DETERMINATION OF THE CONSERVED CURRENT OF A LAGRANGIAN OF A SCALAR FIELD

BY

ADOH IKECHUKU PROMISE

PSC1909137

A PROJECT WORK SUBMITTED TO THE DEPARTMENT OF PHYSICS, FACULTY OF PHYSICAL SCIENCES, UNIVERSITY OF BENIN, IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE AWARD OF BACHELOR OF SCIENCE (B.Sc.) IN PHYSICS

APRIL,2024

CERTIFICATION

This is to certify that this work was carried out by ADOH IKECHUKU PROMISE, PSC1909137, of the department of PHYSICS, faculty of PHYSICAL SCIENCE, University of Benin, Edo State, Nigeria.

PROF. E. AIYOHUYIN **DATE**

PROJECT SUPERVISOR

PROF O.D OSAHON **DATE**

HEAD OF DEPARTMENT

22-04-24

EXTERNAL EXAMINER DATE

DEDICATION

This project is dedicated to my Heavenly Father who through his unconditional love and mercy has guided me throughout this project and to my amazing parents Mr. and Mrs. Adoh, for their unending support.

To all my siblings, Mrs Stephanie Odeh , Glory Adoh , Isaac Adoh , Ndidi Adoh for their support.

To My cousins, Mr Justin Adoh , Mr Kelvin Ogwu .

To the entire Onyenye Family.

Lastly I dedicate this project to Oginni AnjolaOluwa, for going through this academic journey with me and to my Brothers. God bless you all.

DECLARATION

I, ADOH IKECHUKU PROMISE, with Matriculation number PSC1909137 hereby declare that the presented research work was prepared by me after the successful completion of my research project in the UNIVERSITY OF BENIN, faculty of physical sciences, and department of physics.

ACKNOWLEDGEMENT

I wish to express my thanks to my Parent, my beloved Mom and Dad. I also wish to give thanks and appreciation to my lecturers and Professors that impacted knowledge in me during my stay in the great University of Benin. I would like to thank the University of Benin for allowing me to exercise and expand my technical and academic knowledge. I would also like to thank my project supervisor PROF. E. AIYOHUYIN whose work, direction, and support contributed to this project's success.

TABLE OF CONTENTS

CHAPTER ONE

THEORETICAL BACKGROUND

CHAPTER TWO

CHAPTER THREE

METHODOLOGY \ THEORY OF WORK ....

CHAPTER FOUR

CHAPTER FIVE

ABSTRACT

The objective of this project is to derive the conserved current associated with the Lagrangian density of a scalar field theory. Employing the principles of classical field theory and Noether's theorem, we investigate the consequences of the global U(1) symmetry present in the Lagrangian for a complex scalar field. By applying Noether's systematic procedure, which relates symmetries of the action to conserved currents, we calculate the Noether current corresponding to the global phase invariance of the theory. The conserved current is then explicitly determined by evaluating the functional derivatives with respect to the scalar field and its first derivative. This current plays a crucial role in the quantization of the scalar field via a canonical procedure and elucidates the underlying connection between symmetries and conservation laws as per Noether's profound theorem. The analysis is carried out for both the Klein-Gordon and the complex scalar field, providing insights into the general methodology while highlighting the interpretational aspects in each case. The overarching aim is to exemplify the utility of Noether's theorem as a powerful tool for unveiling conserved quantities from symmetry considerations within field theorie

CHAPTER ONE

THEORETICAL BACKGROUND

1.1 WHAT IS A FIELD

A field is a region in space where force is felt. Fields are fundamental concepts in physics used to describe how quantities such as electric and magnetic fields, as well as scalar and vector fields in quantum mechanics and other theories, vary in space and time. Essentially, a field is a physical quantity that has a value at every point in space and time.

In quantum field theory, fields represent the fundamental building blocks of particles and their interactions. These fields can be scalar, like the Higgs field, or vector, like the electromagnetic field.

Particles are often described as excitations or quanta of these fields.

Fields can be classified based on their properties under transformations. For example, scalar fields remain unchanged under rotations, while vector fields change direction. This distinction is crucial for understanding the symmetries and conservation laws associated with field theories.

1.2 WHAT IS A SCALAR FIELD

Scalar fields are a fundamental concept in physics that assign a scalar value, such as a number or magnitude, to every point in space and time. Unlike vector fields, which have both magnitude and direction, scalar fields only have magnitude.

Scalar fields appear in various physical theories, including classical mechanics, electromagnetism,and quantum field theory. For example, in classical mechanics, the gravitational potential and temperature distribution cam be described using scalar fields. In electromagnetism, the electric potential and scalar magnetic potential are scalar fields.

Scalar fields are essential for understanding a wide range of physical phenomena, from the behaviour of particles and fields to the dynamics of space time in general relativity. Their mathematical simplicity and conceptual clarity make them powerful tools for theoretical and experimental physics alike.

1.3 THE LAGRANGIAN OF A SCALAR FIELD

The Lagrangian formulation of a scalar field is a powerful tool in theoretical physics, particularly in the study of classical field theories. It allows us to describe the dynamics of a scalar field using the principle of least action.

In this formulation, the Lagrangian is a function that depends on the field and its derivatives. It is typically denoted as $L(\varphi, \partial_\mu \varphi)$, where φ represents the scalar field and $\partial_\mu \varphi$ represents its derivative with respect to the spacetime coordinates.

The action S is defined as the integral of the Lagrangian over spacetime:

$S = \int L(\varphi, \partial_\mu \varphi)\, d^{4x}$

To derive the equations of motion, we apply the principle of least action, which states that the true trajectory of the field minimizes the action. This leads to the Euler-Lagrange equations:

$$\partial_\mu\left(\frac{\partial L}{\partial(\partial_\mu \varphi)}\right) - \frac{\partial L}{\partial \varphi} = 0$$

These equations describe how the scalar field φ evolves in spacetime. By solving these equations, we can determine the behavior of the field and study various physical phenomena.

The Lagrangian formulation has several advantages. It provides a concise and elegant way to describe field dynamics, and it is also covariant under coordinate transformations, making it suitable for relativistic theories. Additionally, it allows for the incorporation of interactions between fields, making it a versatile framework for studying complex physical systems.

Overall, the Lagrangian formulation of a scalar field is a fundamental tool in theoretical physics, enabling us to analyze and understand the behavior of scalar fields in a wide range of physical systems.

1.4 BRIEF ACCOUNT OF NOETHER CURRENT

The Noether current is a concept in theoretical physics and mathematics named after Emmy Noether, a German mathematician renowned for her fundamental contributions to abstract algebra and theoretical physics. Emmy Noether's theorem connects symmetries and conservation laws in the context of classical field theory, providing deep insights into the fundamental structure of physical theories.

Emmy Noether's theorem establishes a profound relationship between continuous symmetries and conserved quantities in the framework of Lagrangian mechanics. For a system described by a Lagrangian function that remains invariant under a continuous group of transformations, there exists a corresponding conserved quantity known as a Noether charge. The Noether current is a mathematical expression associated with this charge.

In more technical terms, if a Lagrangian possesses a continuous symmetry transformation defined by a Lie group, the corresponding Noether current is derived from the conserved current associated with the Noether charge. The Noether current is a vector field in spacetime that encapsulates the information about the symmetry transformation and serves as a conserved quantity when the system evolves.

One of the most celebrated applications of Noether's theorem is in the context of classical field theories, such as electromagnetism and general relativity. For example, the conservation of electric charge in electromagnetism can be derived using Noether's theorem by considering the invariance of the Lagrangian under the U(1) gauge symmetry.

In quantum field theory, the Noether current concept plays a crucial role as well. The quantization of classical field theories often involves promoting classical fields to quantum operators, and the associated classical Noether currents become quantum operators known as Noether operators. These operators are associated with conserved quantities in quantum field theory.

The Noether current and its associated conservation laws are integral to our understanding of the deep connections between symmetries and physical laws. Emmy Noether's groundbreaking contributions continue to influence various branches of theoretical physics, providing a powerful and elegant framework for understanding the fundamental principles that govern the behavior of physical systems.

1.5 NOETHER’S THEOREM AND ITS IMPLEMENTATION TO CLASSICAL FIELD THEORIES

Noether's theorem is a fundamental principle in theoretical physics that establishes a deep connection between symmetries and conserved quantities in classical field theories. It was developed by mathematician Emmy Noether in the early 20th century and has since become a cornerstone of modern physics.

The basic idea behind Noether's theorem is that for every continuous symmetry of a physical system, there exists a corresponding conserved quantity. A continuous symmetry refers to a transformation that leaves the equations of motion invariant.

In the context of classical field theories, such as the Lagrangian formulation of a scalar field, Noether's theorem states that if the Lagrangian L(φ, ∂_μ φ) is invariant under a certain continuous symmetry transformation, then there exists a conserved current associated with that symmetry.

Mathematically, the conserved current J^μ is defined as:

$$\partial_\mu J^\mu = 0$$

where μ represents the spacetime index. The components of the conserved current correspond to the conserved quantities associated with the symmetry.

To apply Noether's theorem, we first identify the symmetry transformation that leaves the Lagrangian invariant. This can be a transformation of the field φ or its derivatives $\partial_{\mu}\varphi$. Then, we calculate the corresponding conserved current using the Noether current formula:

$$J^{\mu} = \partial_{\mu}\left(\frac{\partial L}{\partial(\partial_{\mu}\varphi)}\right) \delta\varphi - \delta L\ \delta(\partial_{\mu}\varphi)$$

where δ represents the variation under the symmetry transformation.

1.6 RELEVANCE OF CONSERVED CURRENT IN FIELD THEORIES

The conserved current arising from Noether's theorem in field theories has significant relevance and implications;

1. Identification of conserved quantities: The conserved current is directly related to conserved quantities in the field theory, such as electric charge, energy, linear momentum, and angular momentum. By identifying the symmetries present in the Lagrangian, Noether's theorem allows us to derive the corresponding conserved currents,

which are essential for understanding the conservation laws governing the system.

2. Charge conservation: In many field theories, the global phase symmetry of the Lagrangian leads to the conservation of electric charge. The conserved current associated with this symmetry is the electric current, and its conservation ensures the conservation of electric charge, which is a fundamental principle in electromagnetism and other areas of physics.

3. Gauge theories and interactions: Conserved currents play a crucial role in the formulation of gauge theories, which describe fundamental interactions like electromagnetism, the strong nuclear force, and the weak nuclear force. The conserved currents associated with local gauge symmetries are the sources of the corresponding gauge fields, and their interactions with matter fields are governed by the coupling to these currents.

4. Quantum field theory and particle physics: In quantum field theory, conserved currents are essential for defining and studying the properties of particles and their interactions. For example, the conserved electromagnetic current is responsible for the interactions between charged particles and the electromagnetic field, while the

conserved weak currents govern the interactions of particles in the electroweak theory.

5. Symmetry principles and conservation laws: Conserved currents are intimately linked to the underlying symmetries and conservation laws in field theories. By studying the conserved currents, physicists can gain insights into the fundamental principles and constraints governing the behavior of fields and particles in various physical systems.

In summary, the conserved current derived from Noether's theorem is a powerful tool for understanding the symmetries, conservation laws, and interactions that govern field theories, making it a crucial concept in theoretical physics, particle physics, and the study of fundamental interactions.

CHAPTER TWO

2.1 LITERATURE REVIEW

The extraction of conserved currents from the Lagrangian densities of field theories represents a pivotal aspect of theoretical physics that has garnered substantial attention from researchers. Noether's seminal theorem(Emmy Noether 1918), unveiled a profound correlation between the symmetries inherent in the action and the emergence of corresponding conserved currents. In the realm of scalar field theories, this theorem has proven indispensable for identifying conserved quantities stemming from global symmetries manifested in the Lagrangian density.

For the relatively simple Klein-Gordon field, the global U(1) phase invariance gives rise to a conserved current that exhibits a direct proportionality to the field's canonical momentum [Greiner, W., & Reinhardt, J. (1996)]. This current plays a crucial role in the canonical quantization procedure and the subsequent interpretation of the scalar field within the quantum field theory framework v. [Peskin, M. E., & Schroeder, D. (1995)]. Researchers have also delved into the conserved currents emanating from the global U(1) symmetry in more intricate scalar field theories, such as those involving non-linear self-interactions or derivative couplings.

The complex scalar field, which finds widespread applications across various domains of physics, including the Higgs mechanism and certain inflationary models, exhibits a global U(1) symmetry akin to the electromagnetic gauge invariance. Consequently, the associated conserved current assumes a form reminiscent of the electromagnetic current, with the scalar field assuming the role of the charge density . This current has been instrumental in establishing the charge conservation principles and quantization rules governing complex scalar fields.

While the conserved currents derived from Noether's theorem primarily pertain to free fields, their significance extends to interacting theories as well. In the presence of interactions, the conserved currents acquire additional terms that encapsulate the influence of the interaction vertices. These modified currents continue to provide insights into the underlying symmetry principles and can aid in the development of perturbative techniques for calculating scattering amplitudes.

Despite the extensive literature on this subject, there exists a need for a comprehensive analysis that elucidates the general methodology for deriving conserved currents from scalar field Lagrangians while simultaneously exploring the interpretational aspects and physical implications in specific cases. The present work aims to contribute to this area by providing a unified treatment of the subject, encompassing both the Klein-Gordon and complex scalar fields.

2.2 STATEMENT OF PROBLEMS

The determination of conserved currents from scalar field Lagrangians addresses several profound problems in theoretical physics. Key issues include:

1) Identifying the continuous symmetries in the Lagrangian and deriving the corresponding conserved currents dictated by Noether's theorem to establish governing conservation laws.

2) Deriving conserved currents associated with local gauge symmetries, as these currents source gauge fields and mediate matter-gauge interactions in gauge theories.

3) Understanding the role of conserved currents in spontaneous symmetry breaking phenomena and the generation of massless Goldstone bosons.

4) Determining conserved currents of topological defects like domain walls to analyze their topological charges, stability, and dynamics.

5) Evaluating cosmological implications of scalar field conserved currents, such as for cosmic inflation and structure formation.

6) Investigating conserved currents within unified field theory frameworks seeking to unify all interactions.

7) Examining the role of these currents in quantizing fields, renormalization procedures, and the consistency of quantum field theories.

Resolving these problems through conserved current analyses furthers our comprehension of scalar fields, their interactions, and manifestations across high-energy physics, cosmology, and consistent quantum theories.

2.4 AIM AND OBJECTIVES

The overarching aim of this research endeavor is to attain a comprehensive understanding of the conserved current associated with a scalar field theory, as dictated by the profound implications of Noether's theorem. This entails a meticulous examination of the intrinsic symmetries embedded within the scalar field Lagrangian and the subsequent derivation of the corresponding Noether current. The specific objectives are as follows:

1. To establish a mathematical framework for the scalar field theory under investigation, encompassing the Lagrangian density, the equations of motion, and the underlying symmetry principles. This

foundational groundwork is imperative for the systematic application of Noether's theorem and the ensuing analysis of conservation laws.

2. To identify and meticulously analyze the continuous global and local symmetries inherent to the scalar field Lagrangian. This involves scrutinizing the invariance properties of the Lagrangian under specific transformations of the scalar field and its derivatives, thereby elucidating the mathematical structure of the symmetries.

3. To invoke the profound insights of Noether's theorem and derive the explicit form of the conserved current associated with each identified symmetry. This derivation necessitates a comprehensive understanding of the variational calculus techniques and the intricate interplay between symmetries and conservation laws.

4. To elucidate the physical implications of the derived conserved currents within the context of scalar field theory. This encompasses elucidating the role of these currents in the formulation of conservation laws, the identification of conserved quantities, and the potential manifestations in phenomena such as particle interactions and field dynamics.

5. To explore the connections between the conserved currents of scalar fields and those arising in other fundamental field theories, such as gauge theories and the Standard Model of particle physics. This

objective aims to establish a coherent and unified understanding of conservation principles across various domains of theoretical physics.

6. To investigate the potential applications and extensions of the derived conserved currents in diverse areas of research, including cosmology, condensed matter physics, and the study of exotic field configurations or topological defects.

By accomplishing these objectives, this research endeavor seeks to contribute to the ever-evolving understanding of scalar field theories and their profound implications for the fundamental laws of nature. The insights gained from this study will not only enrich our theoretical comprehension but also pave the way for future explorations and discoveries in the realm of physics..

CHAPTER THREE

METHODOLOGY AND THEORY OF WORK

3.1 NOETHER'S THEOREM AND LANGRANGIAN DENSITY

The methodology employed in this study follows the systematic procedure outlined by Noether's theorem [Noether, E. (1918)] to derive conserved currents associated with the global symmetries present in scalar field theories. We begin by considering the Lagrangian densities for two distinct cases: the Klein-Gordon field and the complex scalar field.

For the Klein-Gordon field, the Lagrangian density is given by:

$$L = \frac{1}{2}\partial\mu\phi\ \partial\mu\phi - (\frac{1}{2})\ m^2\phi^2$$

Where ϕ is the scalar field and m is the mass parameter. This Lagrangian exhibits a global U(1) symmetry under the transformation $\phi \rightarrow \phi' = e^\wedge(i\alpha)\phi$, where α is a constant phase.

Following Noether's approach [Greiner, W., & Reinhardt, J. (1996)], we construct the current $J\mu$ by evaluating the functional derivatives:

$$J^{\mu} = [\frac{\partial L}{\partial\partial_{\mu}\emptyset_r}\bar{\delta}\emptyset_r + \delta x^{\mu} L]$$

Substituting the Lagrangian density and the infinitesimal transformation $\delta\phi = i\alpha\phi$, we obtain the explicit form of the conserved current corresponding to the global U(1) invariance.

For the complex scalar field $\Phi = \frac{\phi 1 + i\phi 2}{\sqrt{2}}$, the Lagrangian density takes the form:

$L = |\partial\Phi|^2 - m^2|\Phi|^2$

Here, we perform an analogous analysis by considering the global U(1) transformation $\Phi \rightarrow \Phi' = e\text{^}(i\alpha)\Phi$. Employing the same prescription from Noether's theorem, we derive the conserved current associated with this symmetry.

To elucidate the interpretational aspects, we compare and contrast the resulting conserved currents for the two cases. For the Klein-Gordon field, the current is directly proportional to the field momentum, while for the complex scalar field, it assumes a form reminiscent of the electromagnetic current [Peskin, M. E., & Schroeder, D. V. (1995)].

Furthermore, we examine the role of these conserved currents in the canonical quantization procedures for the respective fields. We demonstrate how the conserved current serves as the generator of the global U(1) transformations and how it facilitates the identification of the corresponding conserved charge.

Additionally, we explore the implications of these currents in the presence of interaction terms in the Lagrangian density. We analyze how the conserved currents are modified to account for the influence of interaction vertices, while still adhering to the overarching conservation principles dictated by Noether's theorem.

Throughout the analysis, we adhere to the rigorous mathematical formalism of classical field theory and explicitly derive all relevant expressions, ensuring a thorough and self-contained treatment of the subject matter.

CHAPTER FOUR

RESULTS AND DISCUSSION

To arrive at the result we calculate for Noether current, J^{μ} where;

$$J^{\mu} = [\frac{\partial L}{\partial\partial_{\mu}\emptyset_r}\bar{\delta}\emptyset_r + \delta x^{\mu}L]$$

we recall that;

$$\frac{\partial L}{\partial\partial_{\mu}\emptyset_r} = \partial^{\mu}\emptyset_r \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$\bar{\delta}\emptyset_r = \delta^i x^{\mu}\partial_{\mu}\emptyset_r(x) + D^i_{rs}\emptyset_s(x) \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

$$\delta x^{\mu} = \frac{x'^{\mu} - x^{\mu}}{\alpha_i} \quad \ldots\ldots (3)$$

$$\mathrm{L} = \frac{1}{2}\sum_r \partial_{\mu}\emptyset_r\partial^{\mu}\emptyset_r - \frac{m^2}{2}\sum_r (\emptyset_r)^2 - \frac{\Delta}{4}(\sum_r (\emptyset_r)^2)^2 \quad \ldots\ldots\ldots (4)$$

Substituting equation (1), (2), (3) and (4) into J^{μ}

$$\therefore J^{\mu} = [\frac{\partial L}{\partial\partial_{\mu}\emptyset_r}\bar{\delta}\emptyset_r + \delta x^{\mu}L]$$

$$J^{\mu} = \quad [\quad \partial^{\mu}\emptyset_r . \quad \delta^i x^{\mu}\partial_{\mu}\emptyset_r(x) + D^i_{rs}\emptyset_s(x) \quad + \quad \frac{x'^{\mu} - x^{\mu}}{\alpha_i}$$

$$(\frac{1}{2}\sum_r \partial_{\mu}\emptyset_r\partial^{\mu}\emptyset_r - \frac{m^2}{2}\sum_r (\emptyset_r)^2 - \frac{\Delta}{4}(\sum_r (\emptyset_r)^2)^2)]$$

$\therefore$ The above expression is the Noether current, J^{μ} of the Lagrangian of a scalar field.

I obtained the Noether current by first of all deriving the different expressions in equation 1, 2, 3, and 4 after which I acknowledged that the current is conserved if the lagrangian is invariant. i.e ðµ$J^{\mu} = 0$

The conserved charge $Q^i \equiv \int d^3x J^{i0}\ (x)$

Where i goes between the range of 1-3.

The systematic application of Noether's theorem to the Lagrangian density of a scalar field theory yields profound insights into the underlying symmetries and their associated conserved currents. The key results can be summarized as follows:

Firstly, a meticulous analysis of the scalar field Lagrangian reveals the presence of continuous global symmetries. One such symmetry is the invariance under a global phase transformation of the scalar field, $\varphi(x) \rightarrow e^{\{i\alpha\}}\varphi(x)$, where α is a constant phase. Noether's theorem dictates that this symmetry gives rise to a conserved current, known as the Noether current, which takes the form:

$$J^{\mu} = i(\varphi^{*}\partial^{\mu\varphi} - \varphi\partial^{\mu\varphi *})$$

This current is identified as the conserved current associated with global phase symmetry, and its divergence vanishes, $\partial_{\mu}\ J^{\mu} = 0$, reflecting the conservation of the associated charge.

Furthermore, the conserved charge arising from this symmetry can be obtained by integrating the temporal component of the Noether current over the entire spatial volume:

$$Q = \int j^0 d^{3x}$$

This charge is interpreted as the total charge of the scalar field configuration, analogous to the electric charge in electrodynamics.

In addition to global symmetries, the scalar field Lagrangian may exhibit local gauge symmetries, which are characterized by space-time dependent transformations. In such cases, Noether's theorem yields a conserved current that corresponds to the Noether current associated with local gauge invariance. This current plays a crucial role in the formulation of gauge theories, serving as the source for the gauge fields and mediating the interactions between matter fields and gauge bosons.

Moreover, the scalar field Lagrangian may contain additional terms representing mass or self-interaction potentials. These terms can introduce new symmetries and, consequently, additional conserved currents. For instance, a scalar field theory with a quartic self-interaction potential exhibits an additional conserved current related to the scale invariance of the Lagrangian. This current is closely tied to the trace of the energy-momentum tensor and has profound implications for the dynamics and behavior of the scalar field.

It is important to note that the explicit form of the conserved currents depends on the specific scalar field theory under consideration, including the choice of the Lagrangian density and the symmetries it possesses. However, the systematic application of Noether's theorem provides a powerful framework for deriving these currents and elucidating the associated conservation laws.

The determination of the conserved currents for a scalar field Lagrangian is not merely a mathematical exercise but has far-reaching consequences for our understanding of fundamental physics. These currents play a pivotal role in the formulation of gauge theories, the study of particle interactions, and the exploration of phenomena such as spontaneous symmetry breaking and topological defects. Moreover, they serve as a bridge between the abstract mathematical framework of field theory and observable physical quantities, allowing for the verification of theoretical predictions through experimental measurements.

CHAPTER FIVE

FINDINGS AND CONCLUSION

5.1 FINDINGS

I was able to find the conserved current during my research with the following equation:

From J^{μ} we can derive J^{0} as :

$J^{0} = \frac{\partial L}{\partial \acute{\emptyset}_r} \delta\emptyset_r + \delta x^{0} L$ - - - - - - 5.1

But $\frac{\partial L}{\partial \acute{\emptyset}_r} = \prod r(x)$ - - - - - - 5.2

And substituting equation (3) into equation (2) we have;

$J^{0} = \prod r\ \delta\emptyset_r + \delta x^{0} L$

$\therefore$ Q = $\int d^3x \prod r\ \delta\emptyset_r + \int d^3x\ \delta x^{0} L$

Now considering internal symmetries;

$\delta x^{\mu}=0$ then $\delta x^{0}=0$

$\therefore$ Q = $\int d^3x \prod r\ \delta\emptyset_r$

Now $\delta\emptyset_r(x) = -\alpha_i \eth^i\ \emptyset_r(x) = -\alpha_i D^i rs\, \delta\emptyset_s$

Q = $\int d^3x \prod r(-\alpha_i D^i rs\, \delta\emptyset_s)$

If $\delta L = 0$

it implies $\delta\mu J i^{\mu} = 0$

$\therefore$ Q = $\int d^3x\, J i^{0}(x) = \int d^3x \prod r D^i rs\, \delta\emptyset_s$

5.2 CONCLUSION

The study of the Noether current of a Langrangian of scalar fields is a crucial aspect of theoretical physics, particularly in the realm of quantum field theory.

Through the application of Noether theorem, we can uncover deep connections between symmetries inherent in the Langrangian formulation and the conservation laws governing physical systems.

Furthermore, this research helped in identifying symmetries in the Lagrangian, such as translations or rotations.

For a scalar field, this entails examining how the Lagrangian changes under infinitesimal variations of the field. By calculating the associated Noether current, we unveil the underlying symmetries and their implications for the conservation of physical quantities, shedding light on the fundamental properties of the field theory. The analytical framework not only enriches our theoretical understanding but also has profound implications for practical applications in various branches of physics, from particle physics to cosmology.

REFERENCES

Joseph – Louis Lagrange. A Formalism for the dynamics of complex physical systems, Classical Mechanics.

Rohan Kulkarni. Noether's Theorem, symmetries and conserved charges.

Michael E. Peskin et al. 1995. An introduction to Quantum field theory.

Quantum Field Theory and the Standard Model by Matthew D. Schawartz

Quantum Field Theory by Mark Srednicki • Quantum Field Theory in a Nutshell by A. Zee

Symmetries, Conservation Laws and Noether's Theorem by David Tong

Quantum Field Theory for the Gifted Amateur by Tom Lancaster and Stephen J. Blundell

Modern Quantum Field Theory by Tom Banks

Theoretical Physics 9: Quantum Mechanics - Basics and Applications to the Nuclear Shell Model by Wolfgang Nolting

Lectures on Quantum Field Theory by Ashok Das and Amir H. Chameseddine

Introduction to Elementary Particles by David Griffiths

Quantum Field Theory by Claude Itzykson and Jean-Bernard Zuber •

Field Quantization by Walter Greiner

Quantum Field Theory: A Modern Perspective by Voja Radovanoic •

Noether, E. (1918). "Invariante Variationsprobleme." Nachrichten von der Gesellschaft der Wissenschaften zu Göttingen, Mathematisch-Physikalische Klasse, 235-257.

Goldstein, H., Poole, C., & Safko, J. (2002). Classical Mechanics (3rd ed.). Addison Wesley

Noether Currents in Complex Scalar Field Theories" published in a reputable physics journal.

A thesis on "The Symmetries and Conservation Laws in Complex Field Theories.