from drilling a well, unconventional plays require stimulation to unlock the oil and gas that is in the rock itself. In fact, over 90% of the wells drilled today are hydraulically-fractured. Unconventional natural gas and oil reserves are constantly being discovered and developed which has vastly increased our domestic reserves.

During hydraulic fracturing, pressurized fluid is injected through a well into a subsurface rock layer in order to open the fractures.

There are many factors affecting hydraulic fracturing which includes reservoir characteristics, fracture parameters, and fracturing treatment parameters.

This work will be considering the fracturing parameters and their effects on the productivity index of a fractured well.

To obtain the best stimulation result, optimization of theses parameters is conducted such as fracture area optimization, fracture geometry optimization, fracture spacing optimization and proppant distribution optimization.

1.1 Statement of Problem

Unconventional oil and gas reservoirs are being explored significantly around the globe nowadays. The economical production of hydrocarbons from these unconventional oil and gas reservoirs requires very advanced and cost effective technologies. Hydraulic fracturing is such a technology which is being used in the oil and gas industry for many decades to create highly conductive channels in the formations having very low permeability values. Multistage hydraulic fracturing along with horizontal drilling has been proved to be a great achievement in oil and gas industry to enhance the production from unconventional reservoirs and massive shale gas production in the US is a successful example of it. An effective hydraulic fracturing design is a key to achieve the expected results in terms of production from unconventional reservoirs such as tight gas, shale gas, coal bed methane or other very low permeability reservoirs. There are many factors which must be considered while designing and executing hydraulic fracturing operation. These factors are not only limited to reservoir characteristics, fracture parameters and fracture treatment design parameters also play a significant role on the amount of hydrocarbon recovered(productivity index). These parameters can vary significantly at different locations around the globe. There is no universal method of hydraulic fracturing which can be applied anywhere in the world without proper formation evaluation of underground formations containing hydrocarbons. The high cost of the fracturing process and the serious need for a technique that can help in evaluating the benefits of the continously increasing number of fractures are the two motivating factors to develop the productivity index models for fractured formations. Therefore, an effective hydraulic fracturing design that puts into consideration the effects of these hydraulic fracture parameters on the productivity index of a fractured well is presented at the end of this study.

1.2 Aim and Objectives of the Study

1.2.1 AIM:

The primary aim is to review and show the effect of fracture parameter (fracture areal extent more precisely) on well productivity index.

1.2.2 OBJECTIVES:

1. Investigate the effect of fracture parameters on hydrocarbon recovery from a fractured well.
2. Show a relationship between fracture area and productivity index.
3. Investigate how increase in fracture area result in increased productivity index.

1.2.3 Scope of the Study

This study is focused on field data analysis obtained from four wells of different fields that was hydraulically fractured. Pressure drop, reservoir permeability, porosity, viscosity of injected fluid, rock compressibility, duration of stimulation, and fracture clearance data was obtained to carry out this study. All of These wells were hydraulically fractured to increase the low recovery as a result of low permeability. There are some concerns regarding how the hydraulic fracturing operation performed on these wells affect the production output obtained, which ultimately are the objectives of this study. There are precisely three objectives of this project work which have been discussed in the objectives section above.

CHAPTER TWO
LITERATURE REVIEW

2.1 Definition of Hydraulic Fracturing

Hydraulic fracturing is an oil and gas production techniques used in tight or low permeability geologic formations that involves injecting chemicals and liquids at high pressure to fracture the rock and release hydrocarbons.

Hydraulic fracture treatments are used to increase the productivity index of a producing well or the injectivity index of an injection well. The productivity index is a critical parameter in the oil and gas production and its management. Regardless of the type of formation and the type of wellbore, the productivity index defines the volumes of oil or gas that can be produced at a given pressure differential between the reservoir and the wellbore (United States Environmental Protection Agency, 2004).

$$PRODUCTIVITY\ INDEX = PI = J = \frac{flowrate}{drawdown} = \frac{q}{p} = \frac{q}{p_e - p_{wf}}$$

For a horizontal well with multiple hydraulic fractures, the productivity index is influenced by several factors such as the fracture area, number of fracture and the spacing between them. Reservoir permeability, and reservoir fluid properties also have great influence on the productivity index as well as the geometry of the drainage area.

There are many different objectives of Hydraulic Fracturing depending upon certain situations. For instance, Hydraulic Fracturing is used to:

i. Create deep-penetrating reservoir fractures to improve the productivity of a well.

ii. Overcome well bore damage.

iii. Assist in the injection or disposal of brine and industrial waste material

iv. Increase the flow rate of oil and/or gas from wells that have been damaged

v. Aiding secondary recovery operations

vi. Connect the natural fractures in a formation to the wellbore

vii. Decrease the pressure drop around the wellbore

viii. Increase the area of drainage or the amount of formation in contact with the wellbore

ix. Connect the full vertical extent of the formation to the wellbore

2.2 Hydraulic Fracturing Process

Hydraulic fracturing jobs are carried out at well sites, using heavy equipment including truckmounted pumps, blenders, fluid tanks, and proppant tanks. A simplified equipment layout in hydraulic fracturing treatments of oil and gas wells is illustrated in figure 2.1 below.

A hydraulic fracturing job is divided into two stages: the pad stage and the slurry stage as shown in figure 2.2. In the pad stage, only fracturing fluid is injected into the well to break down the formation and to create a pad. The pad is created because the fracturing fluid injection rate is higher than the flow rate at which the fluid can escape into the formation. After the pad grows to a desirable size, the slurry stage is started. During the slurry stage, the fracturing fluid is mixed with sand/proppant in a blender and the mixture is injected into the pad/fracture. After filling the fracture with sand/proppant, the fracturing job is over and the pump is shut down. Apparently, to reduce the injection rate requirement a low leak-off fracturing fluid is essential. Proppants are used to keep the fractures open and should have a compressive strength that is high enough to bear stresses from the formation. (Boyun Guo, 2007).

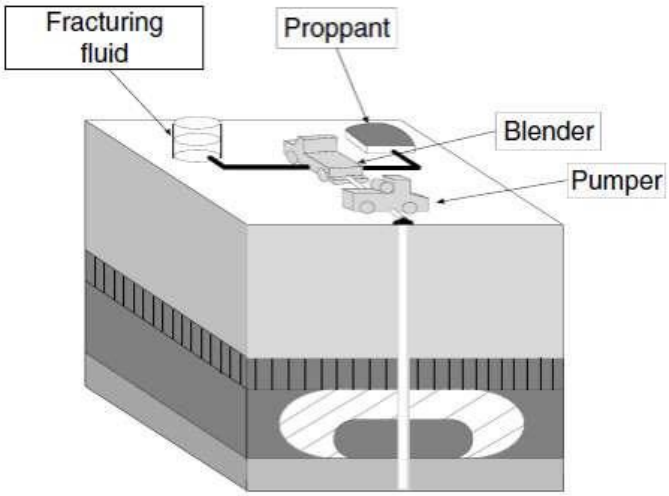


FIG 2.1; A Schematic to show the equipment layout in hydraulic fracturing treatments of oil and gas wells (Boyun Guo, 2007)

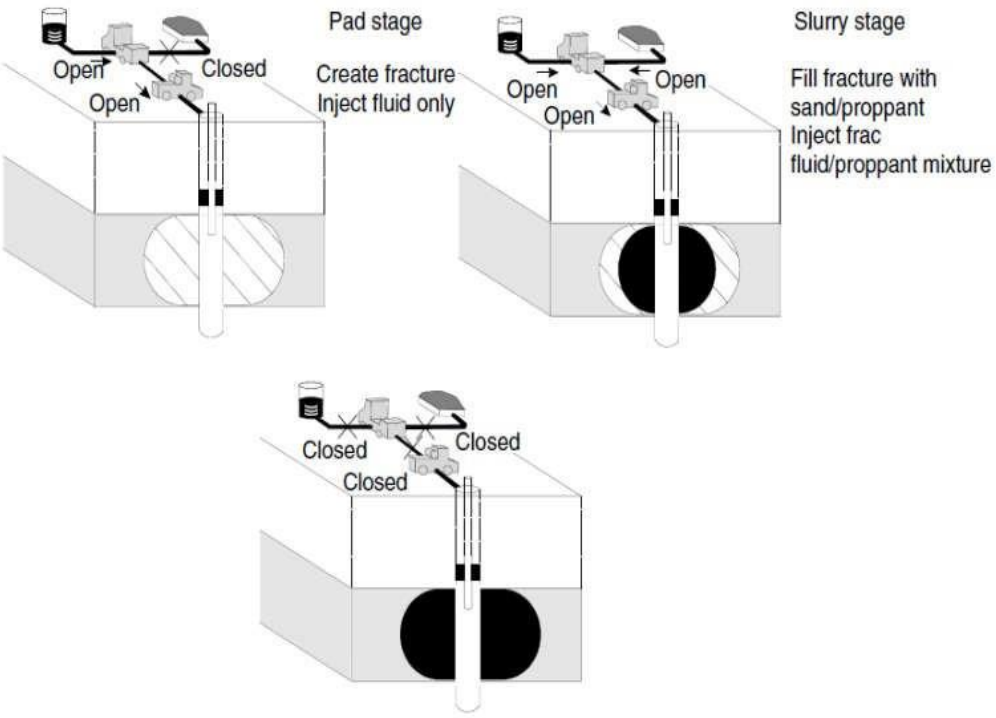


FIG 2.2; A Schematic to show the procedure of hydraulic fracturing treatments of oil and gas wells

2.3. Hydraulic Fracturing Technologies

The main technologies being used in the industry can be dived into four major categories as described below.

i. Hydraulic fracturing

Water based, foam based, oil based, acid based, alcohol based, emulsion based and cryogenic liquid based technologies are all included in the category of hydraulic fracturing.

ii. Pneumatic Fracturing

Air or nitrogen based technologies are termed as pneumatic fracturing technologies.

iii. Fracturing with dynamic loading This category includes explosives based and electric fracturing technologies. iv. Other fracturing techniques

This category includes thermal fracturing, mechanical cutting etc. (Gandossi, 2013).

2.4 Fracture parameters

There are many factors which affects the rate of production from a hydraulically stimulated well, fracture parameters such as fracture geometry, fracture spacing and fracture area are factors that have a great influence on the productivity index of a fractured well. The impact of these parameters cannot be overemphasized. For the purpose of simplicity the effects of the fracture area on production rate was thoroughly dealt with in this study.

2.4.1 Fracture geometry

Shape and size are the two most difficult fracture geometry parameters to establish because there is no reliable direct method for measuring them, even with very strong assumptions. Evidence from laboratory experiments and field observations has demonstrated that for a tensile fracture in an isotropic and homogeneous rock the fracture shape, or at least its initial shape, is indeed circular. However, this circularity can be quickly modified by successive tectonic movements and deformation processes (folding, faulting and jointing), and thus causing induced changes in fracture shapes.

The fracture shape is in reality too complex to be confidently idealized into regular plane shapes, such as circular, elliptic, square or rectangular. General polygons may be a more acceptable conceptual model, but such shapes are difficult for computational idealization, with their varying numbers of vertices and edges, for both convex and concave shapes. In practice, a common solution is to assume that the fractures are either circular as in the FracMan code (Golder Associates, 1993), elliptic, square or rectangular as in the NAPSAC code (Wilcock, 1996) for computational simplicity. However, if a very large number of fractures are involved, for example in a flow analysis, the significance of the fracture shape decreases with an increase in the fracture population size. It is still controversial about whether a single fracture or multiple fractures are created in a hydraulic fracturing job. Whereas both cases have been evidenced based on the information collected from tiltmeters and micro seismic data, it is commonly accepted that each individual fracture is sheet-like. However, the shape of the fracture varies as predicted by different models described below

1. Radial Fracture Model

A simple radial (penny-shaped) crack/fracture was first presented by Sneddon and Elliot (1946). This occurs when there are no barriers constraining height growth or when a horizontal fracture is created. Geertsma and de Klerk (1969) presented a radial fracture model showing that the fracture width at wellbore is given by

$$w_w = 2.56\left[\frac{\mu q_i(1-\nu)R}{E}\right]^{\frac{1}{4}}, \qquad \text{..............................2.2}$$

Where

W_W = fracture width at wellbore, in.

μ = fluid viscosity, cp.

q_i = pumping rate,

bpm

R = the radius of the fracture, ft.

E = Young’s modulus, psi.

Assuming the fracture width drops linearly in the radial direction, the average fracture width may be expressed as

$$\overline{w} = 0.85\left[\frac{\mu q_i(1-\nu)R}{E}\right]^{\frac{1}{4}}.$$

......................................2.3

2. The KGD Model

Assuming that a fixed-height vertical fracture is propagated in a well-confined pay zone (i.e., the stresses in the layers above and below the pay zone are large enough to prevent fracture growth out of the pay zone), Khristianovic and Zheltov (1955) presented a fracture model as shown in Fig.2.3. The model assumes that the width of the crack at any distance from the well is independent of vertical position, which is a reasonable approximation for a fracture with height much greater than its length. Their solution included the fracture mechanics aspects of the fracture tip. They assumed that the flow rate in the fracture was constant, and that the pressure in the fracture could be approximated by a constant pressure in the majority of the fracture body, except for a small region near the tip with no fluid penetration, and hence, no fluid pressure. This concept of fluid lag has remained an element of the mechanics of the fracture tip. Geertsma and de Klerk (1969) gave a much simpler solution to the same problem. The solution is now referred to as the KGD model. The average width of the KGD fracture is expressed as

$$\overline{w} = 0.29\left[\frac{q_i\mu(1-\nu)x_f^2}{Gh_f}\right]^{1/4}\left(\frac{\pi}{4}\right),$$

..................2.4

Where

$\overline{W}$ = average width, in.

$q_{i\ qi}$ = pumping rate,

bpm G = 2(1 þ n),

shear modulus, psia

h_f = fracture height,

ft

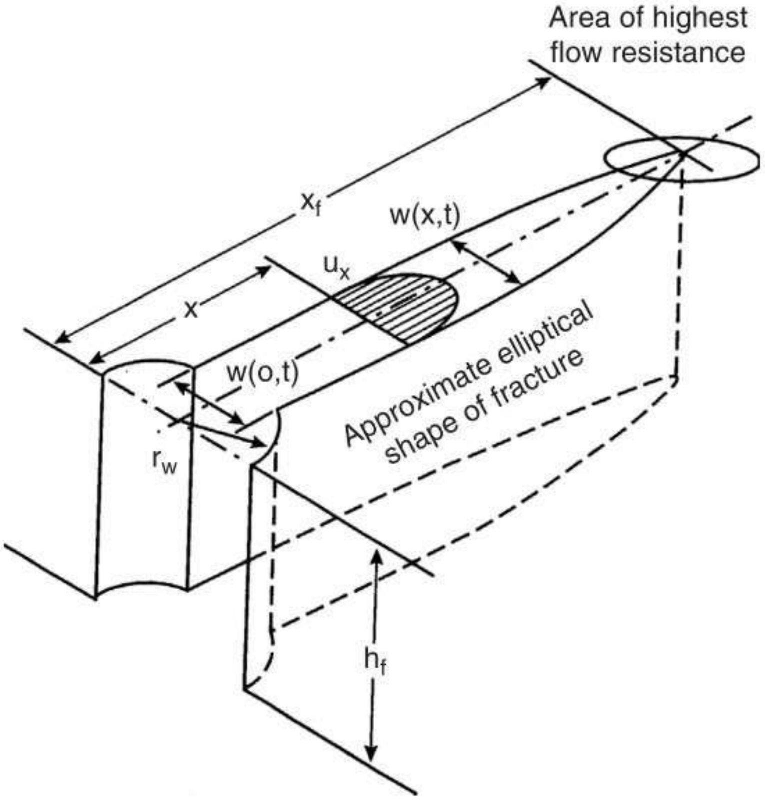


FIG 2.3; KGD fracture geometry

3. The PKN model

Perkins and Kern (1961) also derived a solution for a fixed- height vertical fracture as illustrated in Fig. 2.4 Nordgren (1972) added leak off and storage with in the fracture(due to increasing width) to the Perkins and Kern model, deriving what is now known as the PKN model. The average width of the PKN fracture is expressed as

$$\overline{w} = 0.3\left[\frac{q_i\mu(1-\nu)x_f}{G}\right]^{1/4}\left(\frac{\pi}{4}\gamma\right), \qquad \text{..................................2.5}$$

where = 0:75. It is important to emphasize that even for contained fractures, the PKN solution is only valid when the fracture length is at least three times the height. The three models discussed in this section all assume that the fracture is planar, that is, fracture propagates in a particular direction (perpendicular to the minimum stress), fluid flow is one-dimensional along the length (or radius) of the fracture, and leakoff behavior is governed by a simple expression derived from filtration theory. The rock in which the fracture propagates is assumed to be a continuous, homogeneous, isotropic linear elastic solid, and the fracture is considered to be of fixed height (PKN and KGD) or completely confined in a given layer (radial). The KGD and PKN models assume respectively that the fracture height is large or small relative to length, while the radial model assumes a circular shape. Since these models were developed, numerous extensions have been made, which have relaxed these assumptions.

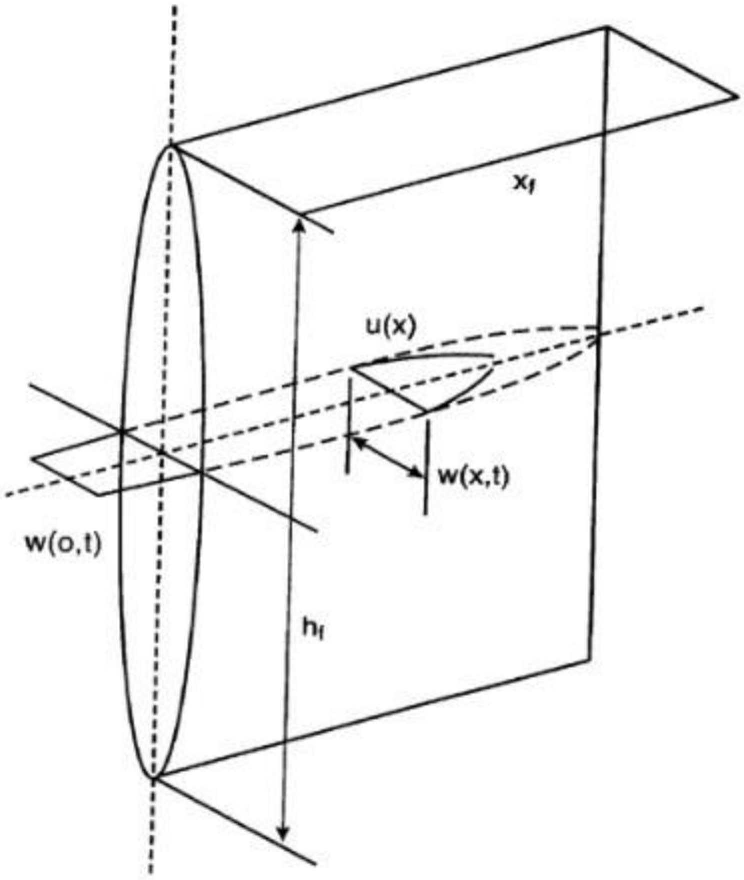


FIG 2.4; The PKN fracture geometry

4. Three-Dimensional and Pseudo-3D Models

The planar 2D models discussed above are deviated with significant simplifying assumptions. Although their accuracies are limited, they are useful for understanding the growth of hydraulic fractures. The power of modern computer allows routine treatment designs to be made with more complex models, which are solved numerically. The biggest limitation of the simple models is the requirement to specify the fracture height or to assume that a radial fracture will develop. It is not always obvious from data such as logs where, or whether, the fracture will be contained. In addition, the fracture height will usually vary from the well to the tip of the fracture, as the pressure varies. There are two major types of pseudo–three-dimensional (P3D) models: lumped and cell based. In the lumped (or elliptical) models, the fracture shape is assumed to consist of two halfellipses joined at the center. The horizontal length and wellbore vertical tip extensions are calculated at each time-step, and the assumed shape is made to match these positions. Fluid flow is assumed to occur along streamlines from the perforations to the edge of the ellipse, with the shape of the streamlines derived from simple analytical solutions. In cell-based models, the fracture shape is not prescribed. The fracture is treated as a series of connected cells, which are linked only via the fluid flow from cell to cell. The height at any cross-section is calculated from the pressure in that cell, and fluid flow in the vertical direction is generally approximated. Lumped models were first introduced by Cleary (1980), and numerous papers have since been presented on their use (e.g., Cleary et al., 1994). As stated in the 1980 paper, ‘‘the heart of the formulae can be extracted very simply by a non dimensionalization of the governing equations; the remainder just involves a good physics-mathematical choice of the undetermined coefficients.’’ The lumped models implicitly require the assumption of a self-similar fracture shape (i.e., one that is the same as time evolves, except for length scale). The shape is generally assumed to consist of two halfellipses of equal lateral extent, but with different vertical extent. In cell-based P3D models, the fracture length is discretized into cells along the length of the fracture. Because only one direction is discretized and fluid flow is assumed to be essentially horizontal along the length of the fracture, the model can be solved much more easily than planar 3D models. Although these models allow the calculation of fracture height growth, the assumptions make them primarily suitable for reasonably contained fractures, with length much greater than height.

2.4.2 FRACTURE SPACING OPTIMIZATION

Fracture spacing index is the number of fractures in one-meter length of drill core.

More than a decade ago, due to the lack of understanding of unconventional resource reservoirs and the limits of completion technologies, the cluster spacing was designed up to 700 ft. in Barnett and Bakken plays. Currently, the cluster spacing is as close as 15 ft. apart in Eagle Ford and DJ Basin, and the operators are testing even closer cluster spacing than 15 ft. The motivation of having tight fracture spacing is to have higher initial production rate. For example, for a horizontal well with 5,000 ft lateral length, we can create 10 fractures with 500 ft fracture spacing, or 100 fractures with 50 ft with the assumption of planar fracture propagation, resulting in 10 times more fracture surface area, the initial production rate can be 10 times different provided that all fractures are created equally with the same dimensions and conductivities. Thus, we want to create as many fractures as possible to achieve the highest possible initial production rate. However, higher initial rate is at the expense of possible higher completion cost and operation implementation complexity.

Since fracture spacing is so critical, it is necessary to spend excessive time and significant resources to optimize the fracture spacing. In theory, we can forecast the well performance by building a proper reservoir model with different fracture spacing, fracture properties, and reservoir properties, and then optimize the fracture spacing by tying that with cost and other economic data, provided we have enough data and properly understand the mechanisms of fracturing propagation and fluid flow within nano-darcy rocks. In 2009, Cipolla et.al performed a series of numerical simulations to study the impact of fracture networks on well performance. The study concluded that more complicated fracture network will enhance the well performance. One can interpret the more complicated fracture network as the higher fracture surface area and/or permeability within SRV.

In 2012, Cheng (2012) published a study on the impacts of number of perforation clusters and cluster spacing on production performance. The study focused more on the stress-shadowing effect on the inner fractures, which may have narrower fracture width or less fracture conductivities. Thus, the less conductive fractures would impair the well performance. The study concluded that the magnitude of mechanical interactions of fractures is closely related to the number of fracture simultaneously created and fracture spacing, and that increasing the number of perforation clusters in one stage does not necessarily increase the initial production rate.

Lu et al. (2016) performed a similar study. However, probably due to different assumptions, the study showed the wider middle fractures (higher fracture conductivity) but less fracture dimensions. For the formation case they studied, it was recommended that optimal cluster spacing is about 120 ft. Jin et al. (2013) performed a thorough fracture spacing study with numerical simulations and developed simple correlations that quantify the required fracture spacing necessary to optimize hydrocarbon recovery for different reservoir permeability and fluid properties. The study focused more on the impact of "*k*/*u*" (reservoir flow mobility ratio). The study used the fracture or cluster efficiency to simplify the impact of the fracture dimensions and conductivity, and no geomechanical impact was directly included in the study. The study concluded that the fracture spacing would range from 33 ft to 273 ft in Eagle Ford depending on matrix permeability and reservoir fluid properties. The study also recommends increasing the number of stages while reducing the number of clusters per stage, which was derived from the consideration of less efficiency with more perforation clusters within a single completion stage.

All of the studies have concluded that the well performance and economics depend on the number of effective fractures created and connected to the wellbore. The optimal fracture spacing depends on the matrix permeability and the effective drainage surface of hydraulic fractures.

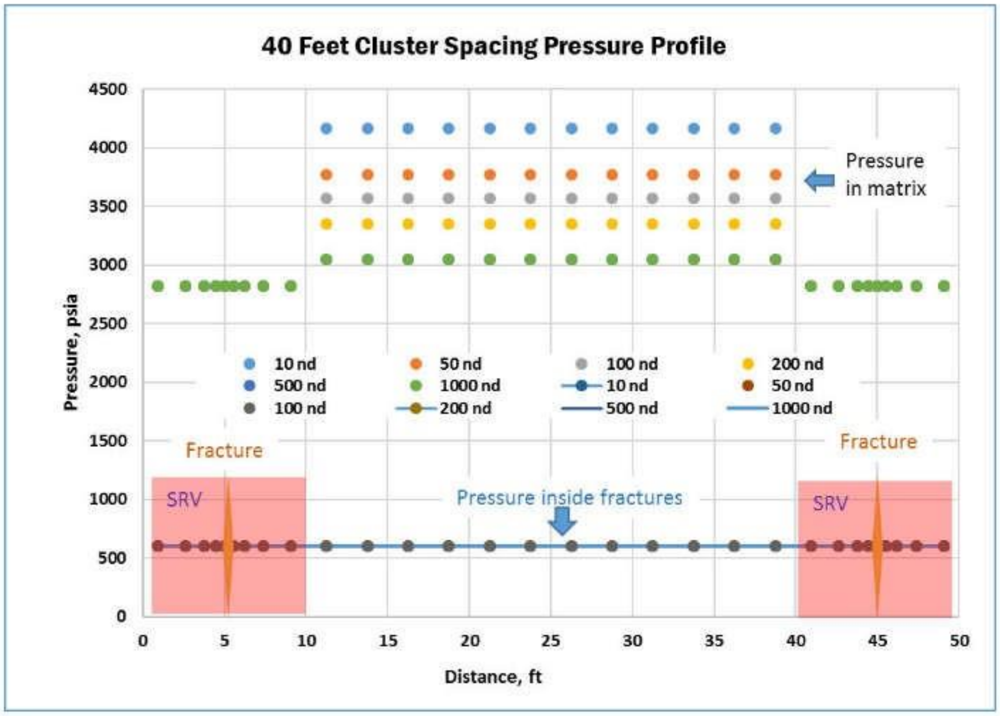


FIG 2.5; Pressure depletion profile for a dual porosity model at end of year 30

Reservoir characterization plays a significant role in cluster/fracture optimization process. For example, we need to know if a single-porosity model fits a given formation or a dual-porosity model fits the situation better.

Based upon the dual-porosity model, the pressure inside fractures is the same as the flowing bottom-hole pressure while the pressure in matrix is hardly depleted (Fig. 2.5) at the end of year 30. The modeling results also tell us that we may want to place the cluster as close as 10 ft, even with higher matrix permeability, which is significantly different from the conclusion based upon the single-porosity model. Therefore, the deep understanding of reservoir characterization dictates the selection of different modeling approaches, which, in turn, will dramatically impact on the selection of the fracture spacing.

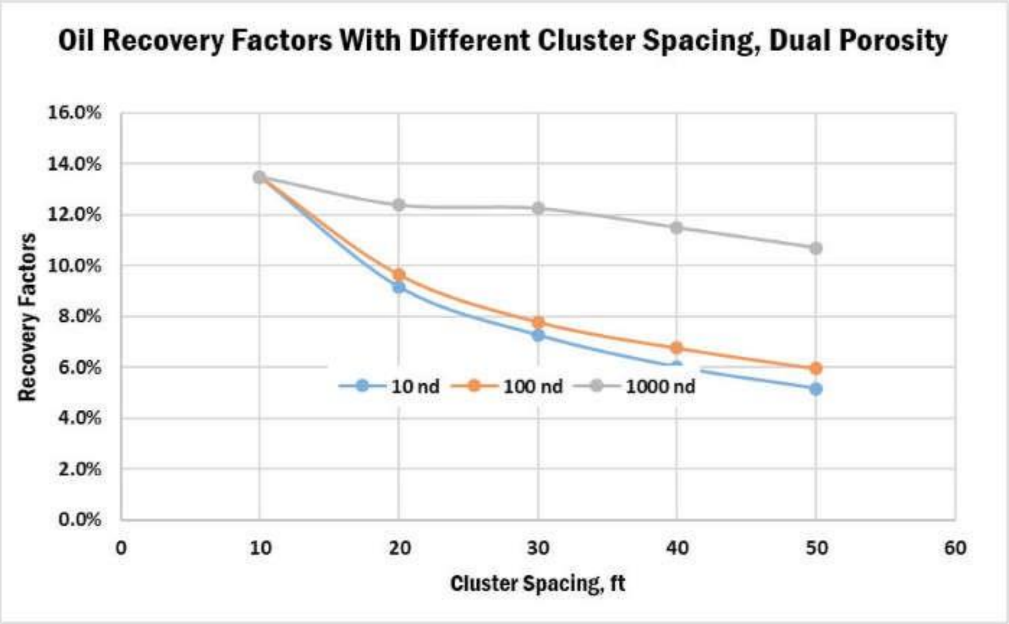


FIG 2.6; Recovery efficiency based upon the dual porosity model at the end of year 30

Effective fracture spacing is critical to well performance. The key to an optimal fracture/cluster spacing decision is reservoir characterization, such as understanding of reservoir matrix permeability, geomechanical properties, and existing natural fracture networks. Great efforts should be taken to understand the reservoir and develop better modeling technologies to reduce the piloting cost and shorten the piloting period to get into full-field development stage. The industry is working in different technology fronts to attack the challenges.

2.4.3 FRACTURE AREA

Analytical and electrical model studies have shown the influence of fracture penetration on both the flush and stabilized production that may be obtained from a given reservoir with a given fracture system. Investigations have shown that a fracturing treatment is influenced not only by the flow capacity of a hydraulically created fracture but also, to a large degree, by the areal extent of the fracture. The benefits derived from fracturing may be classified as flush production and post-flush production.(G.C Howard).

Flush and post-flush production

The effect of horizontal fracture penetration on the flush production immediately following a fracturing treatment has been illustrated by analytical calculations. Results of these calculations are presented in the next chapter. With graphs of production rate vs fracture area. The plots of productivity index vs fracture extent indicates that higher rates of production are maintained for a longer time when deeper fractures are made, and that the fracture penetration have great effect on the initial flush production rate.

Deeply penetrating fractures not only result in sustained increases in flush and post flush production, but also result in increased ultimate recovery.

From available exposures, the distribution of fracture area is obtained from the orientation data and fracture trace data. Rarely, if ever, can the actual areal extent of a fracture be observed directly. The trace length sample must first be corrected for bias, as discussed below. Then the corrected trace length distribution can be used to predict the distribution for fracture area. The relationship between the trace length distribution and the size distribution depends on fracture shape. Normally, it is necessary to make an assumption about the shape of the fracture (e.g., disk-shaped) and then estimate the size distribution from the sample data.

When mapping in adits, it is important to distinguish between fractures caused by blasting and those that reflect in situ conditions. It is also important to characterize the types of fracture intersections. Fractures may either cross or one may terminate against the other. Termination can be quantified simply by noting the percentage of fractures of a given set that terminate against other fractures.

There are four biases affecting the estimation of fracture area: length bias, orientation bias, truncation bias, and censoring. Small fractures will be underrepresented, as there is a lower probability of intersecting smaller fractures than larger fractures (length bias). Fractures parallel to the sampling plane will also be underrepresented (orientation bias). Fractures shorter than a predetermined length are usually not mapped (truncation bias). A censoring bias is introduced because the sample area is finite, and the fracture traces may not be completely visible (Baecher et al., 1977). Censoring bias is most important for longer fractures, which are generally the more conductive fractures. With respect to censoring bias, fracture traces can be divided into three groups: (1) traces with both endpoints visible, (2) traces with one endpoint visible, and (3) traces with no endpoints visible. The trace length distribution of the first group can be determined. These fractures have a maximum length determined by the dimensions of the sample area. For the second group a distribution of minimum lengths can be found. For the third group all that is known is that the fracture traces are larger than the sample dimensions. Some analytical solutions to the censoring problem have been obtained by assuming a distributional form for the trace lengths (Long and Billaux, 1987; Einstein et al., 1979, vol. IV). Similarly, if a distributional form is assumed for the fracture areas, one can use the corrected trace length distribution to estimate the parameters of the area distribution (Long and Billaux, 1987). Dershowitz et al. (1991a) have developed an automated procedure for generating fractures of a given size distribution and sampling the system the same way the field data were sampled. The prospective area distributions can be easily modified until a good (but necessarily unique) match to the field data is found. If only borehole measurements are available, problems associated with obtaining estimates of fracture size can be severe.

CHAPTER THREE
METHODOLOGY

This chapter develops a mathematical model that relates the extent of fracturing i.e. the fracture area, to well productivity index.

The importance of fracture penetration on well productivity as being discussed in previous chapters. The fracture penetration (area) is affected by fracturing fluid characteristics, reservoir characteristics and rock characteristics. To establish this effect, R.D. Carter derived an equation for calculating the areal extent of hydraulically created fractures. A modified form of the equation is used in this project and further modification as it relates to productivity index is derived.

The following assumptions are used in this derivations.

1. The fracture will be considered as having a uniform width.
2. The velocity of flow into the formation at a point on the fracture face depends on the length of time this point have being exposed to flow.
3. The flow of fracturing fluid from the fracture into the formation is linear and the direction of flow is perpendicular to the fracture face.
4. The velocity function, v (t) is the same for every point in the formation. However zero time for a given point is defined individually as the time at which the fracture and, consequently the fracturing fluid reaches that point.
5. The pressure in the fracture is constant and is equal to the proppant injection pressure.

The rate of fluid injection is equal to the sum of the rate at which fluid is lost to the formation and the rate of volume increase of the fracture itself. That is,

$$\mathrm{i} = i_l + Q_f \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.1$$

$$i_l\,(\mathrm{t}) = 2 \int_0^t ff^{(t)} \quad vdA_{ff} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.2$$

The quantity A is a function of time, and the value of v at a time t corresponding to a given element formed at the time λ, is v (t- λ). Also, since is a function of time,

$$_{f\ f} = \frac{d\ A_{f\ f}}{d\lambda}\,\mathrm{d}\,\lambda.$$

Equation 3.2 can therefore be written as

$$_{L}(t) = 2\int_0^t v(t \quad \lambda)\frac{d\ A_{f\ f}}{d\lambda}\mathrm{d}\,\lambda \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.3$$

The rate at which the volume of the fracture is increased is given by

$$Q_{f=W\frac{dA_{ff}}{dt}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.4$$

Where W is the fracture clearance

Substituting Eqs. 3.3 and 3.4 into equation 3.1,

$$_{L}(t) = 2\int_0^t v(t \quad \lambda)\frac{d\ A_{f\ f}}{d\lambda}\mathrm{d}\,\lambda + W\frac{d\ A_{f\ f}}{d\ t} \quad \ldots\ldots\ldots\ldots\ldots\ldots 3.5$$

Equation 3.5 can be solved for A (t) by means of the Laplace transformation, once the forms of v (t) and Q (t) are given. The ease with which a solution is obtained, of course, depends on the form of v (t) and q (t).

Let v (t) $=\frac{C}{\sqrt{t}}$, and i (t) = i =constant. Then, substituting this into equation 3.5 and using the Laplace transform method of solution, results in

$$A_{f\ f}(t) = \frac{i\ W}{4C^2\pi}\left[e\left(\frac{2C\sqrt{\pi\ t}}{W}\right) \quad e\ r\ f\left(\frac{2C\sqrt{\pi\ t}}{W}\right) + \frac{4C\sqrt{t}}{W} \quad 1\right] \ldots\ldots\ldots\ldots 3.6$$

$$\mathrm{A} = \frac{IW}{4\pi C^2}\left(e^{x^2}.erf(x) + \frac{2}{\sqrt{\pi}}x - 1\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.7$$

Where

$X = \frac{2C\sqrt{\pi t}}{W}$……………………………………………………………………3.7

The fracturing fluid coefficient, C defines the three types of linear flow mechanism that are encountered with fracturing fluids and for which equation 3.2 applies. The three types of linear flow mechanism are: (C_{I}) – Viscosity and relative permeability effect; (C_{II}) – Reservoir fluid viscosity – compressibility effect; and (C_{III}) – Wall building effects. The first two mechanisms involve coefficients that can be calculated from reservoir data and fracturing fluid viscosity with the aid of presently available formulas. The third case involves fluid loss coefficient for fluid loss additives, which must be determined experimentally. Although each mechanism is considered to be acting alone by this equation, all may act simultaneously in a fracturing treatment so that the mechanisms may complement each other and increase the fluid's effectiveness.

$C_1 = 0.0469\sqrt{\frac{K\,P}{\mu}, \frac{ft}{m_i n^{\frac{1}{2}}}}$ 3.9

$C_{11} = 0.0374\,p\sqrt{\frac{k\ \ c}{\mu}, \frac{ft}{m_i n^{\frac{1}{2}}}}$ ………………………………………………………..3.10

$\frac{1}{C_1} + \frac{1}{C_{11}} = \frac{1}{C}$…………………………………………………………….....3.11

C_{III} is ignored due to consideration on conditions where leak-off is controlled by fracturing fluid viscosity and reservoir fluid viscosity and compressibility.

On simplifying equation 3.11,

$C = \frac{C_1 C_{11}}{C_1 + C_{11}}$…………………………………………………………………… 3.12

On imputing the the values of C_I and C_{II} into equation 3.12, C can be determined as

$$C = \frac{0.0469\sqrt{\frac{k\,p}{\mu}}\ \ 0.0374\,p\sqrt{\frac{k\,c}{\mu}}}{0.0469\sqrt{\frac{k\,p}{\mu}} + 0.0374\,p\sqrt{\frac{k\,c}{\mu}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.13$$

Substituting Z = $\sqrt{\frac{k}{\ }}$ and simplifying equation 3.13,

$$= \frac{0.8Z\ \ p\sqrt{\ \ p\ }C}{\sqrt{\ \ p}+0.8\sqrt{c}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.14$$

Productivity index which is a parameter that defines the production potential of a well gives an insight to the amount of output to be expected from a particular well. To predict the variation of this parameter with fracture area which is the major focus of this study, a mathematical equation that relates fracture area and productivity index is needed.

Generally, $Productivity\ index = \frac{flowrate}{pressure\ draw\ down}$ ……………………………..3.15

Where $= p = p_r \quad P_{wf}$ ……………………………………………………………3.16

Flowrate (q) is given as

q = A v ……………………………………………………………………………… 3.17

Substituting equation 3.7 and 3.16 into equation 3.15,

$$Productivity\ index = \frac{\frac{iw\left(e^{x^2}erfc(x)+\frac{2}{\sqrt{x}}x-1\right)v}{4\pi C^2}}{P_r-P_{wf}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.18$$

Data for the study

Four hydraulically fractured wells is used as the case study. Table 1-4 shows the different data used for this analysis.

Table 3.1: Data for well 1

Basic parameters	Value
Permeability (k) (md)	20
Pressure draw down ( ) (psi)	1000
Porosity ( )	0.25
Viscosity ( )(cp)	0.57
Rock compressibility (c)	0.00001
Fracture Clearance (W)(inch)	0.2
injection rate (I)(BPM)	25
Velocity of flow (ft./min)	39.37

Table 3.2; Data for well 2

Basic parameters	Value
Permeability (k) (md)	10
Pressure draw down ( ) (psi)	600
Porosity ( )	0.2
Viscosity ( )(cp)	0.57
Rock compressibility (c)	0.0002
Fracture Clearance (W)	0.3
Injection rate (I)(BPM)	30
Velocity of flow(v)(ft./min)	19.69

Table 3.3: Data for well 3

Basic parameters	Value
Permeability (k) (md)	65
Pressure draw down ( ) (psi)	500
Porosity ( )	0.2
Viscosity ( )	0.5
Rock compressibility (c)	0.0015
Fracture Clearance (W)	0.2
Injection rate (I)(BPM)	20
Velocity of flow(v)(ft./min)	24.23

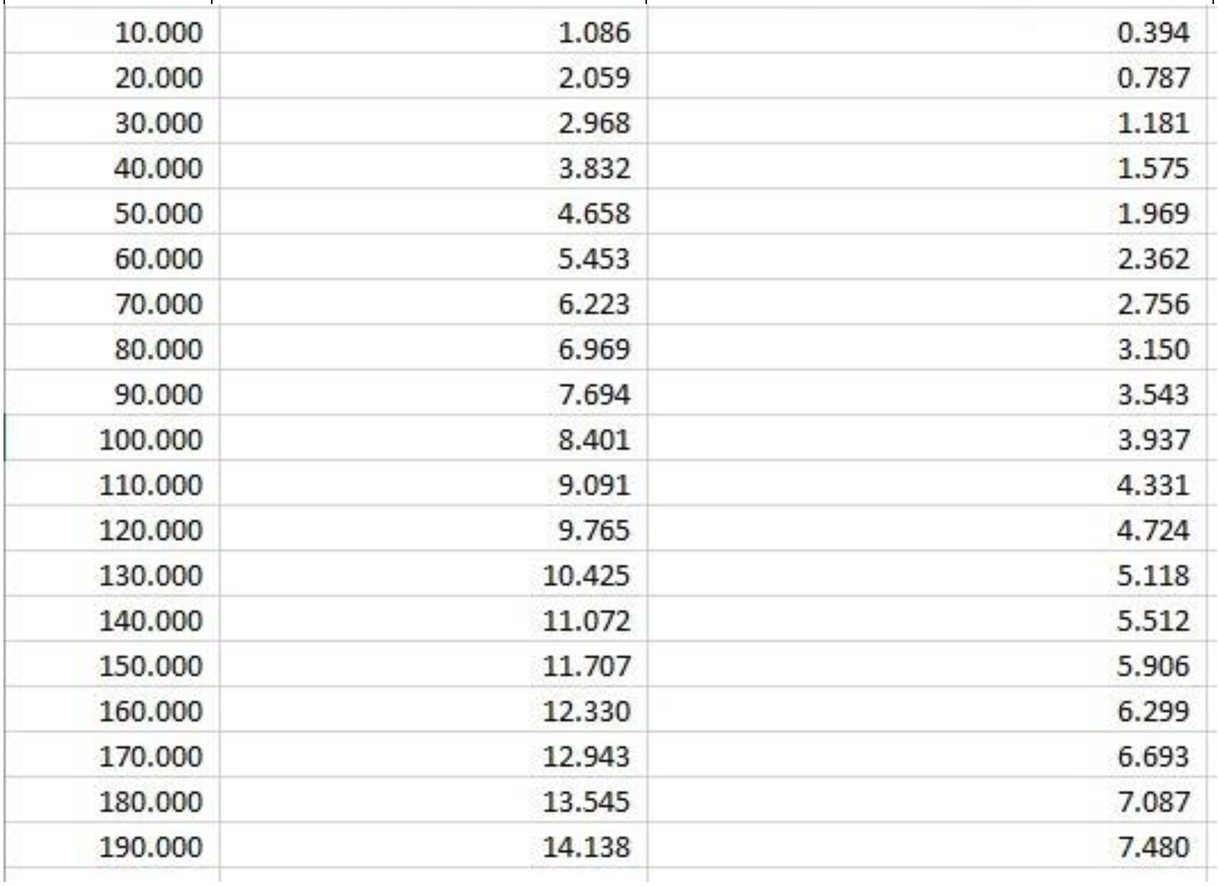
Table 3.4: Data for well 4

Basic parameters	Value
Permeability (k) (md)	100
Pressure draw down ( ) (psi)	750
Porosity ( )	0.3
Viscosity ( )	0.42
Rock compressibility (c)	0.0025
Fracture Clearance (W)	0.25
Injection rate (I)(BPM)	20
Velocity of flow(v)(ft./min)	30

CHAPTER FOUR
RESULTS AND DISCUSSION

The result analysis for four different wells are shown below. The data from the different wells were inserted into our various equations developed in the previous chapter and a relationship between fracture area and productivity index was established through plotted graphs.

Table 4.1: Results obtained for well 1



Time (t)(min)	Fracture Area (in^2)(10^3)	Productivity Index (STB/min/PSI)
10.000	1.086	0.394
20.000	2.059	0.787
30.000	2.968	1.181
40.000	3.832	1.575
50.000	4.658	1.969
60.000	5.453	2.362
70.000	6.223	2.756
80.000	6.969	3.150
90.000	7.694	3.543
100.000	8.401	3.937
110.000	9.091	4.331
120.000	9.765	4.724
130.000	10.425	5.118
140.000	11.072	5.512
150.000	11.707	5.906
160.000	12.330	6.299
170.000	12.943	6.693
180.000	13.545	7.087
190.000	14.138	7.480

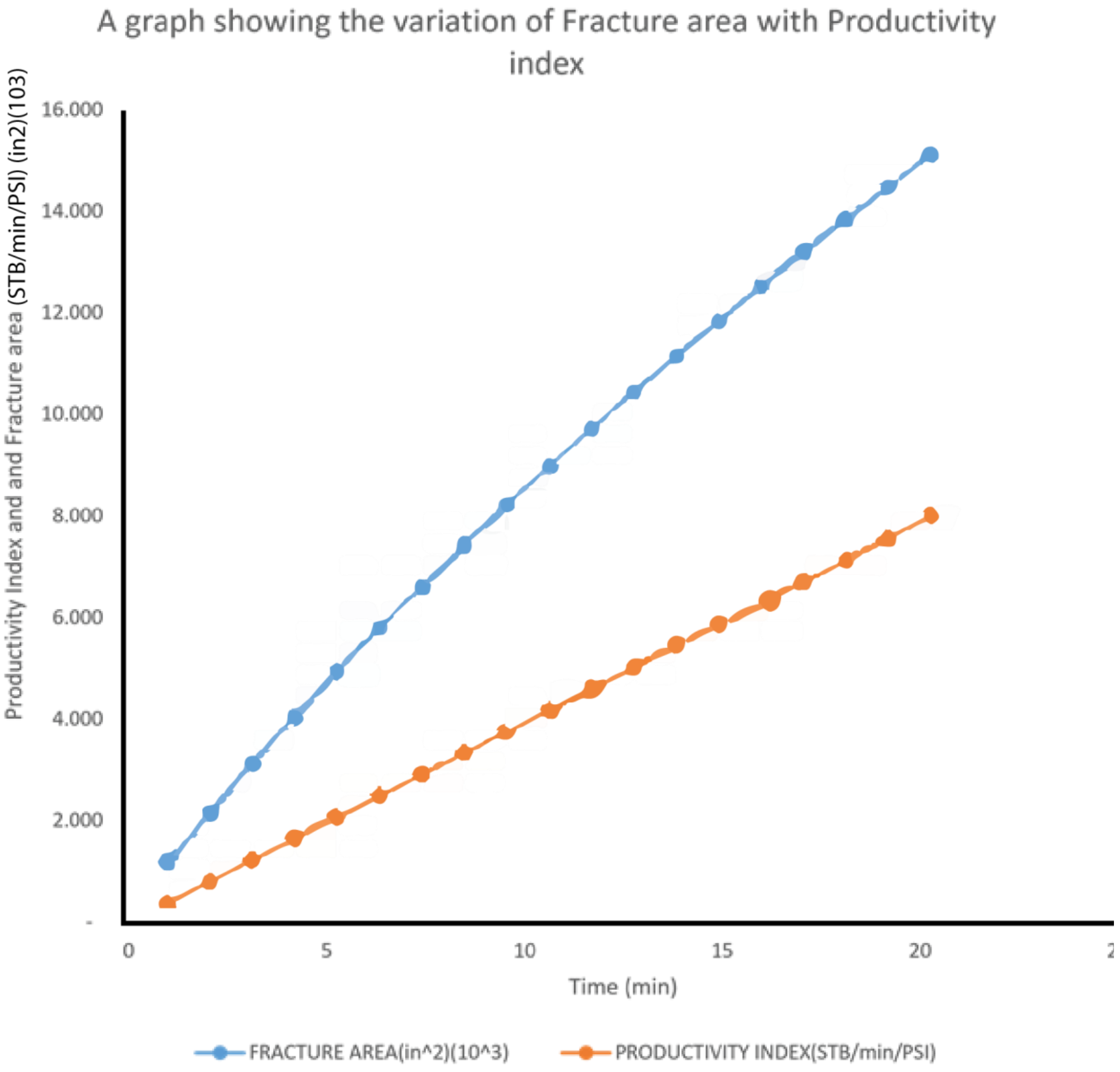


Fig 4.1 the relationship between Fracture Area and productivity index at different time intervals for well 1.

Table 4.2: Results obtained for well 2

Time (t)(min)	Fracture Area (in^2)(10^3)	Productivity Index (STB/min/PSI)
10.000	0.699	0.656
20.000	1.238	1.312
30.000	1.705	1.969
40.000	2.125	2.625
50.000	2.512	3.281
60.000	2.872	3.937
70.000	3.211	4.593
80.000	3.531	5.249
90.000	3.837	5.906
100.000	4.130	6.562
110.000	4.411	7.218
120.000	4.681	7.874
130.000	4.943	8.530
140.000	5.196	9.186
150.000	5.442	9.843
160.000	5.681	10.499
170.000	5.914	11.155
180.000	6.141	11.811
190.000	6.362	12.467

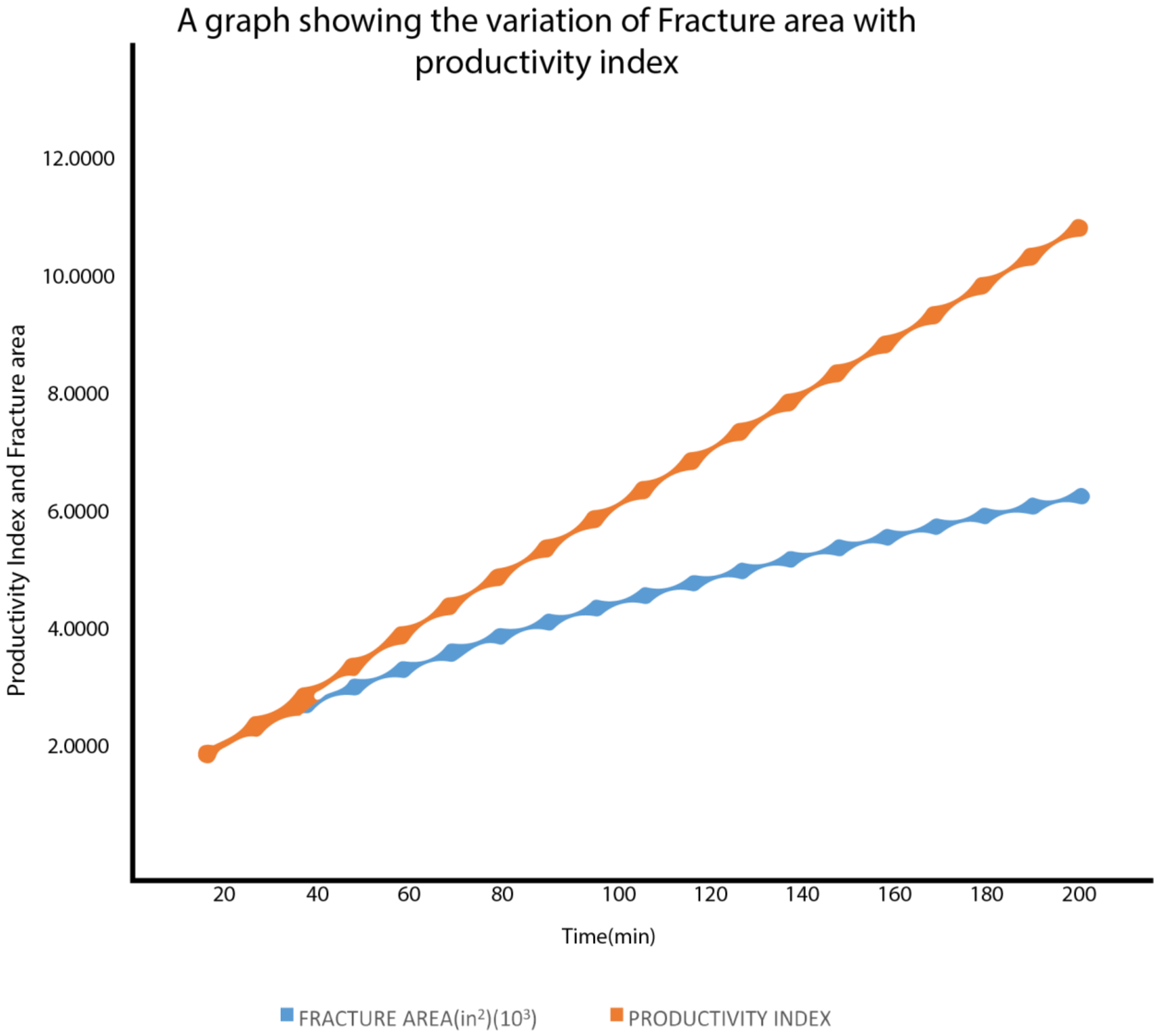


Fig 4.2 the relationship between Fracture Area and productivity index at different time intervals for well 2.

Table 4.3: Result Obtained for well 3

Time (t)(min)	Fracture Area (in^2)(10^3)	Productivity Index (STB/min/PSI)
10.000	0.234	0.787
20.000	0.352	1.575
30.000	0.444	2.362
40.000	0.521	3.150
50.000	0.590	3.937
60.000	0.651	4.724
70.000	0.708	5.512
80.000	0.761	6.299
90.000	0.811	7.087
100.000	0.858	7.874
110.000	0.903	8.661
120.000	0.946	9.449
130.000	0.987	10.236
140.000	1.027	11.024
150.000	1.065	11.811
160.000	1.102	12.598
170.000	1.138	13.386
180.000	1.173	14.173
190.000	1.207	14.961

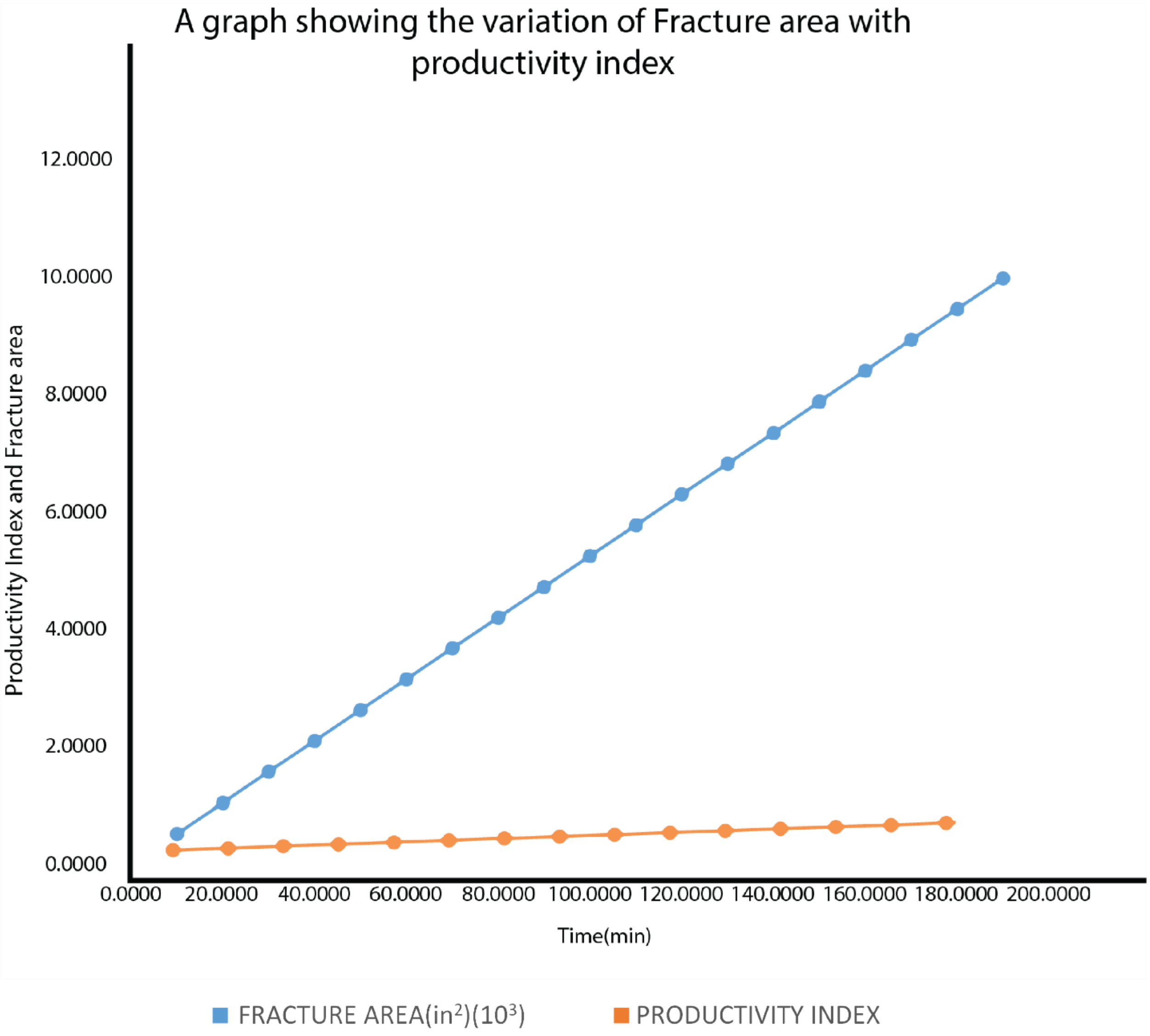
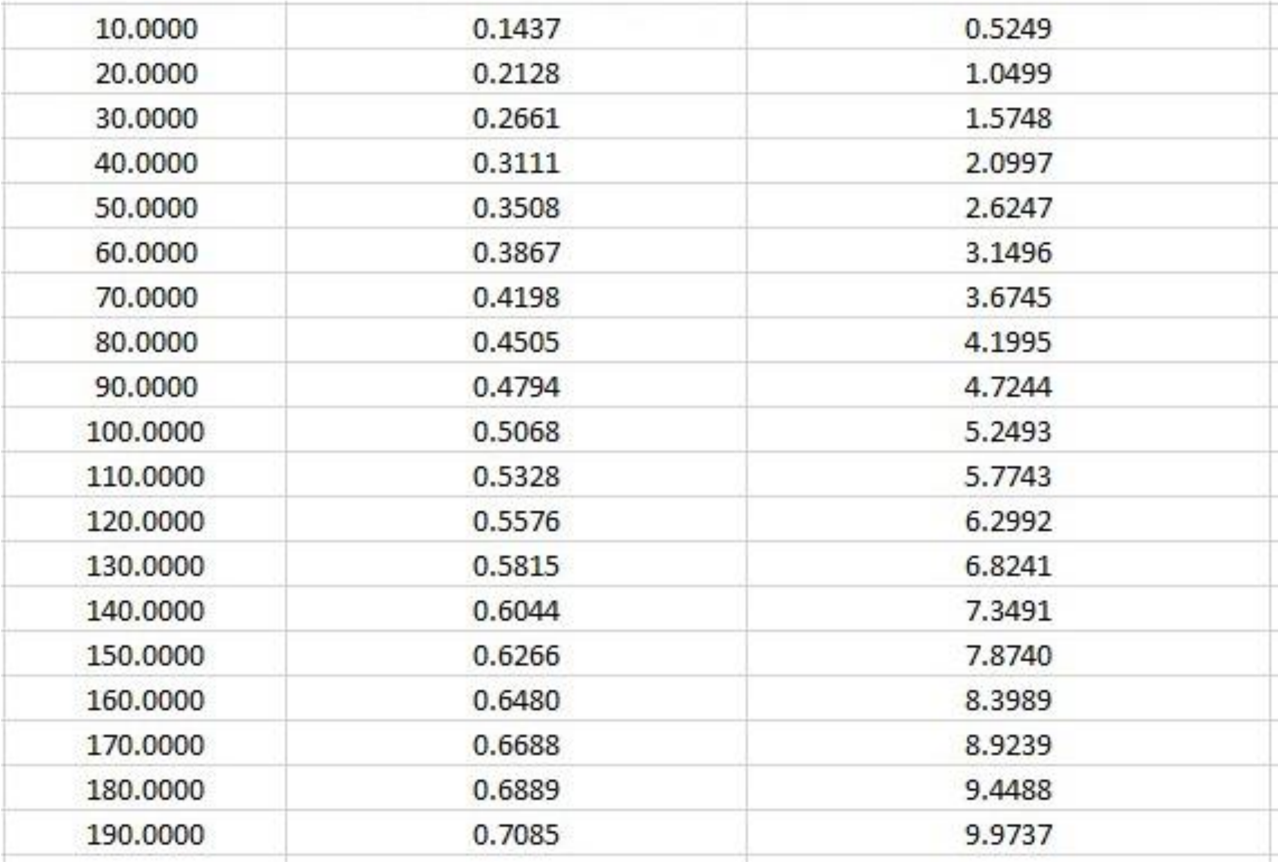


Fig 4.3 the relationship between Fracture Area and productivity index at different time intervals for well 3.

Table 4.4: Result Obtained for well 4



Time (t)(min)	Fracture Area (in^2)(10^3)	Productivity Index (STB/min/PSI)
10.0000	0.1437	0.5249
20.0000	0.2128	1.0499
30.0000	0.2661	1.5748
40.0000	0.3111	2.0997
50.0000	0.3508	2.6247
60.0000	0.3867	3.1496
70.0000	0.4198	3.6745
80.0000	0.4505	4.1995
90.0000	0.4794	4.7244
100.0000	0.5068	5.2493
110.0000	0.5328	5.7743
120.0000	0.5576	6.2992
130.0000	0.5815	6.8241
140.0000	0.6044	7.3491
150.0000	0.6266	7.8740
160.0000	0.6480	8.3989
170.0000	0.6688	8.9239
180.0000	0.6889	9.4488
190.0000	0.7085	9.9737

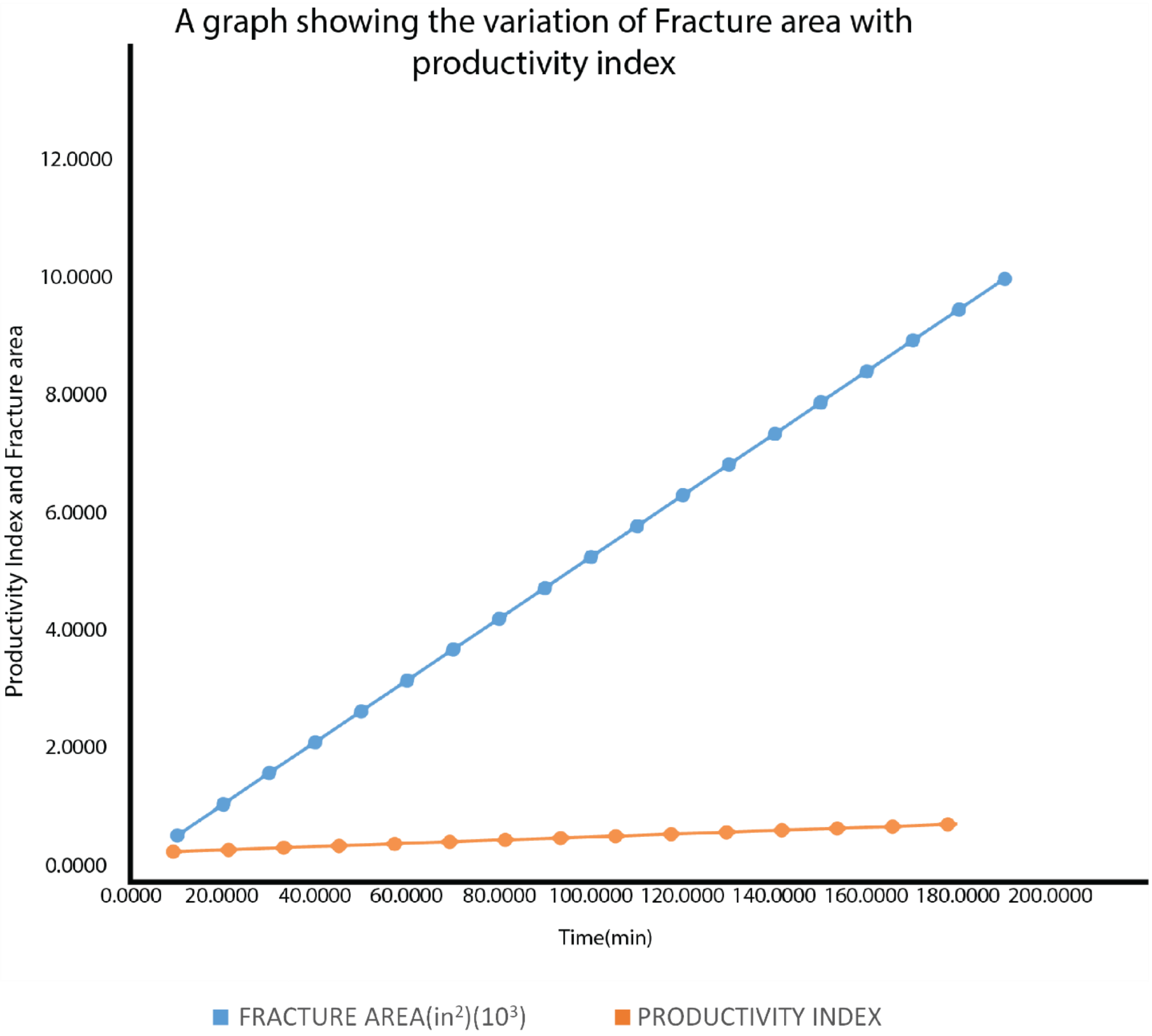


Fig 4.4 the relationship between Fracture Area and productivity index at different time intervals for well 4.

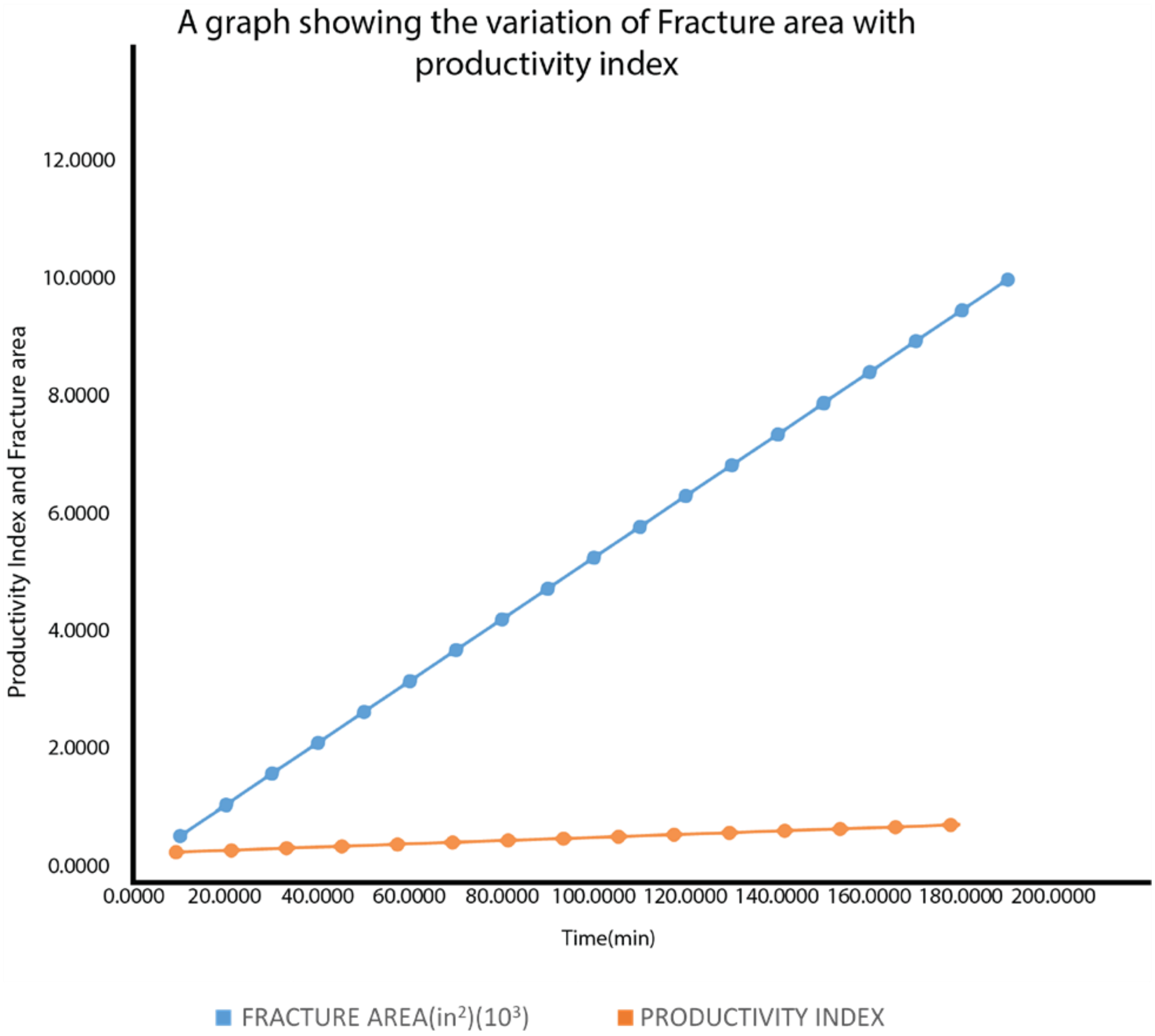

A graph showing the variation of Fracture area with productivity index
Productivity Index and Fracture area
12.0000
10.0000
8.0000
6.0000
4.0000
2.0000
0.0000
0.0000
20.0000
40.0000
60.0000
80.0000
100.0000
120.0000
140.0000
160.0000
180.0000
200.0000
Time(min)
■ FRACTURE AREA(in²)(10³)
■ PRODUCTIVITY INDEX

Table 4.5; Data showing a variation of different fracture area with productivity index at constant flow velocity and at a particular time keeping all other parameters constant.

WELL NO	FRACTURE AREA(in^2)(10^3)	PRODUCTIVITY INDEX
1	5.453	4.724
2	2.872	3.937
3	0.651	3.15
4	0.3867	2.362

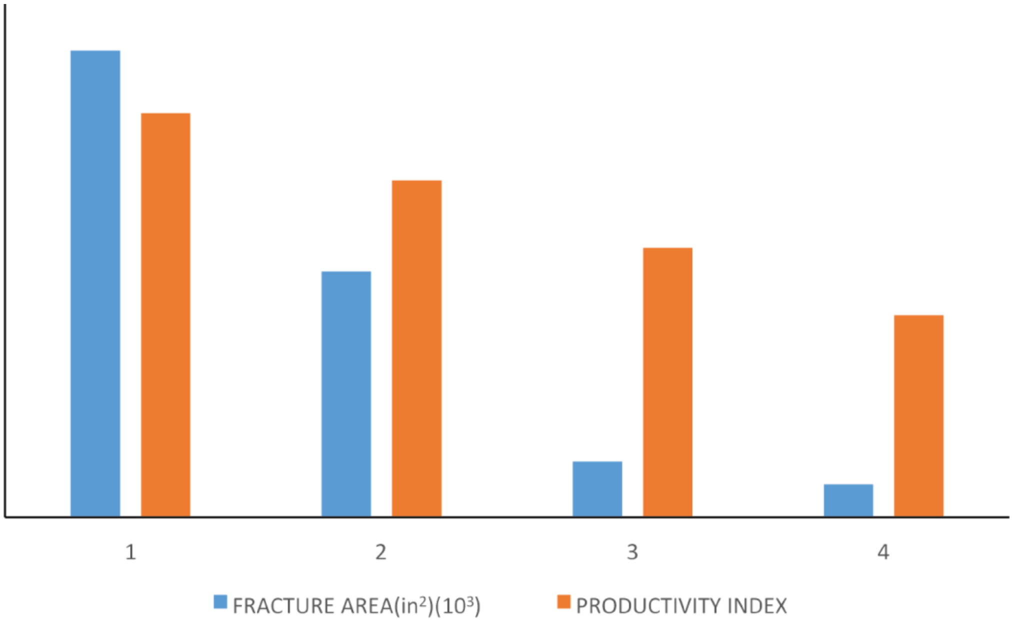


Fig 4.5. A bar chart showing the variation of fracture area and productivity index for well 1-4.

The result analysis presented above have shown the influence of fracture treatment on the production that may be obtained from a given well with a given fracture system. This investigation have shown that a fracturing treatment is not only influenced by the flow capacity of a hydraulically created fracture but also to a large degree by the areal extent of the fracture. Fig 4.1 – 4.4 is a plot of productivity index vs. fracture extent at different injection time interval.

It shows the effect of increased fracture extent on productivity index. This plot reveals that the productivity index increases as the fracture area increases for all well 1-4 at any injection time of the fracturing fluid.

Deeply penetrating fractures which implies increased fracture area not only result in sustained increase in production, but also may result in increased ultimate recovery. This increase has been evidenced by extension of the economic producing life of a well or field, and the stimulation of initial production from field in which the relationship between permeability and differential pressure did not permit economical production prior to fracturing.

Fig 4.5 which is a bar chart showing a direct relationship between fracture area and productivity index at a particular time, was developed keeping the velocity of flow for the different wells constant. It can be observed that well 1 which has the highest fracture area also has the highest productivity index, followed by well 2 which corresponded to a high fracture area relative to well 3 and 4. Well 4 which had the smallest fracture area had also the smallest productivity index. All these are indications that for a particular well, if the fracturing treatment design is to be done, a larger fracture area will result in a higher recovery from such well ignoring economic factors.

CHAPTER FIVE
CONCLUSION AND RECOMMENDATIONS

5.1 CONCLUSION

The study which was focused on effect of fracture parameters (Fracture spacing, Fracture geometry and fracture area) on production rate of a hydraulically fractured wells reveals that the productivity index of a well depends greatly on this parameters. The variation of the effect of this individual parameters is described below.

5.1.1 Fracture Spacing- Effective fracture spacing is critical to well performance. The study have shown that the well performance and economics depend on the number of effective fractures created and connected to the wellbore. An optimal fracture spacing leads to an increased production rate. That is, the more clustered the fractures, the higher the production rate.

5.1.2 Fracture Geometry - The significance of the fracture shape decreases with an increase in the fracture population size. However, the shape of the fracture varies as predicted by different models described above as a result of the heterogeneity of the formation. the different models are used based on the formation evaluation, hence there is no particular fracture model that gives highest productivity index.

5.1.3 Fracture area – The effect of this parameter was established from graphical analysis performed with data from four (4) hydraulically fractured wells that as the fracture area extent increases the productivity index of the well also increases. That is there is a direct proportionality between the fracture area and the rate of production from a fractured well

5.2 RECOMMENDATIONS

This study was carried out without considering cost effect. A larger fracture area will definitely incur more cost, therefore to achieve the highest rate of production means infinity cost i.e. more and more of materials which is not profitable.

It is therefore recommended that further work be done on this that will bring a balance between cost effectiveness and maximum productivity of our wells.

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